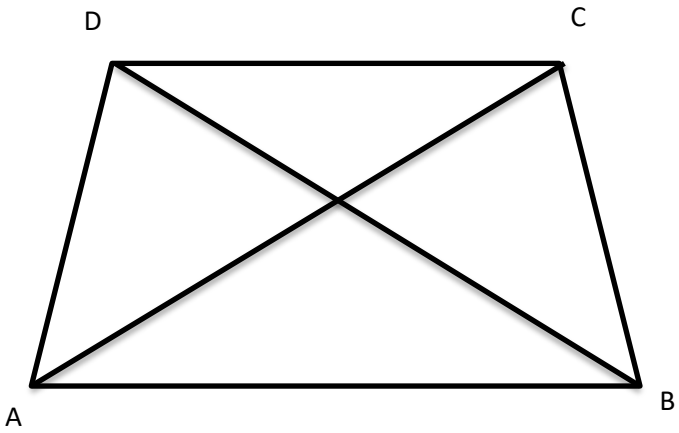
Introduction



Quadrilateral is a polygon bounded by four line segments AB, BC, CD and DA.

Here the points A, B, C, D are known as vertices.

$\angle A, \angle B, \angle C \; and \; \angle D$ are angles of Quad ABCD

The lines AC and BD are diagonals of Quad ABCD

(a) Adjacent sides: Two sides of quadrilateral is said to be adjacent sides, if they have a common end point. In the above figure (AB, BC), (BC, CD), (CD, DA) and (DA, AB) are adjacent sides.

(b) Opposite sides: Two sides of quadrilateral is said to be opposite sides, if they do not have a common end point. In the above figure (AB, CD) and (DA, BC) are opposite sides.

(c) Adjacent angles: Two angles of a quadrilateral are called adjacent angles if they have a common side as an arm. In the above figure $(\angle A, \angle B), (\angle B, \angle C), (\angle C, \angle D) \; and \; (\angle D, \angle A)$ are pairs of adjacent angles.

(d) Opposite angles: Two angles of a quadrilateral are called opposite angles which are not adjacent. In the above figure $(\angle A, \angle C) \; and \; (\angle D, \angle B)$ are pairs of opposite angles.

(e) Angle sum property of a quadrilateral: sum of the angles of a quadrilateral is 360^{o}

Examples

Example 1 – Three angles of a quadrilateral are 54°, 80° and 116°. Find the measure of the fourth angle.

Solution - Given that three angles of quadrilateral are 54°, 80° and 116°

We have to find fourth angle

Let fourth angle be x°

We know that sum of the angles of a quadrilateral = 360°

=> 54 + 80 + 116 + x = 360

=> 250 + x = 360

=> x = 360 – 250 = 110°

Thus, fourth angle = 110°

Example 2 – The four angles of a quadrilateral are in the ratio 2:3:5:8. Find the angles.

Solution - Let the measures of the angle of quadrilateral be (2x) °, (3x) °, (5x) ° and (8x) °

We know that sum of the angles of a quadrilateral = 360°

=> 2x+3x +5x +8x = 360

=> 18x = 360

=> x = 360 /18 = 20°

Thus, measures of angles are (2×20) = 40°, (3×20) = 60°, (5×20) = 100°, (8×20) = 160°

Example 3 – The measures of two angles of a quadrilateral are 115° and 45°, and the other two angles are equal. Find the measure of each of the equal angles.

Solution - Given that two angles of quadrilateral are 115° and 45°

And other two angles of quadrilateral are equal

Let the each of two equal angle be x°

We know that sum of the angles of a quadrilateral = 360°

=> 115+45 +x +x = 360

=>160+ 2x = 360

=> 2x = 360-160 = 200

=> x = 100º

Thus, measure of each equal angle is 100º

Exercise 15

Question 1 – Fill in the blanks:

(a) A quadrilateral has **four** sides

(b) A quadrilateral has **four** angles

(c) A quadrilateral has **four** vertices, no three of which are **collinear**.

(d) A quadrilateral has **two** diagonals

(e) A diagonal of a quadrilateral is a line segment that joins two **opposite** vertices of the quadrilateral.

(f) The sum of the angles of a quadrilateral is **360º**

Question 2 – In the adjoining figure, ABCD is a quadrilateral.

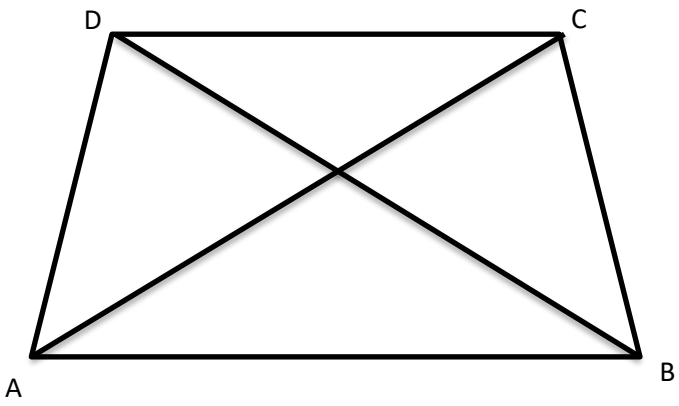
(a) How many pairs of adjacent sides are there? Name them.

(b) How many pairs of opposite sides are there? Name them.

(c) How many pairs of adjacent angles are there? Name them.

(d) How many pairs of opposite angles are there? Name them.

(e) How many diagonals are there? Name them.



Solution - (a) There are four pairs of adjacent sides. (AB, BC), (BC, CD), (CD, DA) and (DA, AB) are adjacent sides.

(b) There are two pairs of opposite sides. (AB, CD) and (DA, BC) are opposite sides.

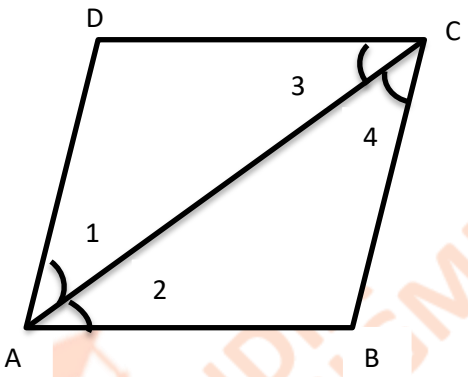
(c) There are four pairs of adjacent angles. $(\angle A, \angle B), (\angle B, \angle C), (\angle C, \angle D)\ and\ (\angle D, \angle A)$ are pairs of adjacent angle.

(d) There are two pairs of opposite angles. $(\angle A, \angle C)\ and\ (\angle D, \angle B)$ are pairs of opposite angle.

(e) There are two diagonals. AC and BD are diagonals.

Question 3 – Prove that the sum of the angles of a quadrilateral is 360°.

Solution - To prove: sum of angles of of a quadrilateral is 360º



Construction: Join AC

Proof: Let ABCD be a quadrilateral with diagonal AC

We know that by angle sum property of triangle, sum of angles of triangle is 180º

In triangle ABC, we have

$\angle 2 + \angle 4 + \angle B = 180 \longrightarrow 1$

In triangle ACD, we have

$\angle 1 + \angle 3 + \angle D = 180 \longrightarrow 2$

Adding 1 and 2, we get

$\angle 2 + \angle 4 + \angle B + \angle 1 + \angle 3 + \angle D = 180+180$

$\Rightarrow (\angle 1+ \angle 2) + (\angle 3+ \angle 4) + \angle B+ \angle D = 360$

=> ∠A+ ∠C+ ∠B+ ∠D = 360º

Thus, sum of angles of a quadrilateral is 360º

Question 4 – The three angles of a quadrilateral are 76°, 54° and 108°. Find the measure of the fourth angle.

Solution - It is given that three angles of quadrilateral are 76º, 54º and 108º

We have to find fourth angle

Let fourth angle be xº

We know that sum of the angles of a quadrilateral = 360º

=> 76 + 54 + 108 + x = 360

=> 238 + x = 360

=> x = 360 – 238 = 122º

Thus, fourth angle = 122º

Question 5 – The angles of a quadrilateral are in the ratio 3:5:7:9. Find the measure of each of these angles.

Solution - Let the measures of the angle of quadrilateral be (3x) º, (5x) º, (7x) º and (9x) º

We know that sum of the angles of a quadrilateral = 360º

=> 3x+5x +7x +9x = 360

=> 24x = 360

=> x = 360 /24 = 15º

Thus, measures of angles are (3×15) = 45º, (5×15) = 75º, (7×15) = 105º, (9×15) = 135º

Question 6 – A quadrilateral has three acute angles, each measuring 75°. Find the measure of the fourth angle.

Solution - It is given that three acute angles of quadrilateral is 75º each

We have to find fourth angle

Let fourth angle be xº

We know that sum of the angles of a quadrilateral = 360º

=> 75 + 75 + 75 + x = 360

=> 225 + x = 360

=> x = 360 – 225 = 135º

Thus, fourth angle = 135º

Question 7 – Three angles of a quadrilateral are equal and the measure of the fourth angle is 120º. Find the measure of each of the equal angles.

Solution - It is given that three angles of a quadrilateral are equal and fourth angle = 120º

Let each of equal angle be xº

We know that sum of the angles of a quadrilateral = 360º

=> x + x + x + 120 = 360

=> 120 + 3x = 360

=> 3x = 360 – 120 = 240º

=> x = 80º

Thus, each equal angle = 80º

Question 8 – Two angles of a quadrilateral measure 85º and 75º respectively. The other two angles are equal. Find the measure of each of these equal angles.

Solution - Given that two angles of quadrilateral are 85º and 75º

And, other two angles of quadrilateral are equal

Let the each of two equal angle be xº

We know that sum of the angles of a quadrilateral = 360º

=> 85+75 +x +x = 360

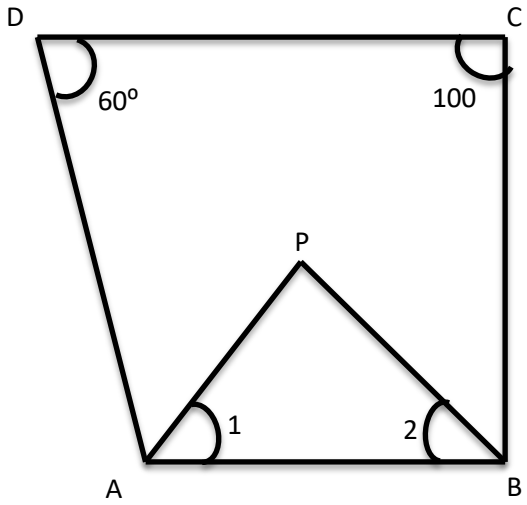
=>160+ 2x = 360

=> 2x = 360-160 = 200

=> x = 100º

Thus, measure of each equal angle is 100º

Question 9 – In the adjacent figure, the bisectors of ∠A and ∠B meet in a point P. If ∠C = 100° and ∠D = 60°, find the measure of ∠APB



Solution - It is given that $\angle C = 100^\circ$ and $\angle D = 60^\circ$

And also given that $\angle 1 = \frac{\angle A}{2}$ and $\angle 2 = \frac{\angle B}{2}$

To find: $\angle APB$

In quadrilateral ABCD, Sum of all angles = 360°

$\Rightarrow \angle A + \angle B + \angle C + \angle D = 360^\circ$

$\Rightarrow \angle A + \angle B + 60 + 100 = 360$

$\Rightarrow \angle A + \angle B + 160 = 360$

$\Rightarrow \angle A + \angle B = 200$

$\Rightarrow \frac{\angle A}{2} + \frac{\angle B}{2} = 100 \longrightarrow 1$

Now, in triangle APB, by angle sum property of triangle we have

$\angle 1 + \angle 2 + \angle APB = 180$

$\Rightarrow \frac{\angle A}{2} + \frac{\angle B}{2} + \angle APB = 180$

$\Rightarrow 100 + \angle APB = 180$ (from 1)

$\Rightarrow \angle APB = 180\text{-}100 = 80^\circ$