Introduction

Basic terms and formulas to be used in this chapter:

1) Curve: A plane figure formed by joining a number of points with the help of pencil without lifting it.

2) Open curve: A curve that does not end at the starting point.

3) Closed curve: A curve that starts and ends at the same point.

4) Polygon: It is a simple closed curve which is made up of only line segments. There are two types of polygons

(a) Concave polygon: A polygon in which at least one angle is more than 180^o

(b) Convex polygon: A polygon in which each angle is less than 180^o

5) Regular polygon: A polygon in which sides and angles are equal.

6) Irregular polygon: Polygons which are not regular are known as irregular polygons.

For a regular polygon of n side

(a) each exterior angle = $\frac{360^0}{n}$
(b) each interior angle = 180^o - (each exterior angle)

In a convex polygon of n sides,

(a) sum of all exterior angles = 4 right angles
(b) sum of all interior angles = (2n -4) right angles

Number of diagonals in a polygons of n sides = $\frac{n(n-3)}{2}$

Examples

Example 1 – Find the measure of each exterior angle of a regular polygon of:

(a) 8 sides

Solution - Each exterior angle of a regular polygon of 8 sides = $\frac{360^0}{8} = 45^0$

(b) 9 sides

Solution - Each exterior angle of a regular polygon of 9 sides = $\frac{360^0}{9} = 40^0$

(c) 12 sides

Solution - Each exterior angle of a regular polygon of 12 sides = $\frac{360^0}{12} = 30^0$

Example 2 – Is it possible to have a regular polygon each of whose exterior angles is 25⁰?

Solution - Given that each of exterior angle of regular polygon = 25^o

We know that each exterior angle = $\frac{360^0}{n}$ where n is the number of sides

=> $\frac{360^0}{n} = 25^0 => n = \frac{360}{25} = 14.4$ is not a whole number

Thus, it is not possible to have a regular polygon each of whose exterior angle is 25^o

Example 3 – Is it possible to have a regular polygon each of whose interior angles is 45⁰?

Solution - Given that each of interior angle of regular polygon = 45^o

Thus, each exterior angle = 180 – 45 = 135^o

We know that each exterior angle = $\frac{360^0}{n}$ where n is the number of sides

=> $\frac{360^0}{n} = 135^0 => n = \frac{360}{135} = 2.67$ is not a whole number

Thus, it is not possible to have a regular polygon each of whose interior angle is 45^o

Example 4 – Find the measure of each interior angle of a regular

(a) Pentagon

Solution - We know that each exterior angle = $\frac{360^0}{n}$ where n is the number of sides

In pentagon, number of sides (n) = 5

Each exterior angle = $\frac{360^0}{5} = 72$

Thus, each interior angle = 180 – 72 = 108^o

(b) Hexagon

Solution - We know that each exterior angle = $\frac{360^0}{n}$ where n is the number of sides

In hexagon, number of sides (n) = 6

Each exterior angle = $\frac{360^0}{6} = 60$

Thus, each interior angle = 180 – 60= 120º

(c) Octagon

Solution - We know that each exterior angle = $\frac{360^0}{n}$ where n is the number of sides

In octagon, number of sides (n) = 8

Each exterior angle = $\frac{360^0}{8} = 45$

Thus, each interior angle = 180 – 45 = 135º

(d) Polygon of 12 sides

Solution - We know that each exterior angle = $\frac{360^0}{n}$ where n is the number of sides

Number of sides (n) = 12

Each exterior angle = $\frac{360^0}{12} = 30$

Thus, each interior angle = 180 – 30 = 150º

Example 5 – What is the minimum interior angle possible for a regular polygon?

Solution - Since we know that with the decrease in number of sides of a regular polygon, each of its exterior angle increases thus each of its interior angle decreases.

And equilateral triangle is a regular polygon with minimum number of sides. We know that each angle of equilateral triangle is 60º so the the minimum interior angle possible for a regular polygon is 60º.

Example 6 – What is the maximum exterior angle possible for a regular polygon?

Solution - Since we know that with the decrease in number of sides of aregular polygon, each of its exterior angle increases.

And equilateral triangle is a regular polygon with minimum number of sides. We know that each exterior angle of equilateral triangle is 120º so the the maximum exterior angle possible for a regular polygon is 120º.

Example 7 – What is the sum of all interior angles of a polygon of

(a) n sides

Solution - We know that sum of all interior angles of a polygon = (2n – 4) right angles

(b) 7 sides

Solution - We know that sum of all interior angles of a polygon = (2n – 4) right angles

= 2(7) – 4 = 14 - 4 = 10 right angles

(c) 8 sides

Solution - We know that sum of all interior angles of a polygon = (2n – 4) right angles

= 2(8) – 4 = 16 - 4 = 12 right angles

(d) 10 sides

Solution - We know that sum of all interior angles of a polygon = (2n – 4) right angles

= 2(10) – 4 = 20 - 4 = 16 right angles

Example 8 – What is the number of diagonals in a

(a) Quadrilateral

Solution - Number of sides = 4

We know that number of diagonals in a polygon of n sides = $\frac{n(n-3)}{2}$

= $\frac{4(4-3)}{2} = 2$

(b) pentagon

Solution - Number of sides = 5

We know that number of diagonals in a polygon of n sides = $\frac{n(n-3)}{2}$

= $\frac{5(5-3)}{2} = 5$

(c) hexagon

Solution - Number of sides = 6

We know that number of diagonals in a polygon of n sides = $\frac{n(n-3)}{2}$

= $\frac{6(6-3)}{2} = 9$

(d) polygon of 10 sides

Solution - Number of sides = 10

We know that number of diagonals in a polygon of n sides = $\frac{n(n-3)}{2}$

= $\frac{10(10-3)}{2} = 35$

Example 9 – Find the number of sides of a regular polygon whose each exterior angle measures 45^o.

Solution - Given each exterior angle of a regular polygon = 45^o

We know that each exterior angle = $\frac{360^0}{n}$ where n is the number of sides

=> $\frac{360^0}{n} = 45 => n = \frac{360}{45} = 8$

Thus, number of sides = 8

Example 10 – What is the measure of

(a) Each exterior angle of a regular hexagon?

Solution - We know that each exterior angle = $\frac{360^0}{n}$ where n is the number of sides

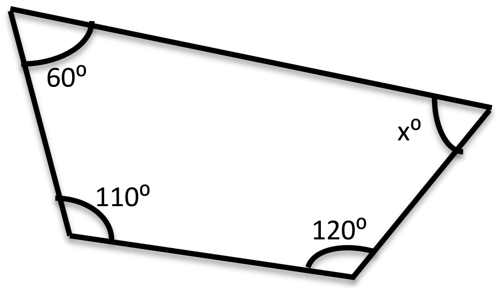
Thus, each exterior angle of a regular hexagon (n= 6) = $\frac{360^0}{6} = 60^0$

(b) Each interior angle of a regular hexagon?

Solution - Each interior angle of a regular hexagon = 180 – 60 = 120^o

Example 11 – Find the angle measure x in each of the following:

(a)



Solution - Number of side, n = 4

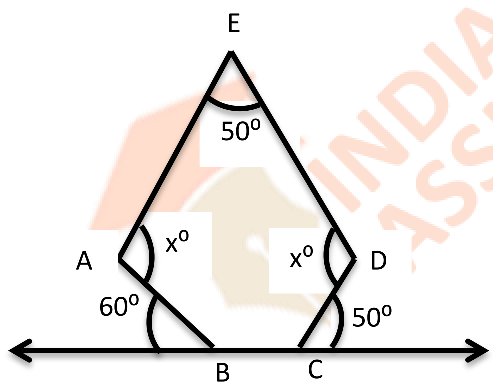
We know that sum of interior angles of a polygon of n sides = (2n – 4) right angles

=> 60 + 110 + 120 + x = 2(4) -4 = 8 – 4 = 4 right angles = 360º

=> 290 + x = 360º

=> x = 360 – 290 = 70º

(b)



Solution - Interior angle *L* ABC = (180 – 60) = 120º

And, interior angle *L* BCD = (180-50) = 130º

Now, in a pentagon,

Sum of all interior angles of a pentagon = (2(5) – 4) right angles

= 10 – 4 = 6 right angles = 6 x 90 = 540º

=> 50 + x + x + 120 + 130 = 540

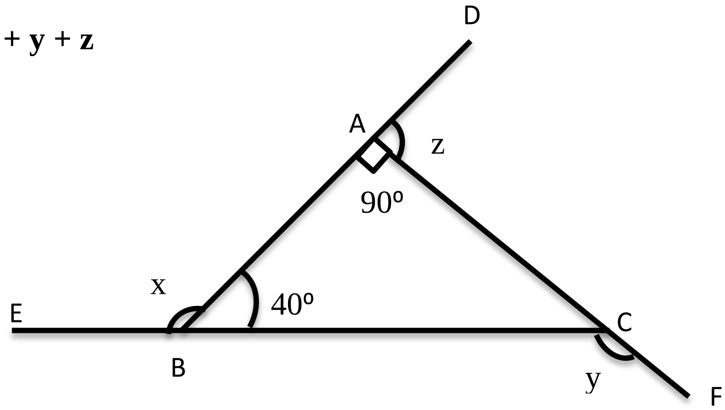
=> 300 + 2x = 540

=> 2x = 540 – 300 = 240

=> x = 120

Example 12 – Look at the figures given below:

(a) Find x + y + z



Solution - To find x + y + z

We know that sum of interior angles of triangle = 2(3) – 4 = 2 right angles = 2(90) = 180

=> 90 + 40 + ∠C = 180

=> 130 + ∠C = 180

=> ∠C = 180 – 130 = 50º

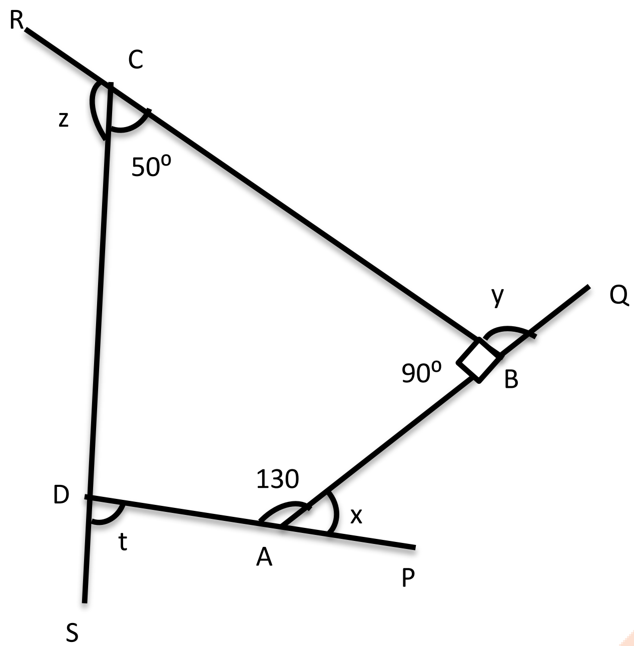
Now, y + 50 = 180 => y = 180 -50 = 130

40+x = 180 => x = 180-40 = 140

90+z = 180 => z = 180 – 90 = 90

Thus, x +y + z = 140 + 130 + 90 = 360º

(b) Find x + y + z + t



Solution - We know that sum of interior angles of quadrilateral = 2(4) – 4 = 4 right angles = 4(90) = 360

=> 90 + 50 + 130 + ∠D = 360

=> 270 + ∠D = 360

=> ∠D = 360 – 270 = 90º

Now, 90 + t = 180 => t = 180-90 = 90

130 + x = 180 => x =180-130 = 50

90 + y = 180 => y = 180-90 = 90

50 + z = 180 => z = 180-50 = 130

Thus, x + y + z + t = 50+130+90+90 = 360º

Exercise 14A

Question 1 – Find the measure of each exterior angle of a regular

(a) Pentagon

Solution - Number of sides in pentagon = 5

Each exterior angle of a pentagon = $\frac{360^0}{5} = 72^0$

(b) Hexagon

Solution - Number of sides in hexagon = 6

Each exterior angle of a hexagon = $\frac{360^0}{6} = 60^0$

(c) Heptagon

Solution - Number of sided in Heptagon = 7

Each exterior angle of a Heptagon = $\frac{360^0}{7} = 51.4^0$

(d) Decagon

Solution - Number of sides in Decagon = 10

Each exterior angle of a Decagon = $\frac{360^0}{10} = 36^0$

Question 2 – Is it possible to have a regular polygon each of whose exterior angle is 50^o?

Solution - Given that each of exterior angle of regular polygon = 50^o

We know that each exterior angle = $\frac{360^0}{n}$ where n is the number of sides

=> $\frac{360^0}{n} = 50^0 => n = \frac{360}{50} = 7.2$ is not a whole number

Thus, it is not possible to have a regular polygon each of whose exterior angle is 50^o

Question 3 – Find the measure of each interior angle of a regular polygon having

(a) 10 sides

Solution - We know that each exterior angle = $\frac{360^0}{n}$ where n is the number of sides

Number of sides (n) = 10

Each exterior angle = $\frac{360^0}{10} = 36$

Thus, each interior angle = 180 – 36 = 144º

(b) 15 sides

Solution - We know that each exterior angle = $\frac{360^0}{n}$ where n is the number of sides.

Number of sides (n) = 15

Each exterior angle = $\frac{360^0}{15} = 24$

Thus, each interior angle = 180 – 24 = 156º

Question 4 – Is it possible to have a regular polygon each of whose interior angles is 100º?

Solution - Given that each of interior angle of regular polygon = 100º

Exterior angle = 180 – 100 = 80º

We know that each exterior angle = $\frac{360^0}{n}$ where n is the number of sides

=> $\frac{360^0}{n} = 80^0 => n = \frac{360}{80} = 4.5$ is not a whole number

Thus, it is not possible to have a regular polygon each of whose interior angle is 100º

Question 5 – What is the sum of all interior angles of a regular

(a) Pentagon

Solution - Number of sides = 5

We know that sum of all interior angles of a polygon = (2n – 4) right angles

= 2(5) – 4 = 10 – 4 = 6 right angles = 6 × 90 = 540º

(b) Hexagon

Solution - Number of sides = 6

We know that sum of all interior angles of a polygon = (2n – 4) right angles

= 2(6) – 4 = 12 – 4 = 8 right angles = 8 × 90 = 720º

(c) Nonagon

Solution - Number of sides = 9

We know that sum of all interior angles of a polygon = (2n – 4) right angles

= 2(9) – 4 = 18 – 4 = 14 right angles = 14 × 90 = 1260º

(d) Polygon of 12 sides

Solution - Number of sides = 12

We know that sum of all interior angles of a polygon = (2n – 4) right angles

= 2(12) – 4 = 24 – 4 = 20 right angles = 20 × 90 = 1800º

Question 6 – What is the number of diagonals in a

(a) Heptagon

Solution - Number of sides = 7

We know that number of diagonals in a polygon of n sides = $\frac{n(n-3)}{2}$

= $\frac{7(7-3)}{2} = 14$

(b) octagon

Solution - Number of sides = 8

We know that number of diagonals in a polygon of n sides = $\frac{n(n-3)}{2}$

= $\frac{8(8-3)}{2} = 20$

(c) polygon of 12 sides

Solution - Number of sides = 12

We know that number of diagonals in a polygon of n sides = $\frac{n(n-3)}{2}$

= $\frac{12(12-3)}{2} = 54$

Question 7 – Find the number of sides of a regular polygon whose each exterior angle measures:

(a) 40⁰

Solution - Given each exterior angle of a regular polygon = 40^0

We know that each exterior angle = $\frac{360^0}{n}$ where n is the number of sides

=> $\frac{360^0}{n} = 40 => n = \frac{360}{40} = 9$

Thus, number of sides = 9

(b) 36⁰

Solution - Given each exterior angle of a regular polygon = 36^0

We know that each exterior angle = $\frac{360^0}{n}$ where n is the number of sides

=> $\frac{360^0}{n} = 36 => n = \frac{360}{36} = 10$

Thus, number of sides = 10

(c) 72⁰

Solution - Given each exterior angle of a regular polygon = 72^0

We know that each exterior angle = $\frac{360^0}{n}$ where n is the number of sides

=> $\frac{360^0}{n} = 72 => n = \frac{360}{72} = 5$

Thus, number of sides = 5

(d) 30⁰

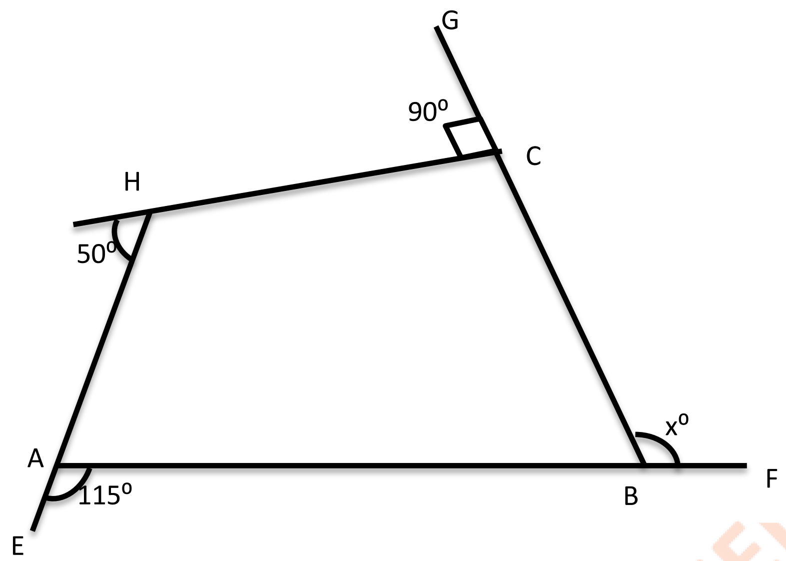
Solution - Given each exterior angle of a regular polygon = 30^0

We know that each exterior angle = $\frac{360^0}{n}$ where n is the number of sides

=> $\frac{360^0}{n} = 30 => n = \frac{360}{30} = 12$

Thus, number of sides = 12

Question 8 – In the given figure, find the angle measure x.



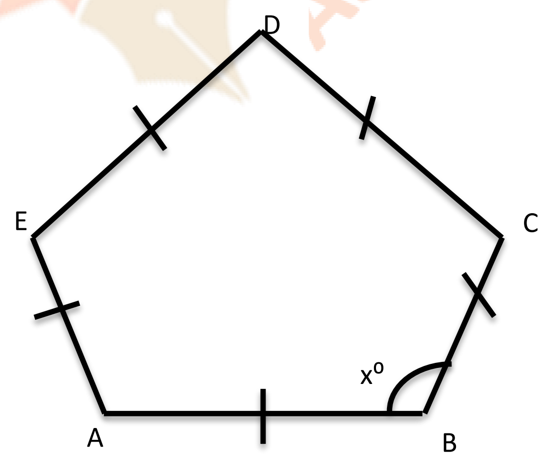
Solution - We know that sum of all exterior angles = 4 right angles = 4 (90) = 360º

=> 90 + 50 + 115 + x = 360

=> 255 + x = 360

=> x = 360- 255 = 105º

Question 9 Find the angle measure x in the given figure



Solution - Since ABCDE is a pentagon

Number of sides = 5

And, we know that that each interior angle = 180 – (each exterior angle)

= 180 – ($\frac{360}{n}$) = 180 – (360/5) = 180 – 72 = 108°

Exercise 14B

Question 1 – How many diagonals are there in a pentagon?

Solution - We know that number of diagonals in a polygon of n sides = $\frac{n(n-3)}{2}$

Thus, in a pentagon, number of diagonals = $\frac{5(5-3)}{2} = 5$

Question 2 – How many diagonals are there in a hexagon?

Solution - We know that number of diagonals in a polygon of n sides = $\frac{n(n-3)}{2}$

Thus, in a hexagon, number of diagonals = $\frac{6(6-3)}{2} = 9$

Question 3 – How many diagonals are there in an octagon?

Solution - We know that number of diagonals in a polygon of n sides = $\frac{n(n-3)}{2}$

Thus, in a octagon, number of diagonals = $\frac{8(8-3)}{2} = 20$

Question 4 – How many diagonals are there in a polygon having 12 sides?

Solution - We know that number of diagonals in a polygon of n sides = $\frac{n(n-3)}{2}$

Thus, in a polygon of sides 12, number of diagonals = $\frac{12(12-3)}{2} = 54$

Question 5 – A polygon has 27 diagonals. How many sides does it have?

Solution - Given that diagonal of polygon = 27

Number of sides =?

We know that number of diagonals in a polygon of n sides = $\frac{n(n-3)}{2}$

=> $\frac{n(n-3)}{2} = 27$

=> n (n-3) = 54

=> $n^2 - 3n = 54$

=> $n^2 - 3n - 54 = 0$

=> $n^2 - 9n + 6n - 54 = 0$

=> $n(n-9) + 6(n-9) = 0$

=> $(n-9)(n+6) = 0$

=> $n = 9 \text{ } or - 6$

But number of sides cannot be negative

Thus, number of sides = 9

Question 6 – The angles of a pentagon are x^o, $(x + 20)^o$, $(x + 40)^o$, $(x + 60)^o$ and $(x + 80)^o$. The smallest angle of the pentagon is?

Solution - Given the angles of pentagon are x^o, $(x + 20)^o$, $(x + 40)^o$, $(x + 60)^o$ and $(x + 80)^o$

We know that sum of interior angles of polygon of n sides = (2n – 4) right angles

Here n = 5

=> $x^o+(x + 20)^o+ (x + 40)^o+ (x + 60)^o +(x + 80)^o$ = 2(5)-4 right angles

=> 5x + 200 = 6(90)

=> 5x + 200 = 540

=> 5x = 540-200 = 340

=> x = 68

Thus, smallest angle of the pentagon is 68^o

Question 7 – The measure of each exterior angle of a regular polygon is 40^o. How many sides does it have?

Solution - Given that each exterior angle of a regular polygon = 40^o

Number of sides =?

We know that each exterior angle of polygon of n sides = $\frac{360^0}{n}$ where n is the number of sides

=> $\frac{360^0}{n} = 40 => n = \frac{360}{40} = 9$

Thus, number of sides = 9

Question 8 – Each interior angle of a polygon is 108^{o}. How many sides does it have?

Solution - Given that each interior angle of a polygon = 108^{o}

Thus, each exterior angle = 180 –108 = 72^{o}

Number of sides =?

We know that each exterior angle of polygon of n sides = $\frac{360^0}{n}$ where n is the number of sides

=> $\frac{360^0}{n} = 72 => n = \frac{360}{72} = 5$

Thus, number of sides = 5

Question 9 – Each interior angle of a polygon is 135^{o}. How many sides does it have?

Solution - Given that each interior angle of a polygon = 135^{o}

Thus, each exterior angle = 180 –135 = 45^{o}

Number of sides =?

We know that each exterior angle of polygon of n sides = $\frac{360^0}{n}$ where n is the number of sides

=> $\frac{360^0}{n} = 45 => n = \frac{360}{45} = 8$

Thus, number of sides = 8

Question 10 – In a regular polygon, each interior angle is thrice the exterior angle. The number of sides of the polygon is?

Solution - Let each exterior angle be x^{o}

Then, each interior angle = 3x^{o}

Number of sides =?

Each interior angle = 180 – x

=> 3x = 180 – x

=> 3x + x = 180

=> 4x = 180

=> x = 45^o

Thus, each exterior angle is 45^o

We know that each exterior angle of polygon of n sides = $\frac{360^0}{n}$ where n is the number of sides

=> $\frac{360^0}{n} = 45 => n = \frac{360}{45} = 8$

Thus, number of sides = 8

Question 11 – Each interior angle of a regular decagon is?

Solution - In a decagon, number of sides = 10

So, each exterior angle = 360/10 = 36^o

Thus, each interior angle = 180 – 36 = 144^o

Question 12 – The sum of all interior angles of a hexagon is?

Solution - In a hexagon, number of sides = 6

We know that sum of all interior angles of a polygon = (2n-4) right angles

= 2(6) – 4 = 8 right angles

Question 13 – The sum of all interior angles of a regular polygon is 1080^o. What is the measure of each of its interior angles?

Solution - Given that sum of all interior angles of a regular polygon = 1080^o

We know that sum of all interior angles of a polygon = (2n-4) right angles

=> (2n – 4) ×90 = 1080

=> 2n – 4 = 1080/90

=> 2n – 4 = 12

=> 2n = 16 => n = 8

Each exterior angle = 360/8 = 45^o

Thus, each interior angle = 180 – 45 = 135^o

Question 14 – The interior angle of a regular polygon exceeds its exterior angle by 108^o. How many sides does the polygon have?

Solution - Let the exterior angle of a regular polygon be x^o

Then, each interior angle = $x + 108^o$

Since, each interior angle = 180 – (each exterior angle)

=> $x + 108 = 180 - x$

=> $x + x = 180 - 108 = 72$

=> $2x = 72$

=> $x = 36^o$

Each exterior angle = 36

=> $360/n = 36^o$

=> $n = 360/36 = 10$