Introduction

Rational Numbers: The numbers which can be written in the form of $\frac{p}{q}$, where p and q are integers and q is not equal to zero.

For example: $\frac{1}{2}$, $\frac{5}{6}$, 0 etc are rational numbers.

Positive Rational Numbers: A rational number is said to be positive if its numerator and denominator are either both positive and both negative.

For example: $\frac{5}{7}$, $\frac{-2}{-5}$ etc.

Negative Rational Numbers: A rational number is said to be negative if its numerator and denominator are of opposite signs.

For example: $\frac{-1}{3}$, $\frac{4}{-5}$ etc.

Properties of Rational number:

1) If $\frac{p}{q}$ is a rational number and m is a non-zero integer then

$$\frac{p}{q} = \frac{p*m}{q*m}$$

Such Rational numbers are equivalent rational numbers.

For example: $\frac{-3}{4} = \frac{-3*2}{4*2}$ etc.

2) If $\frac{p}{q}$ is a rational number and m is a common divisor of p and q, then $\frac{p}{q} = \left(\frac{\frac{p}{m}}{\frac{q}{m}}\right)$

For example: $\frac{20}{25} = \left(\frac{\frac{20}{5}}{\frac{25}{5}}\right) = \frac{4}{5}$.

3) Let $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers. Then, $\frac{a}{b} = \frac{c}{d}$ if and only if

(a*d) = (b*c).

Standard form of a rational number: A rational number $\frac{p}{q}$ is in standard form if p and q are integers having no common divisor other than 1 and b is positive.

General method of comparing Rational numbers

Step 1- Express each rational number with positive denominator.

Step2- Take the LCM of these positive denominators.

Step 3- Express each rational number with this LCM as the common denominator.

Step 4- The number having the greater numerator is greater.

Examples:

Example 1 - Express $\frac{33}{-44}$ in standard form

Solution **-** $\frac{33}{-44} = \frac{33*(-1)}{-44*(-1)} = -\frac{33}{44}$

Now the greatest common divisor of 33 & 44 is 11.

Therefore, $\frac{\frac{-33}{11}}{\frac{44}{11}} = -\frac{3}{4}$

Example 2 - Which of the numbers $\frac{3}{-4}$ or $-\frac{5}{6}$ is greater.

Solution **-** Firstly, we write each number with positive denominator.

One number $= \frac{3}{-4} = \frac{3*(-1)}{-4*(-1)} = -\frac{3}{4}$

Other number $= -\frac{5}{6}$

Now take LCM of 4 and 6 which is 12.

Thus $-\frac{3}{4} = \frac{-3*3}{4*3} = -\frac{9}{12}$

And $-\frac{5}{6} = \frac{-5*2}{6*2} = -\frac{10}{12}$

As -9 > -10

Therefore, $-\frac{3}{4} > -\frac{5}{6}$

Example 3 - Arrange the numbers $-\frac{3}{5}$, $\frac{7}{-10}$ and $-\frac{5}{8}$ in ascending order

Solution **-** Firstly, we write each number with positive denominator.

One number $= -\frac{3}{5}$

Second number $= \frac{7}{-10} = \frac{7*(-1)}{-10*(-1)} = -\frac{7}{10}$

Third number $= -\frac{5}{8}$

Now take LCM of 5, 10 and 8 which is 40.

Thus $-\frac{3}{5} = \frac{-3*8}{5*8} = -\frac{24}{40}$

$-\frac{7}{10} = \frac{-7*4}{10*4} = -\frac{28}{40}$

$-\frac{5}{8} = \frac{-5*5}{8*5} = -\frac{25}{40}$

Clearly, $-\frac{28}{40} < -\frac{25}{40} < -\frac{24}{40}$

Hence, $-\frac{7}{10} < -\frac{5}{8} < -\frac{3}{5}$

Exercise 1A

Question 1 - Express $-\frac{3}{5}$ as a rational number with denominator

(a) 20

Solution - We have $-\frac{3}{5}$ as a rational number

To make denominator as 20, we must multiply by 4 to both numerator and denominator

$\frac{-3*4}{5*4} = -\frac{12}{20}.$

(b) -30

Solution - To make denominator as -30, we must multiply by -6 to both numerator and denominator.

$\frac{-3*(-6)}{5*(-6)} = \frac{18}{-30}$

(c) 35

Solution -To make denominator as 35, we must multiply by 7 to both numerator and denominator.

$$\frac{-3*7}{5*7} = -\frac{21}{35}.$$

(d) -40

Solution **-** To make denominator as -40, we must multiply by -8 to both numerator and denominator.

$$\frac{-3*(-8)}{5*(-8)} = \frac{24}{-40}$$

Question 2 - Express $-\frac{42}{98}$ as a rational number with denominator 7.

Solution **-** To make denominator as 7, we must divide by 14 to both numerator and denominator.

$$\left(\frac{-\frac{42}{14}}{\frac{98}{14}}\right) = -\frac{3}{7}$$

Question 3 - Express $-\frac{48}{60}$ as a rational number with denominator 5.

Solution **-** To make denominator as 5, we must divide by 12 to both numerator and denominator.

$$\left(\frac{-\frac{48}{12}}{\frac{60}{12}}\right) = -\frac{4}{5}$$

Question 4 - Express each of the following rational numbers in standard form:

(a) $-\frac{12}{30}$

Solution **-** Now the greatest common divisor of 12 & 30 is 6.

Therefore, $\frac{-\frac{12}{6}}{\frac{30}{6}} = -\frac{2}{5}$

(b) $-\frac{14}{49}$

Solution **-** Now the greatest common divisor of 14 & 49 is 7.

Therefore, $\frac{\frac{-14}{7}}{\frac{49}{7}} = -\frac{2}{7}$

(c) $\frac{24}{-64}$

Solution $\frac{24}{-64} = \frac{24*(-1)}{-64*(-1)} = -\frac{24}{64}$

Now the greatest common divisor of 24 & 64 is 8.

Therefore $\frac{\frac{24}{8}}{\frac{64}{8}} = -\frac{3}{8}$.

(d)$\frac{-36}{-63}$

Solution $-\frac{36}{-63} = \frac{-36*(-1)}{-63*(-1)} = \frac{36}{63}$

Now the greatest common divisor of 36 & 63 is 9.

Therefore, $\frac{\frac{36}{9}}{\frac{63}{9}} = \frac{4}{7}$

Question 5 - Which of the two rational numbers is greater in the given pair?

(a) $\frac{3}{8}$ or 0

Solution - Since we can write $0 = \frac{0}{8}$

Now we have $\frac{3}{8}$ and $\frac{0}{8}$

And as $3 > 0$

Thus, $\frac{3}{8} > 0$

(b) $\frac{-2}{9}$ or 0

Solution - Since we can write $0 = \frac{0}{9}$

Now we have $-\frac{2}{9}$ and $\frac{0}{9}$

And as $0 > -2$

Thus, $0 > -\frac{2}{9}$

(c) $-\frac{3}{4}$ or $\frac{1}{4}$

Solution - Since $1 > -3$

Thus, $\frac{1}{4} > -\frac{3}{4}$

(d) $-\frac{5}{7}$ **or** $-\frac{4}{7}$

Solution **-** Since -4>-5

Thus, $-\frac{4}{7} > -\frac{5}{7}$

(e) $\frac{2}{3}$ **or** $\frac{3}{4}$

Solution **-** LCM of 3 and 4 is 12

Now, $\frac{2}{3} = \frac{2*4}{3*4} = \frac{8}{12}$

$\frac{3}{4} = \frac{3*3}{4*3} = \frac{9}{12}$

Clearly 9 > 8

Thus, $\frac{3}{4} > \frac{2}{3}$

(f) -1/2 or -1

Solution **-** LCM of 2 and 1 is 2

Now, $-1 = \frac{-1*2}{1*2} = -\frac{2}{2}$

Clearly, -1>-2

Thus, $-\frac{1}{2} > $ -1

Question 6 - Which of the two rational numbers is greater in the given pair?

(a) $-\frac{4}{3}$ **or** $-\frac{8}{7}$

Solution **-** LCM of 3 and 7 is 21

Now $-\frac{4}{3} = \frac{-4*7}{3*7} = -\frac{28}{21}$

$-\frac{8}{7} = \frac{-8*3}{7*3} = -\frac{24}{21}$

Clearly, -24 > -28

Thus, $-\frac{8}{7} > -\frac{4}{3}$

(b) $\frac{7}{-9}$ **or** $-\frac{5}{8}$

Solution ‐ One number $= \frac{7}{-9} = \frac{7*(-1)}{-9*(-1)} = -\frac{7}{9}$

Other number $= -\frac{5}{8}$

LCM of 9 and 8 is 72.

Now, $-\frac{7}{9} = \frac{-7*8}{9*8} = -\frac{56}{72}$

$-\frac{5}{8} = \frac{-5*9}{8*9} = -\frac{45}{72}$

Clearly, -45 > -56

Thus, $-\frac{5}{8} > \frac{7}{-9}$

(c) $\frac{-1}{3}$ **or** $\frac{4}{-5}$

Solution ‐ One number $= \frac{4}{-5} = \frac{4*(-1)}{-5*(-1)} = \frac{-4}{5}$

Other number $= -\frac{1}{3}$

LCM of 5 and 3 is 15.

Now $-\frac{4}{5} = \frac{-4*3}{5*3} = -\frac{12}{15}$

$-\frac{1}{3} = \frac{-1*5}{3*5} = -\frac{5}{15}$

Clearly, -5 > -12

Thus, $-\frac{1}{3} > \frac{4}{-5}$

(d) $\frac{9}{-13}$ **or** $\frac{7}{-12}$

Solution ‐ One number $= \frac{9}{-13} = \frac{9*(-1)}{-13*(-1)} = -\frac{9}{13}$

Other number $= \frac{7}{-12} = \frac{7*(-1)}{-12*(-1)} = -\frac{7}{12}$

LCM of 13 and 12 is 156.

Now, $-\frac{9}{13} = \frac{-9*12}{13*12} = -\frac{108}{156}$

$-\frac{7}{12} = \frac{-7*13}{12*13} = -\frac{91}{156}$

Clearly, -91 > -108

Thus, $\frac{7}{-12} > \frac{9}{-13}$

(e) $\frac{4}{-5}$ or $-\frac{7}{10}$

Solution - One number $= \frac{4}{-5} = \frac{4*(-1)}{-5*(-1)} = -\frac{4}{5}$

Other number $= -\frac{7}{10}$

LCM of 5 and 10 is 10.

Now, $-\frac{4}{5} = \frac{-4*2}{5*2} = -\frac{8}{10}$

Clearly, -7 > -8

Thus, $-\frac{7}{10} > \frac{4}{-5}$

(f) $\frac{-12}{5}$ or -3

Solution - One number $= -\frac{12}{5}$

Other number $= -\frac{3}{1}$

LCM of 5 and 1 is 5.

Now, $-\frac{3}{1} = \frac{-3*5}{1*5} = -\frac{15}{5}$

Clearly, -12 > -15

Thus, $-\frac{12}{5} > $ -3

Question 7 - Fill in the blanks with the correct symbol out of >, = and <

(a) $\frac{-3}{7} - \frac{6}{-13}$

Solution - One number $= -\frac{3}{7}$

Other number $= \frac{6}{-13} = \frac{6*(-1)}{-13*(-1)} = -\frac{6}{13}$

LCM of 7 and 13 is 91.

Now, $-\frac{3}{7} = \frac{-3*13}{7*13} = -\frac{39}{91}$

$-\frac{6}{13} = \frac{-6*7}{13*7} = -\frac{42}{91}$

Clearly, -39 > -42

Thus, $-\frac{3}{7} > \frac{6}{-13}$

(b) $\frac{5}{-13} \ - -\frac{35}{91}$

Solution - One number $= \frac{5}{-13} = \frac{5*(-1)}{-13*(-1)} = -\frac{5}{13}$

Other number $= -\frac{35}{91}$

LCM of 13 and 91 is 91.

Now, $-\frac{5}{13} = \frac{-5*7}{13*7} = -\frac{35}{91}$

Clearly, -35 = -35

Thus, $\frac{5}{-13} = -\frac{35}{91}$

(c) -2 $\_ -\frac{13}{5}$

Solution - One number $= -\frac{2}{1}$

Other number $= -\frac{13}{5}$

LCM of 1 and 5 is 5.

Now, $-\frac{2}{1} = \frac{-2*5}{1*5} = -\frac{10}{5}$

Clearly -10 > -13

Thus, $-2 > -\frac{13}{5}$

(d) $\frac{-2}{3} - \frac{5}{-8}$

Solution - One number $= -\frac{2}{3}$

Other number $= \frac{5}{-8} = \frac{5*(-1)}{-8*(-1)} = -\frac{5}{8}$

LCM of 3 and 8 is 24.

Now, $-\frac{2}{3} = \frac{-2*8}{3*8} = -\frac{16}{24}$

$-\frac{5}{8} = \frac{-5*3}{8*3} = -\frac{15}{24}$

Clearly, $-15 > -16$

Thus, $-\frac{2}{3} < \frac{5}{-8}$

(e) $\mathbf{0} \_ - \frac{3}{-5}$

Solution - One number $= \frac{0}{1}$

Other number $= \frac{-3}{-5} = \frac{-3*(-1)}{-5*(-1)} = \frac{3}{5}$

LCM of 1 and 5 is 5.

Now, $\frac{0}{1} = \frac{0*5}{1*5} = \frac{0}{5}$

Clearly, $3 > 0$

Thus, $0 < -\frac{3}{-5}$

(f) $\frac{-8}{9} - \frac{-9}{10}$

Solution - One number $= \frac{-8}{9}$

Other number $= \frac{-9}{10}$

LCM of 9 and 10 is 90.

Now, $-\frac{8}{9} = \frac{-8*10}{9*10} = -\frac{80}{90}$

$$-\frac{9}{10} = \frac{-9*9}{10*9} = -\frac{81}{9}$$

Clearly, -80 > -81

Thus, $-\frac{8}{9} > -\frac{9}{10}$

Question 8 - Arrange the following rational numbers in ascending order

(a) $\frac{4}{-9}$, $-\frac{5}{12}$, $\frac{7}{-18}$, $-\frac{2}{3}$

Solution **-** One number $= \frac{4}{-9} = \frac{4*(-1)}{-9*(-1)} = -\frac{4}{9}$

Second number $= -\frac{5}{12}$

Third number $= \frac{7}{-18} = \frac{7*(-1)}{-18*(-1)} = -\frac{7}{18}$

Forth number $= -\frac{2}{3}$

LCM of 9, 12, 18, 3 is 36

Now, $-\frac{4}{9} = \frac{-4*4}{9*4} = -\frac{16}{36}$

$-\frac{5}{12} = \frac{-5*3}{12*3} = -\frac{15}{36}$

$-\frac{7}{18} = \frac{-7*2}{18*2} = -\frac{14}{36}$

$-\frac{2}{3} = \frac{-2*12}{3*12} = -\frac{24}{36}$

Clearly, -24 < -16 < -15 < -14

Thus, $-\frac{2}{3} < \frac{4}{-9} < -\frac{5}{12} < \frac{7}{-18}$

(b) $\frac{-3}{4}$, $\frac{5}{-12}$, $-\frac{7}{16}$, $\frac{9}{-24}$

Solution **-** One number $= -\frac{3}{4}$

Second number $= \frac{5}{-12} = \frac{5*(-1)}{-12*(-1)} = -\frac{5}{12}$

Third number $= -\frac{7}{16}$

Forth number $= \dfrac{9}{-24} = \dfrac{9*(-1)}{-24*(-1)} = -\dfrac{9}{24}$

LCM of 4, 12, 16, and 24 is 48

Now, $-\dfrac{3}{4} = \dfrac{-3*12}{4*12} = -\dfrac{36}{48}$

$-\dfrac{5}{12} = \dfrac{-5*4}{12*4} = -\dfrac{20}{48}$

$-\dfrac{7}{16} = \dfrac{-7*3}{16*3} = -\dfrac{21}{48}$

$-\dfrac{9}{24} = \dfrac{-9*2}{24*2} = -\dfrac{18}{48}$

Clearly, -36 < -21 < -20 < -18

Thus, $-\dfrac{3}{4} < -\dfrac{7}{16} < \dfrac{5}{-12} < \dfrac{9}{-24}$

(c) $\dfrac{3}{-5}, -\dfrac{7}{10}, -\dfrac{11}{15}, -\dfrac{13}{20}$

Solution - One number $= \dfrac{3}{-5} = \dfrac{3*(-1)}{-5*(-1)} = -\dfrac{3}{5}$

Second number $= -\dfrac{7}{10}$

Third number $= -\dfrac{11}{15}$

Forth number $= -\dfrac{13}{20}$

LCM of 5, 10, 15, 20 is 60

Now $-\dfrac{3}{5} = \dfrac{-3*12}{5*12} = -\dfrac{36}{60}$

$-\dfrac{7}{10} = \dfrac{-7*6}{10*6} = -\dfrac{42}{60}$

$-\dfrac{11}{15} = \dfrac{-11*4}{15*4} = -\dfrac{44}{60}$

$-\dfrac{13}{20} = \dfrac{-13*3}{20*3} = -\dfrac{39}{60}$

Clearly, -44 < -42 < -39 < -36

Thus, $-\dfrac{11}{15} < -\dfrac{7}{10} < -\dfrac{13}{20} < \dfrac{3}{-5}$

(d) $\dfrac{-4}{7}, -\dfrac{9}{14}, \dfrac{13}{-28}, -\dfrac{23}{42}$

Solution **-** One number $= -\frac{4}{7}$

Second number $= -\frac{9}{14}$

Third number $= \frac{13}{-28} = \frac{13*(-1)}{-28*(-1)} = -\frac{13}{28}$

Forth number $= -\frac{23}{42}$

LCM of 7, 14, 28, 42 is 84

Now $-\frac{4}{7} = \frac{-4*12}{7*12} = -\frac{48}{84}$

$-\frac{9}{14} = \frac{-9*6}{14*6} = -\frac{54}{84}$

$-\frac{13}{28} = \frac{-13*3}{28*3} = -\frac{39}{84}$

$-\frac{23}{42} = \frac{-23*2}{42*2} = -\frac{46}{84}$

Clearly, -54 < -48 < -46 < -39

Thus, $-\frac{9}{14} < -\frac{4}{7} < -\frac{23}{42} < \frac{13}{-28}$

Question 9 - Arrange the following rational numbers in descending order

(a)-2, $-\frac{13}{6}, \frac{8}{-3}, \frac{1}{3}$

Solution **-** One number $= -\frac{2}{1}$

Second number $= -\frac{13}{6}$

Third number $= \frac{8}{-3} = \frac{8*(-1)}{-3*(-1)} = -\frac{8}{3}$

Forth number $= \frac{1}{3}$

LCM of 1, 6, 3, and 3 is 6

Now $-\frac{2}{1} = \frac{-2*6}{1*6} = -\frac{12}{6}$

$-\frac{8}{3} = \frac{-8*2}{3*2} = -\frac{16}{6}$

$\frac{1}{3} = \frac{1*2}{3*2} = \frac{2}{6}$

Clearly, $2 > -12 > -13 > -16$

Thus, $\frac{1}{3} > -2 > -\frac{13}{6} > \frac{8}{-3}$

(b)$\frac{-3}{10}, \frac{7}{-15}, -\frac{11}{20}, \frac{17}{-30}$

Solution **-** One number $= -\frac{3}{10}$

Second number $= \frac{7}{-15} = \frac{7*(-1)}{-15*(-1)} = -\frac{7}{15}$

Third number $= -\frac{11}{20}$

Forth number $= \frac{17}{-30} = \frac{17*(-1)}{-30*(-1)} = -\frac{17}{30}$

LCM of 10, 15, 20, and 30 is 60

Now, $-\frac{3}{10} = \frac{-3*6}{10*6} = -\frac{18}{60}$

$-\frac{7}{15} = \frac{-7*4}{15*4} = -\frac{28}{60}$

$-\frac{11}{20} = \frac{-11*3}{20*3} = -\frac{33}{60}$

$-\frac{17}{30} = \frac{-17*2}{30*2} = -\frac{34}{60}$

Clearly, $-18 > -28 > -33 > -34$

Thus, $-\frac{3}{10} > \frac{7}{-15} > -\frac{11}{20} > \frac{17}{-30}$

(c)$\frac{-5}{6}, -\frac{7}{12}, -\frac{13}{18}, \frac{23}{-24}$

Solution **-** One number $= -\frac{5}{6}$

Second number $= -\frac{7}{12}$

Third number $= -\frac{13}{18}$

Forth number $= \frac{23}{-24} = \frac{23*(-1)}{-24*(-1)} = -\frac{23}{24}$

LCM of 6, 12, 18, and 24 is 72

Now $-\frac{5}{6} = \frac{-5*12}{6*12} = -\frac{60}{72}$

$-\frac{7}{12} = \frac{-7*6}{12*6} = -\frac{42}{72}$

$-\frac{13}{18} = \frac{-13*4}{18*4} = -\frac{52}{72}$

$-\frac{23}{24} = \frac{-23*3}{24*3} = -\frac{69}{72}$

Clearly, -42 > -52 > -60 > -69

Thus, $-\frac{7}{12} > -\frac{13}{18} > -\frac{5}{6} > \frac{23}{-24}$

(d)$\frac{-10}{11}, -\frac{19}{22}, -\frac{23}{33}, -\frac{39}{44}$

Solution - One number $= -\frac{10}{11}$

Second number $= -\frac{19}{22}$

Third number $= -\frac{23}{33}$

Forth number $= -\frac{39}{44}$

LCM of 11, 22, 33, and 44 is 132

Now $-\frac{10}{11} = \frac{-10*12}{11*12} = -\frac{120}{132}$

$-\frac{19}{22} = \frac{-19*6}{22*6} = -\frac{114}{132}$

$-\frac{23}{33} = \frac{-23*4}{33*4} = -\frac{92}{132}$

$-\frac{39}{44} = \frac{-39*3}{44*3} = -\frac{117}{132}$

Clearly, -92 > -114 > -117 > -120

Thus, $-\frac{23}{33} > -\frac{19}{22} > -\frac{39}{44} > -\frac{10}{11}$

Question 10 - Which of the following statements are true and which are false?

(a)Every whole number is a rational number.

True (since all whole numbers can be written in the form of $\frac{p}{q}$)

(b)Every integer is a rational number.

True (since all integers can be written in the form of $\frac{p}{q}$)

(c) 0 is a whole number but it is not a rational number.

False (0 is also a rational number as 0 can be written in the form of $\frac{p}{q}$)

Representation of rational numbers on number line

Let us understand this by examples:

Example 1 - Represent $\frac{1}{2}$ and $\frac{-1}{2}$ on the number line
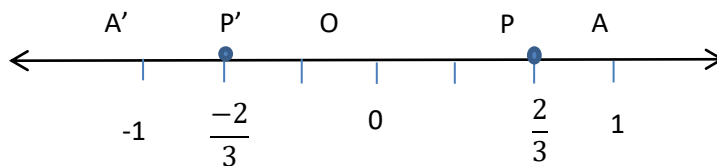
Solution

$\frac{1}{2}$ is a positive number, so it will lie along right side of the zero. We will first divide number line between 0 and 1 in two parts then $\frac{1}{2}$ will be in the middle of 0 and 1. Similarly $-\frac{1}{2}$ will lie between 0 and -1

Example 2 - Represent $\frac{2}{3}$ and $-\frac{2}{3}$ on the number line

Solution - $\frac{2}{3}$ lies between 0 and 1and $-\frac{2}{3}$ lies between 0 and -1.

Here since denominator = 3. So we divide the distance between 0 and 1 into 3equal parts. Points P and P' represents $\frac{2}{3}$ and $-\frac{2}{3}$ respectively.
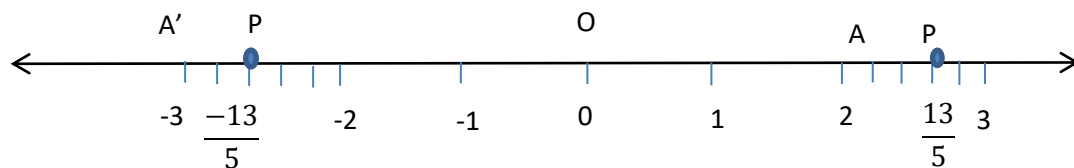

Example 3 - Represent $\frac{13}{5}$ and $-\frac{13}{5}$ on the number line

Solution - Since $\frac{13}{5} = 2\frac{3}{5}$

It is clear that $\frac{13}{5}$ will lie between 2 and 3 and $-\frac{13}{5}$ will lie between -2 and -3.

Now denominator is 5. So we will divide the distance between 2 and 3 into 5 equal parts. Points P and P' represents $\frac{13}{5}$ and $-\frac{13}{5}$ respectively.

Exercise 1B

**Question 1 - Represent each of the following numbers on the number line:**

(a) $\frac{1}{3}$

Solution - $\frac{1}{3}$ is a positive number, so it will lie along right side of the zero. We will first divide number line between 0 and 1 in three parts. P represents $\frac{1}{3}$

(b) $\frac{2}{7}$

Solution - $\frac{2}{7}$ is a positive number, so it will lie along right side of the zero. Since denominator is 7, so we will divide between 0 and 1 into seven equal parts. P represents $\frac{2}{7}$
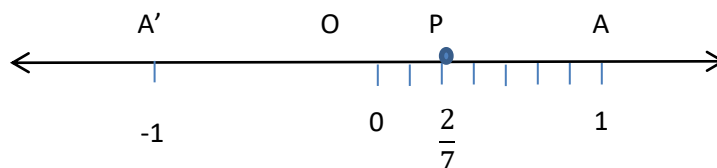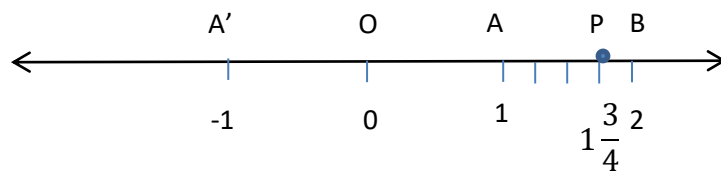
(c) $1\frac{3}{4}$

Solution **-** It is clear that $1\frac{3}{4}$ will lie between 1 and 2

Now denominator is 4 so we will divide the distance between 1 and 2 into 4 equal parts.

P represents $1\frac{3}{4}$

(d) $2\frac{2}{5}$

Solution **-** It is clear that $2\frac{2}{5}$ will lie between 2 and 3.

Now denominator is 5 so we will divide the distance between 2 and 3 into 5 equal parts.
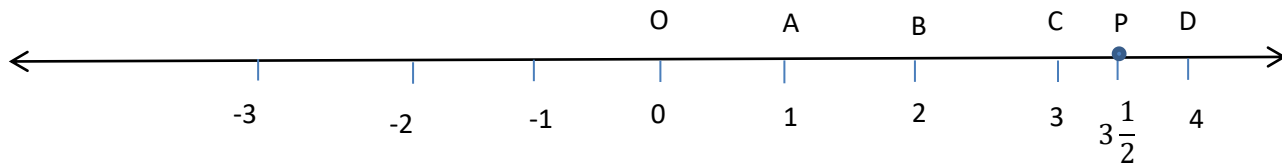
P represents $2\frac{2}{5}$

(e) $3\frac{1}{2}$

Solution **-** It is clear that $3\frac{1}{2}$ will lie between 3 and 4.

Now denominator is 2 so we will divide the distance between 3 and 4 into 2 equal parts.
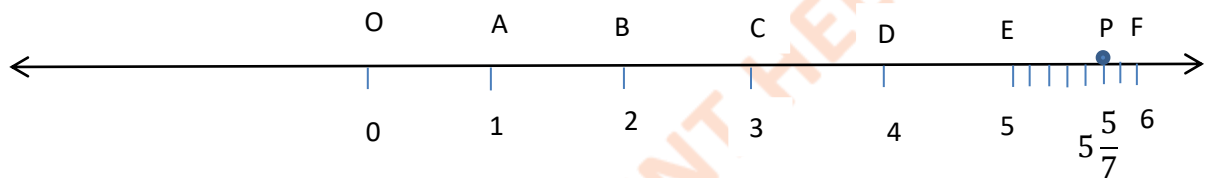
P represents $3\frac{1}{2}$

Number line showing points O, A, B, C, P, D at positions -3, -2, -1, 0, 1, 2, 3, $3\frac{1}{2}$, 4

(f) $5\frac{5}{7}$

Solution - It is clear that $5\frac{5}{7}$ will lie between 5 and 6.

Now denominator is 7 so we will divide the distance between 5 and 6 into 7 equal parts.

P represents $5\frac{5}{7}$

Number line showing points O, A, B, C, D, E, P, F at positions 0, 1, 2, 3, 4, 5, $5\frac{5}{7}$, 6
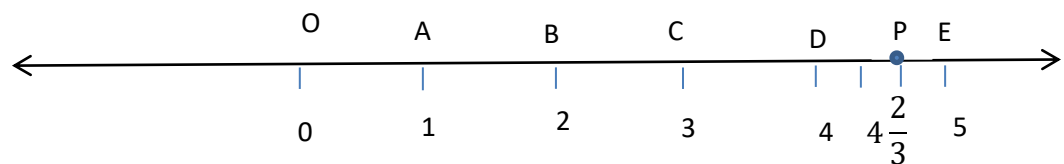
(g) $4\frac{2}{3}$

Solution - It is clear that $4\frac{2}{3}$ will lie between 4 and 5.

Now denominator is 3 so we will divide the distance between 4 and 5 into 3 equal parts.
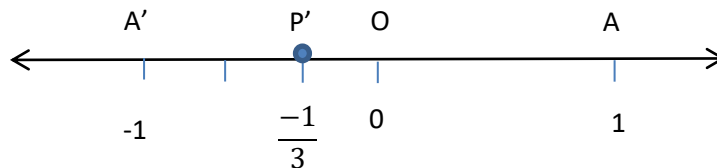
P represents $4\frac{2}{3}$

Number line showing points O, A, B, C, D, P, E at positions 0, 1, 2, 3, 4, $4\frac{2}{3}$, 5

Question 2 - Represent each of the following numbers on the number line:

(a) $\frac{-1}{3}$

Solution - $\frac{-1}{3}$ is a negative number, so it will lie along left side of the zero. We will first divide number line between 0 and -1 in three parts. P' represents $\frac{-1}{3}$

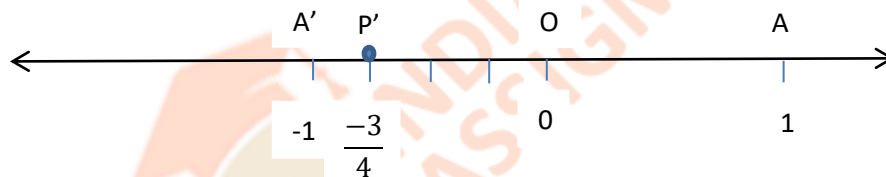A'   P'   O   A

-1 $\frac{-1}{3}$ 0   1

(b) $\frac{-3}{4}$

Solution - $\frac{-3}{4}$ is a negative number, so it will lie along left side of the zero. We will first divide number line between 0 and -1 into four equal parts. P' represents $\frac{-3}{4}$

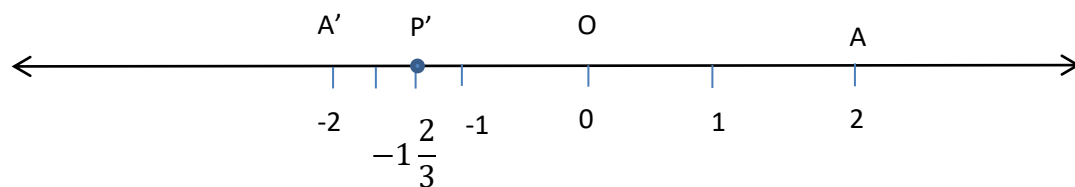A'   P'   O   A

-1 $\frac{-3}{4}$ 0   1

(c) $-1\frac{2}{3}$

Solution - It is clear that $-1\frac{2}{3}$ will lie between -1 and -2.

Now denominator is 3, so we will divide distance between -1 and -2 into 3 equal parts.

P' represents $-1\frac{2}{3}$

A'   P'   O   A

-2   -1   0   1   2

$-1\frac{2}{3}$

(d) $-3\frac{1}{7}$

Solution **-** It is clear that $-3\frac{1}{7}$ will lie between -3 and -4.

Now denominator is 7, so we will divide distance between -3 and -4 into 7 equal parts.

P' represents $-3\frac{1}{7}$

(e) $-4\frac{3}{5}$

Solution **-** It is clear that $-4\frac{3}{5}$ will lie between -4 and -5.

Now denominator is 5, so we will divide distance between -4 and -5 into 5 equal parts.

P' represents $-4\frac{3}{5}$

(f) $-2\frac{5}{6}$

Solution **-** It is clear that $-2\frac{5}{6}$ will lie between -2 and -3.

Now denominator is 6, so we will divide distance between -2 and -3 into 6 equal parts.

P' represents $-2\frac{5}{6}$

(g) -3

Solution **-** P' represents -3

P'   O

(h) $-2\frac{7}{8}$

Solution **-** It is clear that $-2\frac{7}{8}$ will lie between -2 and -3.

Now denominator is 8, so we will divide distance between -2 and -3 into 8 equal parts.
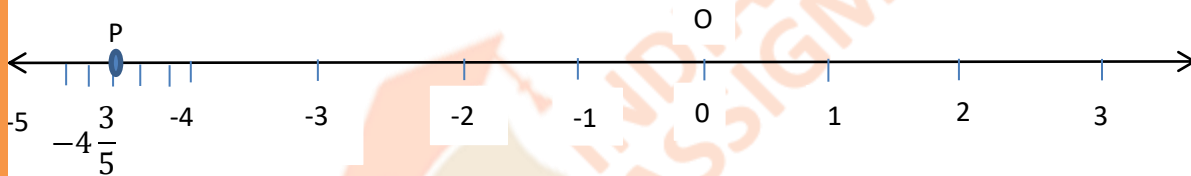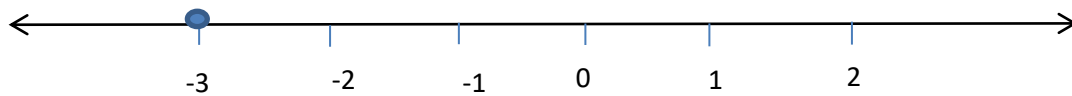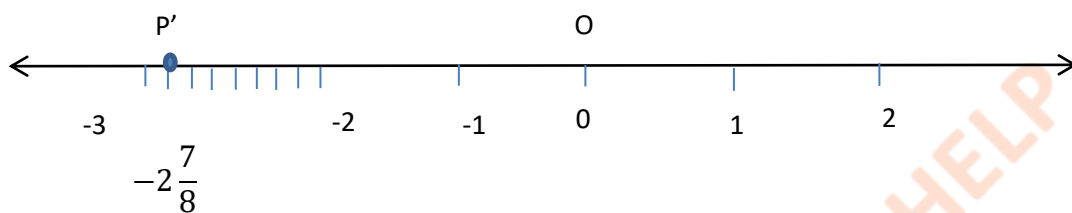
P' represents $-2\frac{7}{8}$

Question 3 - Which of the following statements are true and which are false?

(a) $-\frac{3}{5}$ **Lies to the left of 0 on the number line.**

Solution **-**True (since $-\frac{3}{5}$ is a negative rational number thus it will lie to the left of 0 on the number line)

(b) $-\frac{12}{7}$ **Lies to the right of 0 on the number line.**

Solution **-** False (since $-\frac{12}{7}$ is a negative rational number thus it will lie to the left of 0 on the number line)

(c) The rational numbers $\frac{1}{3}$ **and** $-\frac{5}{2}$ **are on opposite sides of 0 on the number line.**

Solution **-** True (since $\frac{1}{3}$ will lie to right of 0 and $-\frac{5}{2}$ lie on left of 0 on number line)

(d) The rational numbers $\frac{-18}{-13}$ **lies to the left of 0 on the number line.**

Solution **-** False (since $\frac{-18}{-13} = \frac{18}{13}$ is a positive rational number so it will lie to the right of 0 on number line)

Addition of Rational Numbers

Case1 When given numbers have same denominator

$(\frac{a}{b} + \frac{c}{b}) = \frac{(a+c)}{b}$

Example 1 - Find the Sum:

(a) $\frac{7}{9} + \frac{-11}{9}$

Solution $\frac{7+(-11)}{9} = \frac{-4}{9}$

(b) $\frac{8}{-11} + \frac{3}{11}$

Solution $\frac{-8+3}{11} = \frac{-5}{11}$

Case 2 When denominators of given numbers are unequal.

In this case, we take LCM of their denominators and express each number with this LCM as the common denominator.

Example 2 - Find the sum: $\frac{-5}{6} + \frac{4}{9}$

Solution **-** LCM of 6 and 9 is 18

Now -5/6 = (-5*3)/(6*3) = -15/18

4/9 = (4*2)/(9*2) = 8/18

Thus, $\frac{-5}{6} + \frac{4}{9} = \frac{-15}{18} + \frac{8}{18} = \frac{(-15+8)}{18}$ = -7/18

Example 3 - Find the Sum $\frac{-9}{16} + \frac{5}{12}$

Solution **-** LCM of 16 and 12 is 48

Now $\frac{(-9*3)+(5*4)}{48} = \frac{-27+20}{48}$ = -7/48

Properties of Addition of Rational Numbers

Property 1 (Closure Property): The Sum of two rational numbers is always a rational number.

Property 2 (Commutative Law): Two rational numbers can be added in any order.

That is, $(\frac{a}{b} + \frac{c}{d}) = (\frac{c}{d} + \frac{a}{b})$

Property 3 (Associative Law): While adding three rational numbers, they can be grouped in any order.

That is, $(\frac{a}{b} + \frac{c}{d}) + \frac{e}{f} = \frac{a}{b} + (\frac{c}{d} + \frac{e}{f})$

Property 4 (Existence of Additive Identity): 0 is a rational number such that the sum of any rational number and 0 is the rational number itself.

That is, $(\frac{a}{b} + 0) = (0 + \frac{a}{b}) = \frac{a}{b}$

0 is called the additive identity for rational number.

Property 5 (Existence of additive inverse): For every rational number $\frac{a}{b}$ there exists a rational number $\frac{-a}{b}$ such that $\frac{a}{b} + \frac{-a}{b} = 0$.

$\frac{-a}{b}$ is called the additive inverse of $\frac{a}{b}$

Examples:

Example 1 - Find the additive inverse of:

(a) $\frac{5}{9}$

Solution **-** The additive inverse of $\frac{5}{9}$ is $\frac{-5}{9}$.

(b) $\frac{-15}{8}$

Solution - The additive inverse of $\frac{-15}{8}$ is $\frac{15}{8}$.

(c) $\frac{9}{-11}$

Solution **-** The additive inverse of $\frac{9}{-11}$ is $\frac{9}{11}$.

(d) $\frac{-6}{-7}$

Solution - The additive inverse of $\frac{-6}{-7}$ is $-\left(\frac{-6}{-7}\right) = \frac{-6}{7}$.

Example 2-(a) Subtract $\frac{3}{4}$ from $\frac{2}{3}$

Solution $\left(\frac{2}{3} - \frac{3}{4}\right) = \left(\frac{8-9}{12}\right) = \frac{-1}{12}$

(b) Subtract $\frac{-5}{7}$ from $\frac{-2}{5}$

Solution $\left(\frac{-2}{5} - \left(-\frac{5}{7}\right)\right) = \left(\frac{-2}{5} + \frac{5}{7}\right)$

$= \left(\frac{-14+25}{35}\right) = \frac{11}{35}$

Example 3 - The sum of two rational numbers is -5. If one of them is $\frac{-13}{6}$, find the other.

Solution - Let other number be x

Then, $\frac{-13}{6} + x = -5$

$\Rightarrow x = -\frac{5}{1} + \frac{13}{6}$

$\Rightarrow x = \frac{-30+13}{6} = \frac{-17}{6}$

Example 4 - What number should be added to $\frac{-7}{8}$ to get $\frac{4}{9}$?

Solution - Let the required number be x

Then, $\frac{-7}{8} + x = \frac{4}{9}$

$\Rightarrow x = \frac{4}{9} + \frac{7}{8}$

$\Rightarrow x = \frac{32+63}{72}$

$\Rightarrow x = \frac{95}{72}$

Example 5 - Evaluate $\frac{3}{5} + \frac{7}{3} + \frac{-11}{5} + \frac{-2}{3}$

Solution - Using rearrangement property,

$(\frac{3}{5} + \frac{-11}{5}) + (\frac{7}{3} + \frac{-2}{3})$

$= (\frac{3-11}{5}) + (\frac{7-2}{3})$

$= \frac{-8}{5} + \frac{5}{3}$

$= \frac{-24+25}{15} = \frac{1}{15}$

Example 6 - Simplify $(\frac{4}{7} + \frac{-8}{9} + \frac{-5}{21} + \frac{1}{3})$

Solution **-** Using rearrangement property,

$(\frac{4}{7} + \frac{-5}{21}) + (\frac{-8}{9} + \frac{1}{3})$

$= (\frac{12-5}{21}) + (\frac{-8+3}{9})$

$= \frac{7}{21} + \frac{-5}{9}$

$= \frac{21-35}{63} = \frac{-14}{63} = \frac{-2}{9}$

Example 7 - What should be subtracted from $\frac{-5}{7}$ to get -1?

Solution **-** Let the required number be x

Then, $\frac{-5}{7} - x = -1$

$=> \frac{-5}{7} = x - 1$

$=> x = \frac{-5}{7} + 1 = \frac{-5+7}{7} = \frac{2}{7}$

Exercise 1C

Question 1 - Add the following rational numbers:

(a) $\frac{-2}{5} + \frac{4}{5}$

Solution $\frac{(-2+4)}{5} = \frac{2}{5}$

(b) $\frac{-6}{11} + \frac{-4}{11}$

Solution $\frac{(-6-4)}{11} = \frac{-10}{11}$

(c) $\dfrac{-11}{8}+\dfrac{5}{8}$

Solution $\dfrac{(-11+5)}{8}=\dfrac{-6}{8}$

(d) $\dfrac{-7}{3}+\dfrac{1}{3}$

Solution $\dfrac{(-7+1)}{3}=\dfrac{-6}{3}$

(e) $\dfrac{5}{6}+\dfrac{-1}{6}$

Solution $\dfrac{(5-1)}{6}=\dfrac{4}{6}=\dfrac{2}{3}$

(f) $\dfrac{-17}{15}+\dfrac{-1}{15}$

Solution $\dfrac{(-17-1)}{15}=\dfrac{-18}{15}=\dfrac{-6}{5}$

Question 2 - Add the following numbers:

(a) $\dfrac{3}{4}+\dfrac{-3}{5}$

Solution **-** LCM of 4 and 5 is 20

Thus, $\dfrac{3}{4}+\dfrac{-3}{5}=\dfrac{15-12}{20}=\dfrac{3}{20}$

(b) $\dfrac{5}{8}+\dfrac{-7}{12}$

Solution - LCM of 8 and 12 is 24

Thus, $\dfrac{5}{8}+\dfrac{-7}{12}=\dfrac{15-14}{24}=\dfrac{1}{24}$

(c) $\dfrac{-8}{9}+\dfrac{11}{6}$

Solution- LCM of 9 and 6 is 24

Thus, $\dfrac{-8}{9}+\dfrac{11}{6}=\dfrac{-16+33}{18}=\dfrac{17}{18}$

(d) $-\dfrac{5}{16}+\dfrac{7}{24}$

Solution- LCM of 16 and 24 is 48

$\dfrac{-5}{16}+\dfrac{7}{24}=\dfrac{-15+14}{48}=\dfrac{-1}{48}$

(e) $\dfrac{7}{-18} + \dfrac{8}{27}$

Solution- LCM of 18 and 27 is 54

$\dfrac{7}{-18} + \dfrac{8}{27} = \dfrac{-21+16}{54} = -\dfrac{5}{54}$

(f) $\dfrac{1}{-12} + \dfrac{2}{15}$

Solution - LCM of 12 and 15 is 60

$\dfrac{1}{-12} + \dfrac{2}{15} = \dfrac{-5+8}{60} = \dfrac{3}{60} = \dfrac{1}{20}$

(g) -1 $+ \dfrac{3}{4}$

Solution - LCM of 1 and 4 is 4

$-1 + \dfrac{3}{4} = \dfrac{-4+3}{4} = -\dfrac{1}{4}$

(h) 2 $+ \dfrac{-5}{4}$

Solution - LCM of 1 and 4 is 4

$2 + \dfrac{-5}{4} = \dfrac{8-5}{4} = \dfrac{3}{5}$

(i) 0 $+ \dfrac{-2}{5}$

Solution **-** LCM of 1 and 5 is 5

$0 + \dfrac{-2}{5} = \dfrac{0-2}{5} = -\dfrac{2}{5}$

Question 3 - Verify the following

(a) $-\dfrac{12}{5} + \dfrac{2}{7} = \dfrac{2}{7} + \dfrac{-12}{5}$

Solution-LHS: $-\dfrac{12}{5} + \dfrac{2}{7}$

LCM of 5 and 7 is 35

$\dfrac{-84+10}{35} = -\dfrac{74}{35}$

RHS: $\dfrac{2}{7} + \dfrac{-12}{5}$

$$\frac{10-84}{35} = -\frac{74}{35}$$

Hence LHS = RHS

(b) $-\frac{5}{8} + \frac{-9}{13} = -\frac{9}{13} + \frac{-5}{8}$

Solution - LHS: $-\frac{5}{8} + \frac{-9}{13}$

LCM of 8 and 13 is 104

$$\frac{-65-72}{104} = -\frac{137}{104}$$

RHS: $-\frac{9}{13} + \frac{-5}{8}$

$$\frac{-72-65}{104} = -\frac{137}{104}$$

Hence LHS = RHS

(c) $3 + \frac{-7}{12} = -\frac{7}{12} + 3$

Solution - LHS: $3 + \frac{-7}{12}$

LCM of 1 and 12 is 12

$$\frac{36-7}{12} = \frac{29}{12}$$

RHS: $-\frac{7}{12} + 3$

$$\frac{-7+36}{12} = \frac{29}{12}$$

Hence LHS = RHS

(d) $\frac{2}{-7} + \frac{12}{-35} = \frac{12}{-35} + \frac{2}{-7}$

Solution - LHS: $\frac{2}{-7} + \frac{12}{-35}$

LCM of 7 and 35 is 35

$$\frac{-10-12}{35} = \frac{-22}{35}$$

RHS: $\frac{12}{-35} + \frac{2}{-7}$

$$\frac{-12-10}{35} = \frac{-22}{35}$$

Hence LHS = RHS

Question 4 - Verify the following

(a) $(\frac{3}{4} + \frac{-2}{5}) + \frac{-7}{10} = \frac{3}{4} + (\frac{-2}{5} + \frac{-7}{10})$

Solution LHS**:** $(\frac{3}{4} + \frac{-2}{5}) + \frac{-7}{10}$

LCM of 4, 5 is 20

$(\frac{15-8}{20}) + -\frac{7}{10}$

$= \frac{7}{20} - \frac{7}{10}$

LCM of 20 and 10 is 20

$= \frac{7-14}{20} = -\frac{7}{20}$

RHS**:** $\frac{3}{4} + (\frac{-2}{5} + \frac{-7}{10})$

LCM of 5 and 10 is 10

$\frac{3}{4} + (\frac{-4-7}{10}) = \frac{3}{4} + -\frac{11}{10}$

LCM of 4 and 10 is 20

$= \frac{15-22}{20} = -\frac{7}{20}$

Hence LHS = RHS

(b) $(\frac{-7}{11} + \frac{2}{-5}) + \frac{-13}{22} = \frac{-7}{11} + (\frac{2}{-5} + \frac{-13}{22})$

Solution - LHS: $(\frac{-7}{11} + \frac{2}{-5}) + \frac{-13}{22}$

LCM of 11 and 5 is 55

$(\frac{-35-22}{55}) + \frac{-13}{22}$

$= \frac{-57}{55} - \frac{13}{22}$

LCM of 55 and 22 is 110

$$=\frac{-114-65}{110} = -\frac{179}{110}$$

RHS: $\frac{-7}{11} + \left(\frac{2}{-5} + \frac{-13}{22}\right)$

LCM of 5 and 22 is 110

$$-\frac{7}{11} + \left(\frac{-44-65}{110}\right)$$

$$=-\frac{7}{11} - \frac{109}{110}$$

LCM of 11 and 110 is 110

$$=\frac{-70-109}{110} = -\frac{179}{110}$$

Hence LHS = RHS

(c) $-\mathbf{1} + \left(\frac{-2}{3} + \frac{-3}{4}\right) = \left(-\mathbf{1} + \frac{-2}{3}\right) + \frac{-3}{4}$

Solution - LHS: $-1 + \left(-\frac{2}{3} + \frac{-3}{4}\right)$

LCM of 3 and 4 is 12

$$= -1 + \left(\frac{-8-9}{12}\right) = -1 - \frac{17}{12}$$

LCM of 1 and 12 is 12

$$= \left(\frac{-12-17}{12}\right) = -\frac{29}{12}$$

RHS: $\left(-1 + \frac{-2}{3}\right) + \frac{-3}{4}$

LCM of 1 and 3 is 3

$$\left(\frac{-3-2}{3}\right) - \frac{3}{4} = -\frac{5}{3} - \frac{3}{4}$$

LCM of 3 and 4 is 12

$$\frac{-20-9}{12} = -\frac{29}{12}$$

Hence LHS = RHS

Question 5 - Fill in the blanks.

(a) $\left(-\frac{3}{17}\right) + \left(-\frac{12}{5}\right) = \left(-\frac{12}{5}\right) + (....)$

Solution - By using commutative property

$\left(-\frac{3}{17}\right) + \left(-\frac{12}{5}\right) = \left(-\frac{12}{5}\right) + \left(-\frac{3}{17}\right)$

(b) $-9 + \frac{-21}{8} = (....) + (-9)$

Solution - By using commutative property

$-9 + \frac{-21}{8} = \left(\frac{-21}{8}\right) + (-9)$

(c) $\left(-\frac{8}{13} + \frac{3}{7}\right) + \left(-\frac{13}{4}\right) = (....) + \left[\frac{3}{7} + \left(-\frac{13}{4}\right)\right]$

Solution - By using Associative property

$\left(-\frac{8}{13} + \frac{3}{7}\right) + \left(-\frac{13}{4}\right) = \left(-\frac{8}{13}\right) + \left[\frac{3}{7} + \left(-\frac{13}{4}\right)\right]$

(d) $-12 + \left(\frac{7}{12} + \frac{-9}{11}\right) = \left(-12 + \frac{7}{12}\right) + (...)$

Solution - By using Associative property

$-12 + \left(\frac{7}{12} + \frac{-9}{11}\right) = \left(-12 + \frac{7}{12}\right) + \left(-\frac{9}{11}\right)$

(e) $\frac{19}{-5} + \left(\frac{-3}{11} + \frac{-7}{8}\right) = \left\{\frac{19}{-5} + (...)\right\} + \frac{-7}{8}$

Solution - By using Associative property

$\frac{19}{-5} + \left(\frac{-3}{11} + \frac{-7}{8}\right) = \left\{\frac{19}{-5} + \left(-\frac{3}{11}\right)\right\} + \frac{-7}{8}$

(f) $\frac{-16}{7} + ... = ... + \frac{-16}{7} = \frac{-16}{7}$

Solution - Since 0 is the additive identity

We have $\frac{-16}{7} + 0 = 0 + \frac{-16}{7} = \frac{-16}{7}$

Question 6 - Find the additive inverse of each of the following

(a) $\frac{1}{3}$

Solution **-** Additive inverse of $\frac{1}{3}$ is $\frac{-1}{3}$

(b) $\frac{23}{9}$

Solution **-** Additive inverse of $\frac{23}{9}$ is $\frac{-23}{9}$

(c) -18

Solution **-** Additive inverse of -18 is 18

(d) $\frac{-17}{8}$

Solution **-** Additive inverse of $\frac{-17}{8}$ is $\frac{17}{8}$

(e) $\frac{15}{-4}$

Solution **-** Additive inverse of $\frac{15}{-4}$ is $\frac{15}{4}$

(f) $\frac{-16}{-5}$

Solution **-** Additive inverse of $\frac{-16}{-5}$ is $\frac{-(-16)}{-5} = -\frac{16}{5}$

(g) $-\frac{3}{11}$

Solution **-** Additive inverse of $\frac{-3}{11}$ is $\frac{3}{11}$

(h) 0

Solution - Additive inverse of 0 is 0

(i) $\frac{19}{-6}$

Solution **-** Additive inverse of $\frac{19}{-6}$ is $\frac{19}{6}$

(j) $\frac{-8}{-7}$

Solution - Additive inverse of $\frac{-8}{-7}$ is $\frac{-(-8)}{-7} = -\frac{8}{7}$

Question 7 - Subtract

(a) $\frac{3}{4}$ from $\frac{1}{3}$

Solution $(\frac{1}{3} - \frac{3}{4}) = (\frac{4-9}{12})$

$= \frac{-5}{12}$

(b) $-\frac{5}{6}$ from $\frac{1}{3}$

Solution $(\frac{1}{3} - (-\frac{5}{6})) = (\frac{1}{3} + \frac{5}{6})$

$= (\frac{2+5}{6}) = \frac{7}{6}$

(c) $-\frac{8}{9}$ from $-\frac{3}{5}$

Solution $(-\frac{3}{5} - (-\frac{8}{9})) = (-\frac{3}{5} + \frac{8}{9})$

$= (\frac{-27+40}{45}) = \frac{13}{45}$

(d) $-\frac{9}{7}$ from -1

Solution $(-1 - (-\frac{9}{7})) = (-1 + \frac{9}{7})$

$= (\frac{-7+9}{7}) = \frac{2}{7}$

(e) $-\frac{18}{11}$ from 1

Solution $(1 - (-\frac{18}{11})) = (1 + \frac{18}{11})$

$= (\frac{11+18}{11}) = \frac{29}{11}$

(f) $-\frac{13}{9}$ from 0

Solution $(0 - (-\frac{13}{9})) = (0 + \frac{13}{9})$

$$= \frac{13}{9}$$

(g) $-\frac{32}{13}$ **from** $-\frac{6}{5}$

Solution $(-\frac{6}{5} - (-\frac{32}{13})) = (-\frac{6}{5} + \frac{32}{13})$

$= (\frac{-78+160}{65}) = \frac{82}{65}$

(h) -7 from $-\frac{4}{7}$

Solution $(-\frac{4}{7} - (-7)) = (\frac{-4}{7} + 7)$

$= (\frac{-4+49}{7}) = \frac{45}{7}$

Question 8 - Using rearrangement property, find the sum:

(a) $\frac{4}{3} + \frac{3}{5} + \frac{-2}{3} + \frac{-11}{5}$

Solution - $(\frac{4}{3} + \frac{-2}{3}) + (\frac{3}{5} + \frac{-11}{5})$

$= (\frac{4-2}{3}) + (\frac{3-11}{5})$

$= \frac{2}{3} + \frac{-8}{5} = (\frac{10-24}{15}) = -\frac{14}{15}$

(b) $\frac{-8}{3} + \frac{-1}{4} + \frac{-11}{6} + \frac{3}{8}$

Solution - $(\frac{-8}{3} + \frac{-11}{6}) + (\frac{-1}{4} + \frac{3}{8})$

$= (\frac{-16-11}{6}) + (\frac{-2+3}{8})$

$= \frac{-27}{6} + \frac{1}{8} = (\frac{-108+3}{24}) = -\frac{105}{24} = -\frac{35}{8}$

(c) $\frac{-13}{20} + \frac{11}{14} + \frac{-5}{7} + \frac{7}{10}$

Solution - $(\frac{-13}{20} + \frac{7}{10}) + (\frac{11}{14} + \frac{-5}{7})$

$= (\frac{-13+14}{20}) + (\frac{11-10}{14})$

$= \frac{1}{20} + \frac{1}{14} = (\frac{7+10}{140}) = \frac{17}{140}$

(d) $\frac{-6}{7} + \frac{-5}{6} + \frac{-4}{9} + \frac{-15}{7}$

Solution - $(\frac{-6}{7} + \frac{-15}{7}) + (\frac{-5}{6} + \frac{-4}{9})$

$= (\frac{-6-15}{7}) + (\frac{-15-8}{18})$

$= \frac{-21}{7} + \frac{-23}{18} = -3 + \frac{-23}{18}$

$(\frac{-54-23}{18}) = -\frac{77}{18}$

Question 9 - The sum of two rational numbers is -2. If one of the numbers is $\frac{-14}{5}$, find the other.

Solution - Let other number be x

Then x + $\frac{-14}{5}$ = -2

$\Rightarrow$ x - $\frac{14}{5}$ = -2

$\Rightarrow$ x = -2 + $\frac{14}{5}$

$\Rightarrow$ x = $\frac{-10+14}{5} = \frac{4}{5}$

Question 10 - The sum of two rational numbers is $\frac{-1}{2}$. If one of the numbers is $\frac{5}{6}$, find the other.

Solution - Let other number be x

Then x + $\frac{5}{6} = \frac{-1}{2}$

$\Rightarrow$ x = $\frac{-1}{2}$ - $\frac{5}{6}$

$\Rightarrow$ x = $\frac{-3-5}{6} = \frac{-8}{6} = \frac{-4}{3}$

Question 11 - What number should be added to $\frac{-5}{8}$ so as to get $\frac{-3}{2}$?

Solution - Let the required number be x

Then $\frac{-5}{8}$ + x = $\frac{-3}{2}$

$$\Rightarrow x = \frac{-3}{2} + \frac{5}{8}$$

$$\Rightarrow x = \frac{-12+5}{8} = \frac{-7}{8}$$

Question 12 - What number should be added to -1 so as to get $\frac{5}{7}$?

Solution **-** Let the required number be x

Then $-1 + x = \frac{5}{7}$

$$\Rightarrow x = \frac{5}{7} + 1$$

$$\Rightarrow x = \frac{5+7}{7} = \frac{12}{7}$$

Question 13 - what number should be subtracted from $\frac{-2}{3}$ to get $\frac{-1}{6}$?

Solution **-** Let the required number be x

Then, $\frac{-2}{3} - x = -\frac{1}{6}$

$$\Rightarrow \frac{-2}{3} = \frac{-1}{6} + x$$

$$\Rightarrow x = \frac{-2}{3} + \frac{1}{6}$$

$$\Rightarrow x = \frac{-4+1}{6} = \frac{-3}{6} = \frac{-1}{2}$$

Question 14 (a) which rational number is its own additive inverse?

Ans - 0

(b) Is the difference of two rational numbers a rational number?

Ans - yes

(c) Is addition commutative on rational numbers?

Ans - yes

(d) Is addition associative on rational numbers?

Ans - yes

(e) Is subtraction commutative on rational numbers?

Ans - No

(f) Is subtraction associative on rational numbers?

Ans - No

(g) What is the negative of a negative rational?

Ans - Number itself

Multiplication of rational numbers:

$$\left(\frac{a}{b} * \frac{c}{d}\right) = \frac{a*c}{b*d}$$

Example 1 - Find each of the following products:

(a) $\left(\frac{2}{3} * \frac{-5}{7}\right)$

Solution $\frac{2*(-5)}{3*7} = \frac{-10}{21}$

(b) $\left(\frac{-7}{8} * \frac{3}{5}\right)$

Solution $\frac{-7*3}{8*5} = \frac{-21}{40}$

(c) $\left(\frac{-15}{4} * \frac{-3}{8}\right)$

Solution $\frac{-15*(-3)}{4*8} = \frac{45}{32}$

Example 2 - Find each of the following products:

(a) $\left(\frac{-3}{7} * \frac{14}{5}\right)$

Solution $\frac{-3*14}{7*5} = \frac{-42}{35} = \frac{-6}{5}$ (as 7 is the common factor)

(b) $\left(\frac{13}{6} * \frac{-18}{91}\right)$

Solution $\frac{13*(-18)}{6*91} = \frac{-3}{7}$ (as 7 is common factor between 13 and 91 & 3 is common between 18 and 6)

(c) $\left(\frac{-11}{9} * \frac{-51}{44}\right)$

Solution $\frac{-11*(-51)}{9*44} = \frac{17}{12}$ (as 4 is common factor between 11 and 44 & 3 is common between 51 and 9)

Properties of multiplication of rational numbers

Property 1 (closure property): The product of two rational numbers is always a rational number.

Property 2 (commutative law): Two rational numbers can be multiplied in any order.

$$\left(\frac{a}{b} * \frac{c}{d}\right) = \left(\frac{c}{d} * \frac{a}{b}\right)$$

Property 3 (associative law): While multiplying three or more rational numbers, they can be grouped in any order.

$$\left(\frac{a}{b} * \frac{c}{d}\right) * \frac{e}{f} = \frac{a}{b} * \left(\frac{c}{d} * \frac{e}{f}\right)$$

Property 4 (Existence of multiplicative identity):

$$\left(\frac{a}{b} * 1\right) = \left(1 * \frac{a}{b}\right) = \frac{a}{b} \qquad \text{(1 is the multiplicative identity)}$$

Property 5 (Existence of multiplicative inverse):

$$\left(\frac{a}{b} * \frac{b}{a}\right) = \left(\frac{b}{a} * \frac{a}{b}\right) = 1 \qquad (\frac{b}{a} \text{ is the multiplicative inverse of} \frac{a}{b})$$

Property 6 (Distributive law of multiplication over addition):

$$\frac{a}{b} * (\frac{c}{d} + \frac{e}{f}) = \left(\frac{a}{b} * \frac{c}{d}\right) + \left(\frac{a}{b} * \frac{e}{f}\right)$$

Property 7 (Multiplicative property of 0):

$$(\frac{a}{b} * 0) = (0 * \frac{a}{b}) = 0$$

Examples:

Example 1 - Find the reciprocal of each of the following:

(a) 12

Solution **-** Reciprocal of 12 is $\frac{1}{12}$

(b) -8

Solution **-** Reciprocal of -8 is $\frac{-1}{8}$

(c) $\frac{5}{13}$

Solution - Reciprocal of $\frac{5}{13}$ is $\frac{13}{5}$

(d) $\frac{-14}{17}$

Solution - Reciprocal of $\frac{-14}{17}$ is $\frac{-17}{14}$

Example 2 - Verify that:

(a) $\left(\frac{-3}{16} * \frac{8}{15}\right) = \left(\frac{8}{15} * \frac{-3}{16}\right)$

Solution - LHS**:** $\left(\frac{-3}{16} * \frac{8}{15}\right)$

$= \left(\frac{-3*8}{16*15}\right) = \frac{-1}{10}$ (as 2 is common factor between 8 and 16 & 3 is common between 3 and 15)

RHS: $\left(\frac{8}{15} * \frac{-3}{16}\right)$

$= \left(\frac{8*(-3)}{15*16}\right) = \frac{-1}{10}$

(b) $\frac{2}{3} * \left(\frac{6}{7} * \frac{-14}{15}\right) = \left(\frac{2}{3} * \frac{6}{7}\right) * \frac{-14}{15}$

Solution - LHS: $\frac{2}{3} * \left(\frac{6}{7} * \frac{-14}{15}\right)$

$= \frac{2}{3} * \left(\frac{6*(-14)}{7*15}\right) = \frac{2}{3} * \frac{-4}{5} = \frac{-8}{15}$

(As 2 is common factor between 14 and 7 & 3 is common between 6 and 15)

RHS: $\left(\frac{2}{3} * \frac{6}{7}\right) * \frac{-14}{15}$

$= \left(\frac{2*6}{3*7}\right) * \frac{-14}{15} = \frac{4}{7} * \frac{-14}{15} = \frac{-8}{15}$

(As 3 is common factor between 6 and 3 & 7 is common between 14 and 7)

Hence LHS = RHS.

(c) $\frac{5}{6} * (\frac{-4}{5} + \frac{-7}{10}) = (\frac{5}{6} * \frac{-4}{5}) + (\frac{5}{6} * \frac{-7}{10})$

Solution - LHS $\frac{5}{6} * (\frac{-4}{5} + \frac{-7}{10})$

$= \frac{5}{6} * (\frac{-8-7}{10}) = \frac{5}{6} * (\frac{-15}{10}) = \frac{-75}{60} = \frac{-5}{4}$ (as 15 is common factor between 75 and 60)

RHS $\left(\frac{5}{6} * \frac{-4}{5}\right) + \left(\frac{5}{6} * \frac{-7}{10}\right)$

$= \frac{-2}{3} + \frac{-7}{12} = \frac{-8-7}{12} = \frac{-15}{12} = \frac{-15}{12} = \frac{-5}{4}$ (as 3 is common factor between 15 and 12)

Exercise 1D

Question 1 - Find each of the following products:

(a) $\frac{3}{5} * \frac{-7}{8}$

Solution $\frac{3*(-7)}{5*8} = \frac{-21}{40}$

(b) $\frac{-9}{2} * \frac{5}{4}$

Solution $\frac{(-9)*5}{2*4} = \frac{-45}{8}$

(c) $\frac{-6}{11} * \frac{-5}{3}$

Solution $\frac{(-6)*(-5)}{11*3} = \frac{10}{11}$ (as 3 is common factor of 6 and 3)

(d) $\frac{-2}{3} * \frac{6}{7}$

Solution $\frac{(-2)*(6)}{3*7} = \frac{-4}{7}$ (as 3 is common factor of 6 and 3)

(e) $\frac{-12}{5} * \frac{10}{-3}$

Solution $\frac{(-12)*10}{5*(-3)} = 8$ (as 3 is common factor of 12 and 3 & 5 is common factor of 10 and 5)

(f) $\frac{25}{-9} * \frac{3}{-10}$

Solution $\frac{(25)*3}{-9*(-10)} = \frac{5}{6}$ (as 3 is common factor of 3 and 9 & 5 is common factor of 25 and 10)

(g) $\frac{5}{-18} * \frac{-9}{20}$

Solution $\frac{5*(-9)}{(-18)*(20)} = \frac{1}{8}$ (as 9 is common factor of 9 and 18 & 5 is common factor of 5 and 20)

(h) $\frac{-13}{15} * \frac{-25}{26}$

Solution $\frac{-13*(-25)}{(15)*(26)} = \frac{5}{6}$ (as 13 is common factor of 13 and 26 & 5 is common factor of 25 and 15)

(i) $\frac{16}{-21} * \frac{14}{5}$

Solution $\frac{16*14}{(-21)*(5)} = \frac{-32}{15}$ (as 7 is common factor of 14 and 21)

(j) $\frac{-7}{6} * 24$

Solution -28 (as 6 is common factor of 24 and 6)

(k) $\frac{7}{24} * (-48)$

Solution -14 (as 24 is common factor of 48 and 24)

(l) $\frac{-13}{5} * (-10)$

Solution 26 (as 5 is common factor of 10 and 5)

Question 2 - Verify each of the following

(a) $\left(\frac{3}{7} * \frac{-5}{9}\right) = \left(\frac{-5}{9} * \frac{3}{7}\right)$

Solution - LHS $\left(\frac{3}{7} * \frac{-5}{9}\right)$

$= \frac{3*(-5)}{7*9} = \frac{-5}{21}$

RHS $\left(\frac{-5}{9} * \frac{3}{7}\right)$

$= \frac{(-5)*3}{9*7} = \frac{-5}{21}$

Hence LHS = RHS

(b) $\left(\frac{-8}{7} * \frac{13}{9}\right) = \left(\frac{13}{9} * \frac{-8}{7}\right)$

Solution - LHS $\left(\frac{-8}{7} * \frac{13}{9}\right)$

$= \frac{-8*13}{7*9} = \frac{-104}{63}$

RHS $\left(\frac{13}{9} * \frac{-8}{7}\right)$

$= \frac{13*(-8)}{9*7} = \frac{-104}{63}$

Hence LHS = RHS

(c) $\left(\frac{-12}{5} * \frac{7}{-36}\right) = \left(\frac{7}{-36} * \frac{-12}{5}\right)$

Solution - LHS $\left(\frac{-12}{5} * \frac{7}{-36}\right)$

$= \frac{-12*7}{5*(-36)} = \frac{7}{15}$

RHS $\left(\frac{7}{-36} * \frac{-12}{5}\right)$

$= \frac{7*(-12)}{(-36)*5} = \frac{7}{15}$

Hence LHS = RHS

(d) $\left(-8 * \frac{-13}{12}\right) = \left(\frac{-13}{12} * (-8)\right)$

Solution - LHS $\left(-8 * \frac{-13}{12}\right)$

$= \frac{-8*(-13)}{1*12} = \frac{26}{3}$

RHS $\left(\frac{-13}{12} * (-8)\right)$

$= \frac{-13*(-8)}{12*1} = \frac{26}{3}$

Hence LHS = RHS

Question 3 - Verify each of the following:

(a) $\left(\frac{5}{7} * \frac{12}{13}\right) * \frac{7}{18} = \frac{5}{7} * \left(\frac{12}{13} * \frac{7}{18}\right)$

Solution - LHS $\left(\frac{5}{7} * \frac{12}{13}\right) * \frac{7}{18}$

$= \frac{60}{91} * \frac{7}{18} = \frac{10}{39}$

RHS $\frac{5}{7} * \left(\frac{12}{13} * \frac{7}{18}\right)$

$= \frac{5}{7} * \frac{14}{39} = \frac{10}{39}$

Hence LHS = RHS

(b) $\frac{-13}{24} * \left(\frac{-12}{5} * \frac{35}{36}\right) = \left(\frac{-13}{24} * \frac{-12}{5}\right) * \frac{35}{36}$

Solution - LHS $\frac{-13}{24} * \left(\frac{-12}{5} * \frac{35}{36}\right)$

$= \frac{-13}{24} * \frac{-7}{3} = \frac{91}{72}$

RHS $\left(\frac{-13}{24} * \frac{-12}{5}\right) * \frac{35}{36}$

$= \frac{13}{10} * \frac{35}{36} = \frac{91}{72}$

Hence LHS = RHS

(c) $\left(\frac{-9}{5} * \frac{-10}{3}\right) * \frac{21}{-4} = \frac{-9}{5} * \left(\frac{-10}{3} * \frac{21}{-4}\right)$

Solution - LHS $\left(\frac{-9}{5} * \frac{-10}{3}\right) * \frac{21}{-4}$

$= 6 * \frac{21}{-4} = \frac{-63}{2}$

RHS $\frac{-9}{5} * \left(\frac{-10}{3} * \frac{21}{-4}\right)$

$= \frac{-9}{5} * \frac{35}{2} = \frac{-63}{2}$

Hence LHS = RHS

Question 4 - Fill in the blanks:

(a) $\frac{-23}{17} * \frac{18}{35} = \frac{18}{35} * (....)$

Solution - Using commutative law,

$$\frac{-23}{17} * \frac{18}{35} = \frac{18}{35} * \left(\frac{-23}{17}\right)$$

(b) -38 * $\frac{-7}{19}$ = $\frac{-7}{19}$ * (….)

Solution - Using commutative law,

$$-38 * \frac{-7}{19} = \frac{-7}{19} * (-38)$$

(c) $\left(\frac{15}{7} * \frac{-21}{10}\right) * \frac{-5}{6}$ = (….)*$\left(\frac{-21}{10} * \frac{-5}{6}\right)$

Solution - Using associative law,

$$\left(\frac{15}{7} * \frac{-21}{10}\right) * \frac{-5}{6} = \left(\frac{15}{7}\right) * \left(\frac{-21}{10} * \frac{-5}{6}\right)$$

(d) $\frac{-12}{5}$ * $\left(\frac{4}{15} * \frac{25}{-16}\right)$ = $\left(\frac{-12}{5} * \frac{4}{15}\right)$*(….)

Solution - Using associative law,

$$\frac{-12}{5} * \left(\frac{4}{15} * \frac{25}{-16}\right) = \left(\frac{-12}{5} * \frac{4}{15}\right) * \left(\frac{25}{-16}\right)$$

Question 5 - Find the multiplicative inverse of:

(a) $\frac{13}{25}$

Solution - The multiplicative inverse of $\frac{13}{25}$ is $\frac{25}{13}$.

(b) $\frac{-17}{12}$

Solution - The multiplicative inverse of $\frac{-17}{12}$ is $\frac{-12}{17}$.

(c) $\frac{-7}{24}$

Solution - The multiplicative inverse of $\frac{-7}{24}$ is $\frac{-24}{7}$.

(d) 18

Solution - The multiplicative inverse of 18 is $\frac{1}{18}$

(e) -16

Solution - The multiplicative inverse of -16 is $\frac{-1}{16}$

(f) $\frac{-3}{-5}$

Solution - The multiplicative inverse of $\frac{-3}{-5}$ is $\frac{-5}{-3} = \frac{5}{3}$

(g) -1

Solution - The multiplicative inverse of -1 is 1

(h) $\frac{0}{2}$

Solution - The multiplicative inverse of $\frac{0}{2}$ does not exist

(i) $\frac{2}{-5}$

Solution - The multiplicative inverse of $\frac{2}{-5}$ is $\frac{-5}{2}$

(j) $\frac{-1}{8}$

Solution - The multiplicative inverse of $\frac{-1}{8}$ is $\frac{-8}{1} = $ -8

Question 6 - Find the value of:

(a) $\left(\frac{5}{8}\right)^{-1}$

Solution - Here, we have to find multiplicative inverse

The multiplicative inverse of $\frac{5}{8}$ is $\frac{8}{5}$

(b) $\left(\frac{-4}{9}\right)^{-1}$

Solution - The multiplicative inverse of $\frac{-4}{9}$ is $\frac{-9}{4}$

(c) $(-7)^{-1}$

Solution - The multiplicative inverse of -7 is $\frac{-1}{7}$

(d) $\left(\frac{1}{-3}\right)^{-1}$

Solution **-** The multiplicative inverse of $\frac{1}{-3}$ is -3

Question 7 - Verify the following:

(a) $\frac{3}{7} * \left(\frac{5}{6} + \frac{12}{13}\right) = \left(\frac{3}{7} * \frac{5}{6}\right) + \left(\frac{3}{7} * \frac{12}{13}\right)$

Solution - LHS $\frac{3}{7} * \left(\frac{5}{6} + \frac{12}{13}\right)$

$= \frac{3}{7} * \left(\frac{65+72}{78}\right) = \frac{3}{7} * \left(\frac{137}{78}\right) = \frac{137}{182}$ (as 3 is common factor between 3 and 78)

RHS $\left(\frac{3}{7} * \frac{5}{6}\right) + \left(\frac{3}{7} * \frac{12}{13}\right)$

$= \frac{5}{14} + \frac{36}{91} = \frac{65+72}{182} = \frac{137}{182}$ (as 3 is common factor between 3 and 6)

Hence LHS = RHS

(b) $\frac{-15}{4} * \left(\frac{3}{7} + \frac{-12}{5}\right) = \left(\frac{-15}{4} * \frac{3}{7}\right) + \left(\frac{-15}{4} * \frac{-12}{5}\right)$

Solution - LHS $\frac{-15}{4} * \left(\frac{3}{7} + \frac{-12}{5}\right) = \frac{-15}{4} * \left(\frac{3}{7} - \frac{12}{5}\right)$

$= \frac{-15}{4} * \left(\frac{15-84}{35}\right) = \frac{-15}{4} * \left(\frac{-69}{35}\right) = \frac{207}{28}$ (as 5 is common factor between 15 and 35)

RHS $\left(\frac{-15}{4} * \frac{3}{7}\right) + \left(\frac{-15}{4} * \frac{-12}{5}\right)$

$= \frac{-45}{28} + \frac{9}{1} = \frac{-45+252}{28}$ (as 5 is common factor between 15 and 5 & 4 is common factor of 12 and 4)

$= \frac{207}{28}$

Hence LHS = RHS

(c) $\left(\frac{-8}{3} + \frac{-13}{12}\right) * \frac{5}{6} = \left(\frac{-8}{3} * \frac{5}{6}\right) + \left(\frac{-13}{12} * \frac{5}{6}\right)$

Solution - LHS $\left(\frac{-8}{3} + \frac{-13}{12}\right) * \frac{5}{6} = \left(\frac{-8}{3} - \frac{13}{12}\right) * \frac{5}{6}$

$= \left(\frac{-32-13}{12}\right) * \frac{5}{6} = \frac{-45}{12} * \frac{5}{6} = \frac{-225}{72}$

RHS $\left(\frac{-8}{3} * \frac{5}{6}\right) + \left(\frac{-13}{12} * \frac{5}{6}\right)$

$$= \frac{-40}{18} + \frac{-65}{72} = \frac{-40}{18} - \frac{65}{72}$$

$$= \frac{-160 - 65}{72} = \frac{-225}{72}$$

Hence LHS = RHS

(d) $\frac{-16}{7} * (\frac{-8}{9} + \frac{-7}{6}) = \left(\frac{-16}{7} * \frac{-8}{9}\right) + \left(\frac{-16}{7} * \frac{-7}{6}\right)$

Solution - LHS $\frac{-16}{7} * (\frac{-8}{9} + \frac{-7}{6})$

$$= \frac{-16}{7} * (\frac{-8}{9} - \frac{7}{6}) = \frac{-16}{7} * (\frac{-16 - 21}{18})$$

$$= \frac{-16}{7} * (\frac{-37}{18}) = \frac{296}{63}$$

RHS $\left(\frac{-16}{7} * \frac{-8}{9}\right) + \left(\frac{-16}{7} * \frac{-7}{6}\right)$

$$= \frac{128}{63} + \frac{8}{3} = (\frac{128 + 168}{63})$$

$$= \frac{296}{63}$$

Hence LHS = RHS

Question 8 - Name the property of multiplication illustrated by each of the following statements:

(a) $\frac{-15}{8} * \frac{-12}{7} = \frac{-12}{7} * \frac{-15}{8}$

Commutative property

(b) $\left(\frac{-2}{3} * \frac{7}{9}\right) * \frac{-9}{5} = \frac{-2}{3} * \left(\frac{7}{9} * \frac{-9}{5}\right)$

Associative property

(c) $\frac{-3}{4} * (\frac{-5}{6} + \frac{7}{8}) = \left(\frac{-3}{4} * \frac{-5}{6}\right) + \left(\frac{-3}{4} * \frac{7}{8}\right)$

Distributive Property

(d) $\frac{-16}{9} * 1 = 1 * \frac{-16}{9} = \frac{-16}{9}$

Multiplicative identity property

(e) $\frac{-11}{15} * \frac{15}{-11} = \frac{15}{-11} * \frac{-11}{15} = 1$

Multiplicative inverse property

(f) $\frac{-7}{5} * 0$

Multiplicative property of 0

Question 9 - Fill in the blanks:

(a) The product of a rational number and its reciprocal is ……

Solution **-** 1

(b) Zero has ……reciprocal

Solution **-** No

(c)The numbers …. And …. Are their own reciprocals.

Solution **-** 1 and -1

(d) Zero is ….the reciprocal of any number.

Solution **-** Not

(e) The reciprocal of a, where a ≠ 0, is …..

Solution $\frac{1}{a}$

(f) The reciprocal of$\frac{1}{a}$, where a ≠ 0, is ….

Solution **-** a

(g) The reciprocal of a positive rational number is …..

Solution **-** Positive

(h) The reciprocal of a negative rational number is …..

Solution **-** Negative

Division of rational numbers

For two rational number $\frac{a}{b}$ and $\frac{c}{d}$ such that $\frac{c}{d} \neq 0$, we define $(\frac{a}{b} \div \frac{c}{d}) = (\frac{a}{b} * \frac{d}{c})$

Here, $\frac{a}{b}$ is called dividend, $\frac{c}{d}$ is called divisor and the result is known as quotient.

Properties of division:

Property 1 (closure property): For two rational number $\frac{a}{b}$ and $\frac{c}{d}$ such that $\frac{c}{d} \neq 0$, $(\frac{a}{b} \div \frac{c}{d})$ is also a rational number.

Property 2: For each rational number $\frac{a}{b}$, $(\frac{a}{b} \div 1) = \frac{a}{b}$

Property 3: For each non zero rational number $\frac{a}{b}$, $(\frac{a}{b} \div \frac{a}{b}) = 1$

Examples:

Example 1 - Divide

(a) $\frac{9}{16}$ by $\frac{5}{8}$

Solution **-** Let us covert $\div$ to $\times$,

$$\frac{9}{16} \div \frac{5}{8} = \frac{9}{16} \times \frac{8}{5} = \frac{72}{80}$$

As 8 is common divisor of 72 and 80, so we divide by 8 into lowest terms

We get $\frac{72}{80} = \frac{9}{10}$

(b) $\frac{-6}{25}$ by $\frac{3}{5}$

Solution **-** Let us covert $\div$ to $\times$,

$$\frac{-6}{25} \div \frac{3}{5} = \frac{-6}{25} \times \frac{5}{3} = \frac{-30}{75}$$

As 15 is common divisor of 30 and 75, so we divide by 15 into lowest terms

We get $\frac{-30}{75} = \frac{-2}{5}$

(c) $\frac{11}{24}$ by $\frac{-5}{8}$

Solution **-** Let us covert $\div$ to $\times$,

$$\frac{11}{24} \div \frac{-5}{8} = \frac{11}{24} \times \frac{8}{-5} = \frac{88}{-120}$$

As 8 is common divisor of 88 and 120, so we divide by 8 into lowest terms

We get $\frac{88}{-120} = \frac{11}{-15} = \frac{-11}{15}$

(d) $\frac{-9}{40}$ **by** $\frac{-3}{8}$

Solution - Let us covert $\div$ to $\times$,

$$\frac{-9}{40} \div \frac{-3}{8} = \frac{-9}{40} \times \frac{8}{-3} = \frac{-72}{-120} = \frac{72}{120}$$

As 24 is common divisor of 72 and 120, so we divide by 24 into lowest terms.

We get $\frac{72}{120} = \frac{3}{5}$

Example 2 - The product of two numbers is $\frac{-28}{27}$. If one of the numbers is $\frac{-4}{9}$, find the other.

Solution - Let the other number be x

Now it is given that:

One number $= \frac{-4}{9}$

Product of two numbers $= \frac{-28}{27}$

$$\Rightarrow \frac{-4}{9} * x = \frac{-28}{27}$$

$$\Rightarrow x = \frac{-28}{27} \div \frac{-4}{9} = \frac{-28}{27} * \frac{9}{-4} = \frac{-252}{-108} = \frac{252}{108}$$

As 36 is common divisor of 252 and 108, so we divide by 36 into lowest terms.

We get $x = \frac{252}{108} = \frac{7}{3}$

Example 3 - Fill in the blanks: $\frac{27}{16} \div (....) = \frac{-15}{8}$

Solution - Let missing number be x and $x = \frac{a}{b}$

Thus, $\frac{27}{16} \div x = \frac{-15}{8}$

$$\Rightarrow \frac{27}{16} \div \frac{a}{b} = \frac{-15}{8}$$

Let us covert $\div$ to $\times$,

$$\Rightarrow \frac{27}{16} * \frac{b}{a} = \frac{-15}{8}$$

$$\Rightarrow \frac{b}{a} = \frac{-15}{8} \div \frac{27}{16}$$

$$=> \frac{b}{a} = \frac{-15}{8} * \frac{16}{27} = \frac{-240}{216}$$

As 24 is common divisor of 240 and 216, so we divide by 24 into lowest terms.

We get $\frac{b}{a} = \frac{-240}{216} = \frac{-10}{9}$

But x $= \frac{a}{b} = \frac{9}{-10} = \frac{-9}{10}$

Exercise 1E

Question 1 - Simplify

(a) $\frac{4}{9} \div \frac{-5}{12}$

Solution **-** Let us covert ÷ to ×,

$$\frac{4}{9} \div \frac{-5}{12} = \frac{4}{9} \times \frac{12}{-5} = \frac{48}{-45} = \frac{-48}{45}$$

As 3 is common divisor of 48 and 45, so we divide by 3 into lowest terms.

We get $\frac{-48}{45} = \frac{-16}{15}$

(b) -8 $\div \frac{-7}{16}$

Solution - Let us covert ÷ to ×,

$$-8 \div \frac{-7}{16} = \text{-8} \times \frac{16}{-7} = \frac{-128}{-7} = \frac{128}{7}$$

(c) $\frac{-12}{7} \div$ **(-18)**

Solution **-** Let us covert ÷ to ×,

$$\frac{-12}{7} \div (\text{-18}) = \frac{-12}{7} \times \frac{1}{-18} = \frac{-12}{-126} = \frac{12}{126}$$

As 6 is common divisor of 12 and 126, so we divide by 6 into lowest terms.

We get $\frac{12}{126} = \frac{2}{21}$

(d) $\frac{-1}{10} \div \frac{-8}{5}$

Solution **-** Let us covert ÷ to ×,

$$\frac{-1}{10} \div \frac{-8}{5} = \frac{-1}{10} \times \frac{5}{-8} = \frac{-5}{-80} = \frac{5}{80}$$

As 5 is common divisor of 5 and 80, so we divide by 5 into lowest terms.

We get $\frac{5}{80} = \frac{1}{16}$

(e) $\frac{-16}{35} \div \frac{-15}{14}$

Solution **-** Let us covert $\div$ to $\times$,

$\frac{-16}{35} \div \frac{-15}{14} = \frac{-16}{35} \times \frac{14}{-15} = \frac{-224}{-525} = \frac{224}{525}$

As 7 is common divisor of 224 and 525, so we divide by 7 into lowest terms.

We get $\frac{224}{525} = \frac{32}{75}$

(f) $\frac{-65}{14} \div \frac{13}{7}$

Solution **-** Let us covert $\div$ to $\times$,

$\frac{-65}{14} \div \frac{13}{7} = \frac{-65}{14} \times \frac{7}{13} = \frac{-455}{182}$

As 91 is common divisor of 455 and 182, so we divide by 91 into lowest terms.

We get $\frac{-455}{182} = \frac{-5}{2}$

Question 2 - Verify whether the given statement is true or false:

(a) $\frac{13}{5} \div \frac{26}{10} = \frac{26}{10} \div \frac{13}{5}$

Solution - LHS $\frac{13}{5} \div \frac{26}{10}$

Let us covert $\div$ to $\times$,

$\frac{13}{5} \div \frac{26}{10} = \frac{13}{5} \times \frac{10}{26} = \frac{130}{130} = 1$

RHS $\frac{26}{10} \div \frac{13}{5}$

Let us covert $\div$ to $\times$,

$\frac{26}{10} \div \frac{13}{5} = \frac{26}{10} \times \frac{5}{13} = \frac{130}{130} = 1$

Since LHS = RHS

Hence the statement is true.

(b) -9 ÷ $\frac{3}{4}$ = $\frac{3}{4}$ ÷ (-9)

Solution - LHS -9 ÷ $\frac{3}{4}$

Let us covert ÷ to ×,

-9 ÷ $\frac{3}{4}$ = -9 × $\frac{4}{3}$ = $\frac{-36}{3}$ = -12

RHS $\frac{3}{4}$ ÷ (-9)

Let us covert ÷ to ×,

$\frac{3}{4}$ ÷ (-9) = $\frac{3}{4}$ × $\frac{1}{-9}$ = $\frac{3}{-36}$ = $\frac{1}{-12}$ = $\frac{-1}{12}$

Since LHS ≠ RHS

Hence the statement is false.

(c) $\frac{-8}{9}$ ÷ $\frac{-4}{3}$ = $\frac{-4}{3}$ ÷ $\frac{-8}{9}$

Solution - LHS $\frac{-8}{9}$ ÷ $\frac{-4}{3}$

Let us covert ÷ to ×,

$\frac{-8}{9}$ ÷ $\frac{-4}{3}$ = $\frac{-8}{9}$ × $\frac{3}{-4}$ = $\frac{-24}{-36}$ = $\frac{24}{36}$

As 12 is common divisor of 24 and 36, so we divide by 12 into lowest terms.

We get $\frac{24}{36}$ = $\frac{2}{3}$

RHS $\frac{-4}{3}$ ÷ $\frac{-8}{9}$

Let us covert ÷ to ×,

$\frac{-4}{3}$ ÷ $\frac{-8}{9}$ = $\frac{-4}{3}$ × $\frac{9}{-8}$ = $\frac{-36}{-24}$ = $\frac{36}{24}$

As 12 is common divisor of 36 and 24, so we divide by 12 into lowest terms.

We get $\frac{36}{24}$ = $\frac{3}{2}$

Since LHS ≠ RHS

Hence the statement is false.

(d) $\frac{-7}{24} \div \frac{3}{-16} = \frac{3}{-16} \div \frac{-7}{24}$

Solution - LHS $\frac{-7}{24} \div \frac{3}{-16}$

Let us covert $\div$ to $\times$,

$\frac{-7}{24} \div \frac{3}{-16} = \frac{-7}{24} \times \frac{-16}{3} = \frac{112}{72}$

As 8 is common divisor of 112 and 72, so we divide by 8 into lowest terms.

We get $\frac{112}{72} = \frac{14}{9}$

RHS $\frac{3}{-16} \div \frac{-7}{24}$

Let us covert $\div$ to $\times$,

$\frac{3}{-16} \div \frac{-7}{24} = \frac{3}{-16} \times \frac{24}{-7} = \frac{72}{112}$

As 8 is common divisor of 72 and 112, so we divide by 8 into lowest terms.

We get $\frac{72}{112} = \frac{9}{14}$

Since LHS $\neq$ RHS

Hence the statement is false.

Question 3 - Verify whether the given statement is true or false:

(a) $(\frac{5}{9} \div \frac{1}{3}) \div \frac{5}{2} = \frac{5}{9} \div (\frac{1}{3} \div \frac{5}{2})$

Solution - LHS $(\frac{5}{9} \div \frac{1}{3}) \div \frac{5}{2}$

Let us convert $\div$ to $\times$,

$(\frac{5}{9} \div \frac{1}{3}) \div \frac{5}{2} = (\frac{5}{9} * \frac{3}{1}) \div \frac{5}{2} = \frac{15}{9} \div \frac{5}{2}$

Again convert $\div$ to $\times$,

$\frac{15}{9} \div \frac{5}{2} = \frac{15}{9} * \frac{2}{5} = \frac{30}{45}$

As 15 is common divisor of 30 and 45, so we divide by 15 into lowest terms.

We get $\frac{30}{45} = \frac{2}{3}$

RHS $\frac{5}{9} \div (\frac{1}{3} \div \frac{5}{2})$

Let us covert $\div$ to $\times$,

$\frac{5}{9} \div (\frac{1}{3} \div \frac{5}{2}) = \frac{5}{9} \div (\frac{1}{3} * \frac{2}{5})$

$\frac{5}{9} \div \frac{2}{15}$

Again convert $\div$ to $\times$,

$\frac{5}{9} * \frac{15}{2} = \frac{75}{18}$

As 3 is common divisor of 75 and 18, so we divide by 3 into lowest terms.

We get $\frac{75}{18} = \frac{25}{6}$

Since LHS $\neq$ RHS

Hence the statement is false.

(b) $\{(-16) \div \frac{6}{5}\} \div \frac{-9}{10} = (-16) \div \{\frac{6}{5} \div \frac{-9}{10}\}$

Solution - LHS $\{(-16) \div \frac{6}{5}\} \div \frac{-9}{10}$

Let us convert $\div$ to $\times$,

$\{(-16) \div \frac{6}{5}\} \div \frac{-9}{10} = \{(-16) * \frac{5}{6}\} \div \frac{-9}{10}$

$\frac{-80}{6} \div \frac{-9}{10}$

Again convert $\div$ to $\times$,

$\frac{-80}{6} * \frac{10}{-9} = \frac{-800}{-54} = \frac{800}{54}$

As 2 is common divisor of 800 and 54, so we divide by 2 into lowest terms.

We get $\frac{800}{54} = \frac{400}{27}$

RHS $(-16) \div \{\frac{6}{5} \div \frac{-9}{10}\}$

Let us convert $\div$ to $\times$,

$(-16) \div \{\frac{6}{5} \div \frac{-9}{10}\} = (-16) \div \{\frac{6}{5} * \frac{10}{-9}\}$

$(-16) \div \frac{60}{-45}$

Again convert $\div$ to $\times$,

$(-16) * \frac{-45}{60} = \frac{720}{60} = \frac{72}{6}$

As 6 is common divisor of 72 and 6, so we divide by 6 into lowest terms.

We get $\frac{72}{6} = 12$

Since LHS $\neq$ RHS

Hence the statement is false.

(c) $(\frac{-3}{5} \div \frac{-12}{35}) \div \frac{1}{14} = \frac{-3}{5} \div (\frac{-12}{35} \div \frac{1}{14})$

Solution - LHS $(\frac{-3}{5} \div \frac{-12}{35}) \div \frac{1}{14}$

Let us convert $\div$ to $\times$,

$(\frac{-3}{5} \div \frac{-12}{35}) \div \frac{1}{14} = (\frac{-3}{5} * \frac{35}{-12}) \div \frac{1}{14} = \frac{-105}{-60} \div \frac{1}{14}$

Again convert $\div$ to $\times$,

$\frac{105}{60} \div \frac{1}{14} = \frac{105}{60} * 14 = \frac{1470}{60} = \frac{147}{6}$

As 3 is common divisor of 147 and 6, so we divide by 3 into lowest terms.

We get $\frac{147}{6} = \frac{49}{2}$

RHS $\frac{-3}{5} \div (\frac{-12}{35} \div \frac{1}{14})$

Let us covert $\div$ to $\times$,

$\frac{-3}{5} \div (\frac{-12}{35} \div \frac{1}{14}) = \frac{-3}{5} \div \left(\frac{-12}{35} * \frac{14}{1}\right)$

$\frac{-3}{5} \div (\frac{-168}{35})$

Again convert $\div$ to $\times$,

$\frac{-3}{5} * \frac{35}{-168} = \frac{-105}{-840} = \frac{105}{840}$

As 105 is common divisor of 105 and 840, so we divide by 105 into lowest terms.

We get $\frac{105}{840} = \frac{1}{8}$

Since LHS $\neq$ RHS

Hence the statement is false.

Question 4 - The product of two rational numbers is -9. If one of the numbers is -12, find the other.

Solution **-** Let the other number be x

Now it is given that:

One number = -12

Product of two numbers = -9

=> -12* x = -9

=> x = -9 $\div$ (-12)= -9* $\frac{1}{-12} = \frac{-9}{-12} = \frac{9}{12}$

As 3 is common divisor of 9 and 12, so we divide by 3 into lowest terms.

We get x = $\frac{9}{12} = \frac{3}{4}$

Question 5 - The product of two rational numbers is $\frac{-16}{9}$. If one of the numbers is $\frac{-4}{3}$, find the other.

Solution **-** Let the other number be x

Now it is given that:

One number = $\frac{-4}{3}$

Product of two numbers = $\frac{-16}{9}$

=> $\frac{-4}{3}$ * x = $\frac{-16}{9}$

=> x = $\frac{-16}{9} \div \frac{-4}{3} = \frac{-16}{9} * \frac{3}{-4} = \frac{-48}{-36} = \frac{48}{36}$

As 12 is common divisor of 48 and 36, so we divide by 12 into lowest terms.

We get x = $\frac{48}{36} = \frac{4}{3}$

Question 6 - By what rational number should we multiply $\frac{-15}{56}$ to get $\frac{-5}{7}$?

Solution **-** Let the required number be x

Then according to given question,

$\frac{-15}{56} * x = \frac{-5}{7}$

$=> x = \frac{-5}{7} \div \frac{-15}{56}$

$=> x = \frac{-5}{7} * \frac{56}{-15}$

$=> x = \frac{-280}{-105} = \frac{280}{105}$

As 35 is common divisor of 280 and 105, so we divide by 35 into lowest terms.

We get $x = \frac{280}{105} = \frac{8}{3}$

Question 7 - By what rational number should $\frac{-8}{39}$ be multiplied to obtain $\frac{1}{26}$

Solution **-** Let the required number be x

Then according to given question,

$\frac{-8}{39} * x = \frac{1}{26}$

$=> x = \frac{1}{26} \div \frac{-8}{39}$

$=> x = \frac{1}{26} * \frac{39}{-8}$

$=> x = \frac{39}{-208} = \frac{-39}{208}$

As 13 is common divisor of 39 and 208, so we divide by 13 into lowest terms.

We get $x = \frac{-39}{208} = \frac{-3}{16}$

Question 8 - By what number should $\frac{-33}{8}$ be divided to get $\frac{-11}{2}$?

Solution **-** Let the required number be x and $x = \frac{a}{b}$

Then, according to given question

$$\frac{-33}{8} \div x = \frac{-11}{2}$$

$$=> \frac{-33}{8} \div \frac{a}{b} = \frac{-11}{2}$$

$$=> \frac{-33}{8} * \frac{b}{a} = \frac{-11}{2}$$

$$=> \frac{b}{a} = \frac{-11}{2} \div \frac{-33}{8}$$

$$=> \frac{b}{a} = \frac{-11}{2} * \frac{8}{-33} = \frac{-88}{-66} = \frac{88}{66}$$

As 22 is common divisor of 88 and 66, so we divide by 22 into lowest terms.

We get $\frac{b}{a} = \frac{88}{66} = \frac{4}{3}$

But $x = \frac{a}{b} = \frac{3}{4}$

Question 9 - Divide the sum of $\frac{13}{5}$ and $\frac{-12}{7}$ by the product of $\frac{-31}{7}$ and $\frac{1}{-2}$

Solution **-** According to given question,

We will find: $(\frac{13}{5} + \frac{-12}{7}) \div (\frac{-31}{7} * \frac{1}{-2})$

Solving brackets, we get,

$$(\frac{13}{5} - \frac{12}{7}) \div (\frac{-31}{-14})$$

$$= (\frac{91-60}{35}) \div (\frac{31}{14})$$

$$= \frac{31}{35} \div \frac{31}{14}$$

$$= \frac{31}{35} * \frac{14}{31}$$

$$= \frac{14}{35}$$

As 7 is common divisor of 14 and 35, so we divide by 7 into lowest terms.

We get $\frac{14}{35} = \frac{2}{5}$

Question 10 - Divide the sum of $\frac{65}{12}$ and $\frac{8}{3}$ by their difference.

Solution **-** According to given question,

We will find: $(\frac{65}{12}+\frac{8}{3}) \div (\frac{65}{12} -\frac{8}{3})$

Solving brackets, we get,

$=> (\frac{65+32}{12}) \div (\frac{65-32}{12})$

$=> \frac{97}{12} \div \frac{33}{12} = \frac{97}{12} * \frac{12}{33}$

$=> \frac{97}{33}$

Question 11 - Fill in the blanks:

(a) $\frac{9}{8} \div (.....) = \frac{-3}{2}$

Solution - Let missing number be x and $x = \frac{a}{b}$

Thus, $\frac{9}{8} \div x = \frac{-3}{2}$

$=> \frac{9}{8} \div \frac{a}{b} = \frac{-3}{2}$

Let us convert $\div$ to $\times$,

$=> \frac{9}{8} * \frac{b}{a} = \frac{-3}{2}$

$=> \frac{b}{a} = \frac{-3}{2} \div \frac{9}{8}$

$=> \frac{b}{a} = \frac{-3}{2} * \frac{8}{9} = \frac{-24}{18}$

As 6 is common divisor of 24 and 18, so we divide by 6 into lowest terms.

We get $\frac{b}{a} = \frac{-24}{18} = \frac{-4}{3}$

But $x = \frac{a}{b} = \frac{3}{-4} = \frac{-3}{4}$

(b) $(....) \div (\frac{-7}{5}) = \frac{10}{19}$

Solution **-** Let missing number be x

Then, $x \div (\frac{-7}{5}) = \frac{10}{19}$

$\Rightarrow x * \dfrac{5}{-7} = \dfrac{10}{19}$

$\Rightarrow x = \dfrac{10}{19} \div \dfrac{5}{-7}$

$\Rightarrow x = \dfrac{10}{19} * \dfrac{-7}{5}$

$\Rightarrow x = \dfrac{-70}{95}$

As 5 is common divisor of 70 and 95, so we divide by 5 into lowest terms.

We get $x = \dfrac{-70}{95} = \dfrac{-14}{19}$

(c) (….) ÷ (-3) = $\dfrac{-4}{15}$

Solution - Let missing number be x

Then, $x \div (-3) = \dfrac{-4}{15}$

$\Rightarrow x * \dfrac{1}{-3} = \dfrac{-4}{15}$

$\Rightarrow x = \dfrac{-4}{15} \div \dfrac{1}{-3}$

$\Rightarrow x = \dfrac{-4}{15} * \dfrac{-3}{1}$

$\Rightarrow x = \dfrac{12}{15}$

As 3 is common divisor of 12 and 15, so we divide by 3 into lowest terms.

We get $x = \dfrac{12}{15} = \dfrac{4}{5}$

(d) (-12) ÷ (…..) = $\dfrac{-6}{5}$

Solution - Let missing number be x and $x = \dfrac{a}{b}$

Thus, $(-12) \div x = \dfrac{-6}{5}$

$\Rightarrow (-12) \div \dfrac{a}{b} = \dfrac{-6}{5}$

Let us convert ÷ to ×,

$\Rightarrow (-12) * \dfrac{b}{a} = \dfrac{-6}{5}$

$=> \frac{b}{a} = \frac{-6}{5} \div (-12)$

$=> \frac{b}{a} = \frac{-6}{5} * \frac{1}{-12} = \frac{-6}{-60} = \frac{6}{60}$

As 6 is common divisor of 6 and 60, so we divide by 6 into lowest terms.

We get $\frac{b}{a} = \frac{6}{60} = \frac{1}{10}$

But $x = \frac{a}{b} = 10$

Question 12 (a) Are rational numbers always closed under division?

Solution **-** No, rational numbers are not always closed under division.

If we take two numbers 5 and 0, then $\frac{5}{0} = \infty$ which is not defined

(b) Are rational numbers always commutative under division?

Solution **-** No, rational numbers are not always commutative under division.

For ex: we take two numbers say $\frac{2}{3}$ and $\frac{4}{5}$

Then, $\frac{2}{3} \div \frac{4}{5} \neq \frac{4}{5} \div \frac{2}{3}$

Let us prove this

LHS $\frac{2}{3} \div \frac{4}{5} = \frac{2}{3} * \frac{5}{4} = \frac{10}{12} = \frac{5}{6}$

RHS $\frac{4}{5} \div \frac{2}{3} = \frac{4}{5} * \frac{2}{3} = \frac{8}{15}$

LHS $\neq$ RHS

(c) Are rational numbers always associative under division?

Solution **-** No, rational numbers are not always associative under division.

For ex: we take three numbers say $\frac{1}{2}, \frac{2}{3}$ and $\frac{3}{4}$

Then, $(\frac{1}{2} \div \frac{2}{3}) \div \frac{3}{4} \neq \frac{1}{2} \div (\frac{2}{3} \div \frac{3}{4})$

Let us prove this

LHS $(\frac{1}{2} \div \frac{2}{3}) \div \frac{3}{4} = (\frac{1}{2} * \frac{3}{2}) \div \frac{3}{4} = \frac{3}{4} \div \frac{3}{4} = \frac{3}{4} * \frac{4}{3} = 1$

RHS $\frac{1}{2} \div (\frac{2}{3} \div \frac{3}{4}) = \frac{1}{2} \div (\frac{2}{3} * \frac{4}{3}) = \frac{1}{2} \div \frac{8}{9} = \frac{1}{2} * \frac{9}{8} = \frac{9}{16}$

LHS $\neq$ RHS

(d) Can we divide 1 by 0?

Solution **-** No, we cannot divide 1 by 0.

Because if we divide 1 by 0, then $\frac{1}{0} = \infty$ which is not defined.

Methods to find rational numbers between two given rational numbers:

Method 1: If x and y are two rational numbers such that x < y then $\frac{1}{2}$(x + y) is a rational number between x and y. This method is used to find smaller number of rational numbers.

Method 2: If we want to find larger number of rational numbers, then we multiply the numerator and denominator of both the rational number by a constant number in such a way that the denominator remains the same.

Example 1 - Find a rational number lying between $\frac{1}{3}$ and $\frac{1}{2}$

Solution **-** A rational number between $\frac{1}{3}$ and $\frac{1}{2}$ is $\frac{1}{2}(\frac{1}{3} + \frac{1}{2})$

$= \frac{1}{2}(\frac{2+3}{6}) = \frac{1}{2} * \frac{5}{6} = \frac{5}{12}$

Example 2 - Find three rational numbers lying between 3 and 4

Solution **-** A rational number between 3 and 4 is $\frac{1}{2}(3+4) = \frac{1}{2} * 7 = \frac{7}{2}$

Now $3 < \frac{7}{2} < 4$

Also A rational number between 3 and $\frac{7}{2}$ is $\frac{1}{2}(3 + \frac{7}{2}) = \frac{1}{2}(\frac{6+7}{2})$

$= \frac{1}{2} * \frac{13}{2} = \frac{13}{4}$

And a rational number between $\frac{7}{2}$ and 4 is $\frac{1}{2}(\frac{7}{2} + 4) = \frac{1}{2}(\frac{7+8}{2})$

$= \frac{1}{2} * \frac{15}{2} = \frac{15}{4}$

Thus three rational numbers lying between 3 and 4 are $\frac{13}{4}, \frac{7}{2}, \frac{15}{4}$

Example 3 - Find 20 rational numbers between $\frac{-5}{6}$ and $\frac{5}{8}$

Solution **-** Firstly we will make the denominator same by taking LCM of 6 and 8

LCM of 6 and 8 is 24

$$\frac{-5}{6} = \frac{-5*4}{6*4} = \frac{-20}{24}$$

$$\frac{5}{8} = \frac{5*3}{8*3} = \frac{15}{24}$$

Now 20 rational numbers between $\frac{-5}{6}$ and $\frac{5}{8}$ are as follows:

$$\frac{-19}{24}, \frac{-18}{24}, \frac{-17}{24}, \frac{-16}{24}, \frac{-15}{24}, \frac{-14}{24}, \frac{-13}{24}, \frac{-12}{24}, \frac{-11}{24}, \frac{-10}{24}, \frac{-9}{24}, \frac{-8}{24}, \frac{-7}{24}, \frac{-6}{24}, \frac{-5}{24}, \frac{-4}{24}, \frac{-3}{24}, \frac{-2}{24}, \frac{-1}{24}, \frac{1}{24}$$

Example 4 - Find 15 rational numbers between -2 and 0

Solution **-** Here we will multiply and divide by a constant in such a way that the denominator remains the same.

Let constant number be 10

Then $\frac{-2*10}{1*10} = \frac{-20}{10}$

And $\frac{0*10}{1*10} = \frac{0}{10}$

Now 15 rational numbers between -2 and 0 are as follows:

$$\frac{-19}{10}, \frac{-18}{10}, \frac{-17}{10}, \frac{-16}{10}, \frac{-15}{10}, \frac{-14}{10}, \frac{-13}{10}, \frac{-12}{10}, \frac{-11}{10}, \frac{-10}{10}, \frac{-9}{10}, \frac{-8}{10}, \frac{-7}{10}, \frac{-6}{10}, \frac{-5}{10}.$$

Example 5 - Write 9 rational numbers between 1 and 2

Solution **-** Here we will multiply and divide by a constant in such a way that the denominator remains the same.

Let constant number be 10

Then $\frac{1*10}{1*10} = \frac{10}{10}$

And $\frac{2*10}{1*10} = \frac{20}{10}$

Now 9 rational numbers between 1 and 2 are as follows:

$$\frac{11}{10}, \frac{12}{10}, \frac{13}{10}, \frac{14}{10}, \frac{15}{10}, \frac{16}{10}, \frac{17}{10}, \frac{18}{10}, \frac{19}{10}$$

Exercise 1F

Question 1 - Find a rational number between $\frac{1}{4}$ and $\frac{1}{3}$

Solution - A rational number between $\frac{1}{4}$ and $\frac{1}{3}$ is $\frac{1}{2}(\frac{1}{4} + \frac{1}{3})$

$= \frac{1}{2}(\frac{3+4}{12}) = \frac{7}{24}$

Question 2 - Find a rational number between 2 and 3

Solution - A rational number between 2 and 3 is $\frac{1}{2}$(2+3) $= \frac{5}{2}$

Question 3 - Find a rational number between $\frac{-1}{3}$ and $\frac{1}{2}$

Solution - A rational number between $\frac{-1}{3}$ and $\frac{1}{2}$ is $\frac{1}{2}(\frac{-1}{3} + \frac{1}{2})$

$= \frac{1}{2}(\frac{-2+3}{6}) = \frac{1}{2} * \frac{1}{6} = \frac{1}{12}$

Question 4 - Find two rational numbers between -3 and -2

Solution - A rational number between -3 and -2 is $\frac{1}{2}$(-3-2) $= \frac{1}{2}*$ (-5) $= \frac{-5}{2}$

Now -3 $< \frac{-5}{2} <$ -2

Rational number between -3 and $\frac{-5}{2}$ is $\frac{1}{2}$ is $\frac{1}{2}(\frac{-5}{2} + (-3))$

$= \frac{1}{2}(\frac{-5}{2} - \frac{3}{1}) = \frac{1}{2}(\frac{-5-6}{2}) = \frac{1}{2}(\frac{-11}{2}) = \frac{-11}{4}$

Thus two rational numbers between -3 and -2 are as follows:

$\frac{-5}{2}$ And $\frac{-11}{4}$

Question -5 Find three rational numbers between 4 and 5

Solution - A rational number between 4 and 5 is $\frac{1}{2}$(4+5) $= \frac{1}{2}* 9 = \frac{9}{2}$

Now $4 < \frac{9}{2} < 5$

Also A rational number between 4 and $\frac{9}{2}$ is $\frac{1}{2}(4+\frac{9}{2}) = \frac{1}{2}(\frac{8+9}{2})$

$= \frac{1}{2} * \frac{17}{2} = \frac{17}{4}$

And a rational number between $\frac{9}{2}$ and 5 is $\frac{1}{2}(\frac{9}{2} + 5) = \frac{1}{2}(\frac{9+10}{2})$

$= \frac{1}{2} * \frac{19}{2} = \frac{19}{4}$

Thus three rational numbers lying between 3 and 4 are as follows:

$\frac{17}{4}, \frac{9}{2}$ and $\frac{19}{4}$

Question 6 - Find three rational numbers between $\frac{2}{3}$ and $\frac{3}{4}$

Solution - A rational number between $\frac{2}{3}$ and $\frac{3}{4}$ is $\frac{1}{2}(\frac{2}{3} + \frac{3}{4}) = \frac{1}{2}(\frac{8+9}{12}) = \frac{1}{2}(\frac{17}{12}) = \frac{17}{24}$

Now $\frac{2}{3} < \frac{17}{24} < \frac{3}{4}$

Also A rational number between $\frac{2}{3}$ and $\frac{17}{24}$ is $\frac{1}{2}(\frac{2}{3} + \frac{17}{24}) = \frac{1}{2}(\frac{16+17}{24})$

$= \frac{1}{2} * \frac{33}{24} = \frac{33}{48}$

And a rational number between $\frac{17}{24}$ and $\frac{3}{4}$ is $\frac{1}{2}(\frac{17}{24} + \frac{3}{4}) = \frac{1}{2}(\frac{17+18}{24})$

$= \frac{1}{2} * \frac{35}{24} = \frac{35}{4}$

Thus three rational numbers lying between $\frac{2}{3}$ and $\frac{3}{4}$ are as follows:

$\frac{33}{48}, \frac{17}{24}$ and $\frac{35}{4}$

Question 7 - Find 10 rational numbers between $\frac{-3}{4}$ and $\frac{5}{6}$

Solution - Firstly we will make the denominator same by taking LCM of 4 and 6

LCM of 4 and 6 is 12

$\frac{-3}{4} = \frac{-3*3}{4*3} = \frac{-9}{12}$

$\frac{5}{6} = \frac{5*2}{6*2} = \frac{10}{12}$

Now 10 rational numbers between $\frac{-3}{4}$ and $\frac{5}{6}$ are as follows:

$\frac{-8}{12}, \frac{-7}{12}, \frac{-6}{12}, \frac{-5}{12}, \frac{-4}{12}, \frac{-3}{12}, \frac{-2}{12}, \frac{-1}{12}, 0, \frac{1}{12}$

Question 8 - Find 12 rational numbers between -1 and 2

Solution **-** Here we will multiply and divide by a constant in such a way that the denominator remains the same.

Let constant number be 10

Then $\frac{-1*10}{1*10} = \frac{-10}{10}$

And $\frac{2*10}{1*10} = \frac{20}{10}$

Now 12 rational numbers between -1 and 2 are as follows:

$\frac{-9}{10}, \frac{-8}{10}, \frac{-7}{10}, \frac{-6}{10}, \frac{-5}{10}, \frac{-4}{10}, \frac{-3}{10}, \frac{-2}{10}, \frac{-1}{10}, 0, \frac{1}{10}, \frac{2}{10}$

Word problems:

Exercise 1G

Question 1 - From a rope 11 m long, two pieces of lengths $2\frac{3}{5}$ m and $3\frac{3}{10}$ m are cut off. What is the length of the remaining rope?

Solution **-** Total length of rope = 11 m

Total length of cut off pieces = $2\frac{3}{5} + 3\frac{3}{10}$ ($2\frac{3}{5}$ means $\frac{(2*5)+3}{5}$)

$= \frac{13}{5} + \frac{33}{10} = \frac{26+33}{10} = \frac{59}{10}$ (LCM of 5 and 10 is 10)

Thus the length of remaining rope = total length of rope – total cut off length of tope

$$= 11 - \frac{59}{10} = \frac{110-59}{10} = \frac{51}{10} = 5\frac{1}{10} \text{ m}$$

Question 2 - A drum full of rice weighs $40\frac{1}{6}$ kg. If the drum weighs $13\frac{3}{4}$ kg, find the weight of rice in the drum.

Solution - Weight of drum full of rice = $40\frac{1}{6}$ kg

Weight of drum = $13\frac{3}{4}$ kg

Thus weight of rice = weight of drum full of rice - weight of drum

$$= (40\frac{1}{6} - 13\frac{3}{4}) = (\frac{241}{6} - \frac{55}{4})$$

$$= \frac{482-165}{12} = \frac{317}{12} = 26\frac{5}{12} \text{ kg}$$

Question 3 - A basket contains three types of fruits weighing $19\frac{1}{3}$ kg in all. If $8\frac{1}{9}$ kg of these be apples, $3\frac{1}{6}$ kg is oranges and the rest pears, what is the weight of the pears in the basket?

Solution - Weight of all fruits in basket = $19\frac{1}{3}$ kg

Weight of apples = $8\frac{1}{9}$ kg

Weight of oranges = $3\frac{1}{6}$ kg

Weight of pears = Weight of all fruits in basket – (Weight of apples + Weight of oranges)

$$= 19\frac{1}{3} - (8\frac{1}{9} + 3\frac{1}{6})$$

We solve bracket first

$$= \frac{58}{3} - (\frac{73}{9} + \frac{19}{6}) = \frac{58}{3} - (\frac{146+57}{18}) = \frac{58}{3} - \frac{203}{18}$$

$$= \frac{348-203}{18} = \frac{145}{18} = 8\frac{1}{18} \text{ kg}$$

Question 4 - On one day a rickshaw puller earned Rs160. Out of his earnings he spent Rs$26\frac{3}{5}$ on tea and snacks, Rs$50\frac{1}{2}$ on food and Rs$16\frac{2}{5}$ on repairs of the rickshaw. How much did he save on that day?

Solution **-** Total earning of rickshaw puller = Rs160

Spent on tea and snacks = Rs$26\frac{3}{5}$

Spent on food = Rs$50\frac{1}{2}$

Spent on repairs of the rickshaw = Rs$16\frac{2}{5}$

Total expenditure = $(26\frac{3}{5} + 50\frac{1}{2} + 16\frac{2}{5})$

$$=\left(\frac{133}{5}+\frac{101}{2}+\frac{82}{5}\right)=\left(\frac{266+505+164}{10}\right)$$

$$=\frac{935}{10}=\text{Rs}\frac{187}{2}$$

Thus, Savings = Total earning – Total expenditure

$$=(160-\frac{187}{2})=(\frac{320-187}{2})$$

$$=\frac{133}{2}=\text{Rs}66\frac{1}{2}$$

Question -5 Find the cost of $3\frac{2}{5}$ meters of cloth at $Rs63\frac{3}{4}$ per meter.

Solution - Note: when we are given the cost of 1 m and ask to find cost of more meters then we use multiply.

Here cost of 1 meter cloth = $Rs63\frac{3}{4}$

Thus cost of $3\frac{2}{5}$ meter cloth = $63\frac{3}{4}*3\frac{2}{5}$

$$=\frac{255}{4}*\frac{17}{5}=\frac{51*17}{4*1}=\frac{867}{4}=\text{Rs}216\frac{3}{4}\quad(\text{5 is the common divisor of 255 and 5})$$

Question 6 - A car is moving at an average speed of $60\frac{2}{5}$ km/hr. How much distance will it cover in $6\frac{1}{4}$ hours?

Solution - Speed of car = $60\frac{2}{5}$ km/hrs.

Time taken = $6\frac{1}{4}$ hours

Distance covered =?

As we know that speed $=\frac{distance}{time}$

Thus distance = speed * time

Distance = $60\frac{2}{5}*6\frac{1}{4}=\frac{302}{5}*\frac{25}{4}$

$$=\frac{151*5}{1*2}=\frac{755}{2}=377\frac{1}{2}\text{ km}\quad(\text{2 is common divisor of 302 \& 4 and 5 is common divisor of 25 and 5})$$

Question 7 - Find the area of a rectangular park which is $36\frac{3}{5}$ m long and $16\frac{2}{3}$ m broad.

Solution - We are given that Length of park = $36\frac{3}{5}$ m

And breadth of park = $16\frac{2}{3}$ m

Since Area of rectangular park = length * breadth

Therefore, Area = $36\frac{3}{5} * 16\frac{2}{3}$

$= (\frac{183}{5} * \frac{50}{3}) = (61*10) = 610\ m^2$

(3 is common divisor of 183 & 3 and 5 is common divisor of 50 & 3)

Question 8 - Find the area of a square plot of land whose each side measures $8\frac{1}{2}$ meters.

Solution - Side of square plot of land = $8\frac{1}{2}$ m = $\frac{17}{2}$ m

Since area of square = $(side)^2$

Therefore area of square plot of land = $(\frac{17}{2})^2$

$= \frac{17}{2} * \frac{17}{2} = \frac{289}{4} = 72\frac{1}{4}\ m^2$

Question 9 - One litre of petrol costs Rs$63\frac{3}{4}$. What is the cost of 34 litres of petrol?

Solution - Cost of 1 litre of petrol = Rs$63\frac{3}{4}$ = Rs$\frac{255}{4}$

Cost of 34 litres of petrol = $\frac{255}{4} * 34 = \frac{255*17}{2} = \frac{4335}{2} = $ Rs$2167\frac{1}{2}$

(2 is common divisor of 34 and 2)

Question 10 - An aeroplane covers 1020 km in an hour. How much distance will it cover in $4\frac{1}{6}$ hours?

Solution - Distance covered in 1 hour = 1020 km

Distance covered in $4\frac{1}{6}$ hours = $1020 * 4\frac{1}{6}$

$= 1020 * \frac{25}{6} = (170*25) = 4250$ km

(6 is common divisor of 1020 and 6)

Question 11 - The cost of $3\frac{1}{2}$ meters of cloth is Rs$166\frac{1}{4}$. What is the cost of one meter of cloth?

Solution **-** Note: when we are asked to find 1 meter cost and given with cost of more meters then we use divide

Cost of $3\frac{1}{2}$ m cloth = Rs$166\frac{1}{4}$

Cost of 1 m cloth = $166\frac{1}{4} \div 3\frac{1}{2}$

$= \frac{665}{4} \div \frac{7}{2}$

Convert $\div$ by $\times$

$= \frac{665}{4} * \frac{2}{7} = \frac{95*1}{2*1} = \frac{95}{2} = \text{Rs}47\frac{1}{2}$

(7 is common divisor of 665 & 7 and 2 is common divisor of 2 &4)

Question 12 - A cord of length $71\frac{1}{2}$ m has been cut into 26 pieces of equal length. What is the length of each piece?

Solution **-** Total length = $71\frac{1}{2}$ m

Number of pieces = 26

Length of 1 piece = $71\frac{1}{2} \div 26$

$= \frac{143}{2} \div 26$

Convert $\div$ by $\times$

$= \frac{143}{2} * \frac{1}{26} = \frac{11*1}{2*2} = \frac{11}{4} = 2\frac{3}{4}$ m

(13 is common divisor of 143 & 26)

Question 13 - The area of a room is $65\frac{1}{4} \, m^2$. If its breadth is $5\frac{7}{16}$ meters, what is its length?

Solution **-** Area of room = $65\frac{1}{4} \, m^2 = \frac{261}{4} \, m^2$

Length of room = $5\frac{7}{16}$ m = $\frac{87}{16}$m

Breadth of room =?

Since area of room = length * breadth

Breadth = area ÷ length

$$= \frac{261}{4} \div \frac{87}{16}$$

Convert ÷ by ×

$$= \frac{261}{4} \times \frac{16}{87}$$

$= 3 \times 4 = 12$ m   (87 is common divisor of 261 & 87 and 4 is common divisor of 16 & 4)

Question 14 - The product of two fractions is $9\frac{3}{5}$. If one of the fractions is $9\frac{3}{7}$, find the other.

Solution - Product of two fractions $= 9\frac{3}{5} = \frac{48}{5}$

One fraction $= 9\frac{3}{7} = \frac{66}{7}$

Other fraction = Product of two fractions ÷ One fraction

$$= \frac{48}{5} \div \frac{66}{7}$$

Convert ÷ by ×

$= \frac{48}{5} \times \frac{7}{66} = \frac{8*7}{5*11} = \frac{56}{55} = 1\frac{1}{55}$ (6 is common divisor of 48 & 66)

Question 15 - In a school, $\frac{5}{8}$ of the students are boys. If there are 240 girls, find the number of boys in the school.

Solution **-** Let total students in school be x

Number of girls = 240

Number of boys $= \frac{5}{8}$ of total students $= \frac{5}{8}$x

Ratio of girls $= (1 - \frac{5}{8}) = (\frac{8-5}{8}) = \frac{3}{8}$

As number of girls =240

Thus $\frac{3}{8}$x = 240

$X = \frac{240 * 8}{3} = 80 * 8 = 640$

Now, total students = number of boys + number of girls

640 = number of boys + 240

Number of boys = 640 -240 = 400

Question 16 - After reading $\frac{7}{9}$ of a book, 40 pages are left. How many pages are there in the book?

Solution **-** Let number of pages be x

Read pages = $\frac{7}{9}$x

Unread pages = 40

Ratio of unread pages = $(1- \frac{7}{9}) = (\frac{9-7}{9}) = \frac{2}{9}$

As Unread pages =40

Thus $\frac{2}{9}$x = 40

=> x = $\frac{40*9}{2}$ = 20*9 = 180

Therefore, total pages = 180

Question 17 - Rita had Rs300. She spent $\frac{1}{3}$ of her money on notebooks and $\frac{1}{4}$ of the remainder on stationary items. How much money is left with her?

Solution **-** Total money = Rs300

Spent on notebooks = $\frac{1}{3}$ * 300 = Rs100

Left money = 300 – 100 = Rs200

Spent on stationary items = $\frac{1}{4}$ * 200 = Rs50

Money left = 200 – 50 = Rs150

Question 18 - Amit earns Rs32000 per month. He spends $\frac{1}{4}$ of his income on food; $\frac{3}{10}$ of the remainder on house rent and $\frac{5}{21}$ of the remainder on the education of children. How much money is still left with him?

Solution - Total earning per month = Rs32000

Spent on food = $\frac{1}{4}$ * 32000 = Rs8000

Left money = 32000 − 8000 = Rs24000

Spent on house rent = $\frac{3}{10}$ * 24000 = Rs7200

Left money = 24000 − 7200 = Rs16800

Spent on education of children = $\frac{5}{21}$ * 16800 = Rs4000

Lest money = 16800 − 4000 = Rs12800

Thus Rs12800 is still left with him.

Question 19 - If $\frac{3}{5}$ of a number exceeds it's $\frac{2}{7}$ by 44, find the number.

Solution - Let the number be x

According to given question,

$\frac{3}{5}x - \frac{2}{7}x = 44$

$=> \frac{21x - 10x}{35} = 44$

$=> \frac{11x}{35} = 44$

$=> x = \frac{44*35}{11} = 4*35 = 140$

Question 20 - At a cricket test match $\frac{2}{7}$ of the spectators were in a covered place while 15000 were in open. Find the total number of spectators.

Solution - Let total number of spectators be x

Ratio of Spectators in covered place = $\frac{2}{7}$

Ratio of spectators in open place = $1 - \frac{2}{7} = \frac{7-2}{7} = \frac{5}{7}$

Number of spectators in open place = 15000

$=> \frac{5}{7}x = 15000$

$$\Rightarrow x = \frac{15000*7}{5} = 3000*7 = 21000$$

Thus total numbers of spectators are 21000

Exercise 1H

Qustion 1 $(\frac{-5}{16} + \frac{7}{12})$

Solution **-** LCM of 16 and 12 is 48

$$= \frac{-15+28}{48} = \frac{13}{48}$$

Question 2 $(\frac{8}{-15} + \frac{4}{-3})$

Solution $\frac{-8}{15} - \frac{4}{3}$

LCM of 15 and 3 is 15

$$= \frac{-8-20}{15} = \frac{-28}{15}$$

Question 3 $(\frac{7}{-26} + \frac{16}{39})$

Solution $\frac{-7}{26} + \frac{16}{39}$

LCM of 26 and 39 is 78

$$= \frac{-21+32}{78} = \frac{11}{78}$$

Question 4 $(3 + \frac{5}{-7})$

Solution $\frac{3}{1} - \frac{5}{7}$

LCM of 1 and 7 is 7

$$= \frac{21-5}{7} = \frac{16}{7}$$

Question 5 $(\frac{31}{-4} + \frac{-5}{8})$

Solution $\frac{-31}{4} - \frac{5}{8}$

LCM of 4 and 8 is 8

$$= \frac{-62-5}{8} = \frac{-67}{8}$$

Question 6 - What should be added to $\frac{7}{12}$ to get$\frac{-4}{15}$?

Solution **-** Let the required number be x

According to given question,

$$\frac{7}{12} + x = \frac{-4}{15}$$

$$=> x = \frac{-4}{15} - \frac{7}{12}$$

LCM of 15 and 12 is 60

$$=> x = \frac{-16-35}{60} = \frac{-51}{60} = \frac{-17}{20}$$

Question 7 $(\frac{2}{3} + \frac{-4}{5} + \frac{7}{15} + \frac{-11}{20})$

Solution **-** Using rearrangement property,

$$(\frac{2}{3} + \frac{7}{15}) + (\frac{-4}{5} + \frac{-11}{20})$$

$$= (\frac{10+7}{15}) + (\frac{-4}{5} - \frac{11}{20})$$

$$= \frac{17}{15} + (\frac{-16-11}{20})$$

$$= \frac{17}{15} + \frac{-27}{20} = \frac{17}{15} - \frac{27}{20}$$

$$= \frac{68-81}{60} = \frac{-13}{60}$$

Question 8 - The sum of two numbers is$\frac{-4}{3}$. If one of the numbers is -5, what is the other?

Solution **-** Let other number be x

One number = -5

Sum of two numbers = $\frac{-4}{3}$

$$\Rightarrow x + (-5) = \frac{-4}{3}$$

$$\Rightarrow x = \frac{-4}{3} + 5$$

$$\Rightarrow x = \frac{-4+15}{3} = \frac{11}{3}$$

Question 9 - What should be added to $\frac{-5}{7}$ to get $\frac{-2}{3}$?

Solution **-** Let the required number be x

According to given question,

$$\frac{-5}{7} + x = \frac{-2}{3}$$

$$\Rightarrow x = \frac{-2}{3} + \frac{5}{7}$$

$$\Rightarrow x = \frac{-14+15}{21} = \frac{1}{21}$$

Question 10 - What should be subtracted from $\frac{-5}{3}$ to get $\frac{5}{6}$?

Solution **-** Let the required number be x

According to given question,

$$\frac{-5}{3} - x = \frac{5}{6}$$

$$\Rightarrow \frac{-5}{3} = \frac{5}{6} + x$$

$$\Rightarrow x = \frac{-5}{3} - \frac{5}{6}$$

$$\Rightarrow x = \frac{-10-5}{6} = \frac{-15}{6} = \frac{-5}{2}$$

Question 11 $\left(\frac{-3}{7}\right)^{-1}$

Solution The inverse of $\frac{-3}{7}$ is $\frac{-7}{3}$

Question 12 - The product of two rational numbers is $\frac{-28}{81}$. If one of the numbers is $\frac{14}{27}$ then the other one is …

Solution **-** Let the other number be x

One number $= \frac{14}{27}$

Product $= \frac{-28}{81}$

$=> x * \frac{14}{27} = \frac{-28}{81}$

$=> x = \frac{-28}{81} \div \frac{14}{27}$

$=> x = \frac{-28}{81} * \frac{27}{14}$

$=> x = \frac{-2}{3}$ (14 is common divisor of 28 & 14 and 27 is common divisor of 27 & 81)

Question 13 - The product of two numbers is $\frac{-16}{35}$. If one of the numbers is $\frac{-15}{14}$, the other is ….

Solution **-** Let other number be x

One number $= \frac{-15}{14}$

Product $= \frac{-16}{35}$

$=> x * \frac{-15}{14} = \frac{-16}{35}$

$=> x = \frac{-16}{35} \div \frac{-15}{14}$

$=> x = \frac{-16}{35} * \frac{14}{-15}$

$=> x = \frac{32}{75}$ (7 is common divisor of 14 and 35)

Question 14 - What should be subtracted from $\frac{-3}{5}$ to get -2?

Solution **-** Let the required number be x

According to given question,

$\frac{-3}{5} - x = -2$

$\Rightarrow \frac{-3}{5} = -2+x$

$\Rightarrow x = \frac{-3}{5}+2$

$\Rightarrow x = \frac{-3+10}{5} = \frac{7}{5}$

Question 15 - The sum of two rational numbers is -3. If one of them is $\frac{-10}{3}$ then the other one is ….

Solution **-** Let the other number be x

One number $= \frac{-10}{3}$

Sum = -3

$\Rightarrow x + \frac{-10}{3} = -3$

$\Rightarrow x - \frac{10}{3} = -3$

$\Rightarrow x = -3 + \frac{10}{3}$

$\Rightarrow x = \frac{-9+10}{3}$

$\Rightarrow x = \frac{1}{3}$

Question 16 - Which of the following is in standard form?

(a) $\frac{-12}{26}$ (b) $\frac{-49}{71}$ (c) $\frac{-9}{16}$ (d) $\frac{28}{-105}$

Solution $\frac{-9}{16}$ is in standard form.

Question 17 $(\frac{-9}{16} * \frac{8}{15})$

Solution $\frac{-9}{16} * \frac{8}{15} = \frac{-3*1}{2*5} = \frac{-3}{10}$ (3 is common divisor of 9 & 15 and 8 is common divisor of 8 & 16)

Question 18 $(\frac{-5}{9} \div \frac{2}{3})$

Solution Convert ÷ by ×

$= \frac{-5}{9} * \frac{3}{2} = \frac{-15}{18} = \frac{-5}{6}$ (3 is common divisor of 15 and 18)

Q19 $\frac{4}{9} \div ? = \frac{-8}{15}$

Solution **-** Let blank space be x $= \frac{a}{b}$

$=> \frac{4}{9} \div \frac{a}{b} = \frac{-8}{15}$

$=> \frac{4}{9} * \frac{b}{a} = \frac{-8}{15}$

$=> \frac{b}{a} = \frac{-8}{15} \div \frac{4}{9}$

$=> \frac{b}{a} = \frac{-8}{15} * \frac{9}{4} = \frac{-2*3}{5*1} = \frac{-6}{5}$ (4 is common divisor of 8 & 4 and 3 is common divisor of 9 & 15)

Now x $= \frac{a}{b} = \frac{-5}{6}$

Question 20 - Additive inverse of $\frac{-5}{9}$ is ….

Solution **-** Additive inverse of $\frac{-5}{9} = -(\frac{-5}{9}) = \frac{5}{9}$

Question 21 - Reciprocal of $\frac{-3}{4}$ is ….

Solution **-** Reciprocal of $\frac{-3}{4} = \frac{4}{-3} = \frac{-4}{3}$

Question 22 - A rational number between $\frac{-2}{3}$ and $\frac{1}{4}$ is ….

Solution **-** A rational number between $\frac{-2}{3}$ and $\frac{1}{4}$ is $\frac{1}{2}(\frac{-2}{3} + \frac{1}{4})$

$= \frac{1}{2}(\frac{-8+3}{12}) = \frac{1}{2}(\frac{-5}{12}) = \frac{-5}{24}$

Question 23 - The reciprocal of a negative rational number is ….

Solution **-** The reciprocal of a negative rational number is a negative rational number