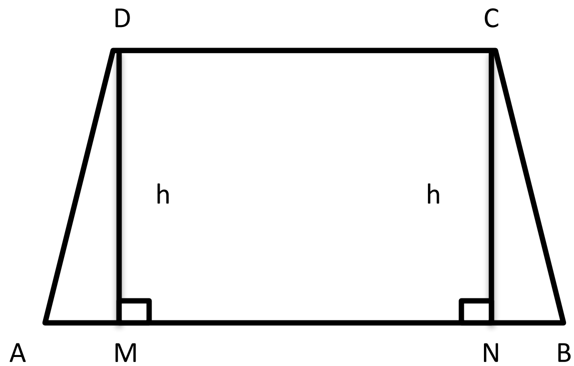
Introduction

Trapezium: A trapezium is a quadrilateral in which only one pair of opposite sides are parallel and the other pair of opposite sides is not parallel.



ABCD is a trapezium with AB//CD

Here, AB and CD are two bases of trap. ABCD

DM and CN are height or altitude of trap. ABCD

Area of trapezium can be computed using the formula:

Area of trapezium = $\frac{1}{2}$ × Sum of parallel sides × Distance between them

Area of trapezium = {$\frac{1}{2}$ × (AB +CD) × h} sq. units

Examples

Example 1 – Two parallel sides of a trapezium are of lengths 27 cm and 19 cm respectively, and the distance between them is 14 cm. Find the area of the trapezium.

Solution **-** It is given that length of two parallel sides is 27 cm and 19 cm respectively

Distance between them (h) = 14 cm

Area of trapezium = $\frac{1}{2}$ × Sum of parallel sides × Distance between them

Area of trapezium = $\frac{1}{2}$ × (27+19) × 14 = $\frac{46\times14}{2} = 322cm^2$

Example 2 – The area of a trapezium is 352 cm^2 and the difference between its parallel sides is 16 cm. If one of the parallel sides is of length 25 cm, find the length of the other.

Solution - Let the length of required side be x cm

It is given that Area of a trapezium = 352 cm^2

One parallel side = 25 cm

Distance between two parallel sides = 16 cm

We know that Area of trapezium = $\frac{1}{2}$ × Sum of parallel sides × Distance between them

352 = $\frac{1}{2}$ × (25 + x) × 16

=> 704/16 = 25+x

=> 44 = 25+x

=> 44-25 = x

=> x = 19 cm

Example 3 – The area of a trapezium is 168 cm^2 and its height is 8 cm. If one of the parallel sides is longer than the other by 6 cm, find the length of each of the parallel sides.

Solution - Let length of one parallel side be x cm

Then, length of other parallel side = (x+6) cm

Height = 8 cm

Area of trapezium = 168 cm^2

We know that Area of trapezium = $\frac{1}{2}$ × Sum of parallel sides × Distance between them

168 = $\frac{1}{2}$ × (x+ x+6) × 8

=> 336/8 = 6+2x

=> 42 = 6+2x

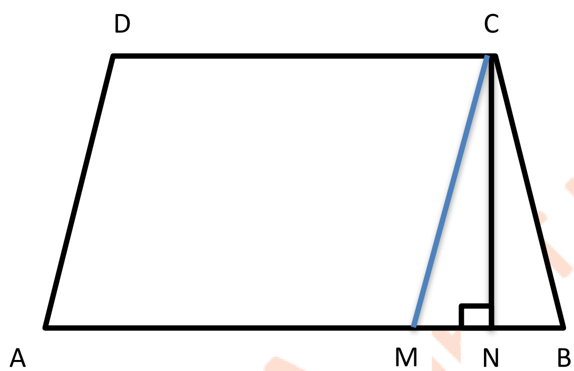
=> 42-6 = 2x

=> x = 36/2 = 18 cm

Thus, lengths of parallel sides are 18 cm and 24 cm

Example 4 – The parallel sides of a trapezium are 25 cm and 13 cm; its nonparallel sides are equal, each being 10 cm. Find the area of the trapezium.

Solution -



ABCD is a trapezium with AB//CD

It is given that AB = 25 cm, CD = 13 cm, AD = BC = 10 cm

We construct CM//AD and CN perpendicular to AB

So, we see that AMCD is a parallelogram.

Since, opposite sides of parallelogram are equal

Thus, AM = CD = 13 cm

MB = AB – AM = 25-13 = 12 cm

Also, AD = CM = 10 cm

Now we have CM = 10 cm, BC = 10 cm

So, Δ CMB is an isosceles triangle and CN is perpendicular on MB

Thus, N is the midpoint of MB

=> MN = NB = MB/2 = 12/2 = 6 cm

Now, in Δ CMN, by Pythagoras theorem,

$CM^2 = CN^2 + MN^2$

=> $10^2 = CN^2 + 6^2$

=> 100 = CN^2 + 36

=> 64 = CN^2

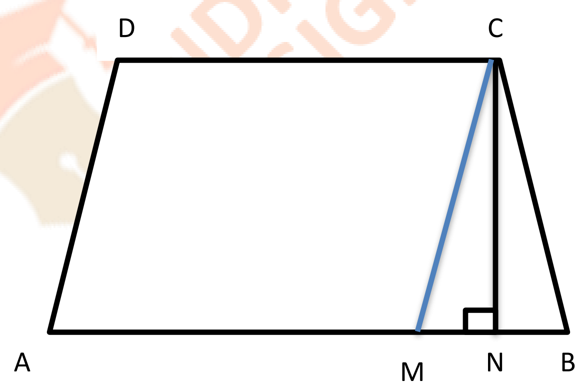
=> CN = 8 cm

Area of trapezium = $\frac{1}{2}$ × Sum of parallel sides × Distance between them

Area of trapezium = $\frac{1}{2}$ × (25+13) × 8 = $\frac{38\times8}{2} = 152cm^2$

Example 5 – ABCD is a trapezium in which AB//DC, AB = 78 cm, CD = 52 cm, AD = 28 cm and BC = 30 cm. Find the area of the trapezium.

Solution -



ABCD is a trapezium with AB//CD

It is given that AB = 78 cm, CD = 52 cm, AD = 28 cm and BC = 30 cm

We construct CM//AD and CN perpendicular to AB

So, we see that AMCD is a parallelogram.

Since opposite sides of parallelogram are equal

Thus, AM = CD = 52 cm and AD = CM = 28 cm

MB = AB – AM = 78-52 = 26 cm

Now, in ΔCMB, CM= 28 cm, MB = 26 cm and BC = 30 cm

Area of ΔCMB = $\sqrt{s(s-a)(s-b)(s-c)}$

Where s = (28+26+30)/2 = 84/2 = 42 cm

Area of ΔCMB = $\sqrt{42(42-28)(42-26)(42-30)}$

Area of ΔCMB = $\sqrt{42 \times 16 \times 14 \times 12}$

= $\sqrt{2 \times 7 \times 3 \times 2 \times 2 \times 2 \times 2 \times 2 \times 7 \times 3 \times 4}$

= 2×2×2×2×3×7 = 336 cm^2

Also, Area of ΔCMB = ½ × MB ×CN

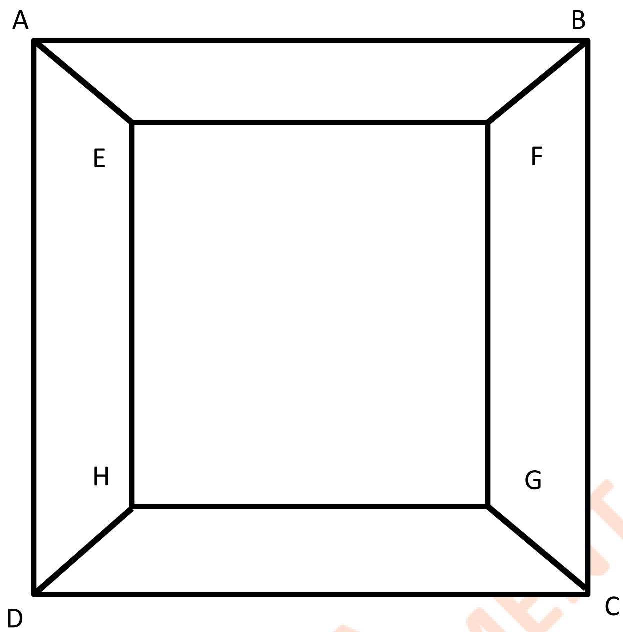
=> 336 = ½ × 26 ×CN

=> CN = 336/13 cm

Area of trapezium = $\frac{1}{2}$ × Sum of parallel sides × Distance between them

Area of trapezium = $\frac{1}{2 \times 13}$ × (78+52) × 336 = $\frac{130 \times 336}{26} = 1680 cm^2$

Example 6 – The adjacent figure shows the diagram of a picture frame having outer dimensions 28 cm × 32 cm and inner dimensions 20 cm × 24 cm. If the width of each section is the same, find the area of each section of the frame.



Solution - It is given that Outer dimensions = 28cm × 32cm = CD × BC

And inner dimensions = 20cm × 24cm = GH × FG

Also, given that width of each section is same

Width of AEHD = Width of BFGC = ½ (CD – GH) = ½ (28 – 20) = 8/2 = 4 cm

Width of ABFE = Width of DCGH = ½ (BC – FG) = ½ (32 – 24) = 8/2 = 4 cm

We can see that each section is a trapezium.

Area of AEHD = Area of BFGC = $\frac{1}{2}$ × Sum of parallel sides × Distance between them

$\frac{1}{2}$ × (24+32) × 4 = 224/2 = 112 cm^2

Also, Area of ABFE = Area of DCGH = $\frac{1}{2}$ × Sum of parallel sides × Distance between them

$\frac{1}{2} \times (20+28) \times 4 = 192/2 = 96\ cm^2$

Exercise 18A

Question 1 – Find the area of a trapezium whose parallel sides are 24 cm and 20 cm and the distance between them is 15 cm.

Solution **-** It is given that length of two parallel sides is 24 cm and 20 cm respectively

Distance between them (h) = 15 cm

Area of trapezium = $\frac{1}{2}$ × Sum of parallel sides × Distance between them

Area of trapezium = $\frac{1}{2} \times (24+20) \times 15 = \frac{44\times15}{2} = 330cm^2$

Question 2 – Find the area of a trapezium whose parallel sides are 38.7 cm and 22.3 cm, and the distance between them is 16 cm.

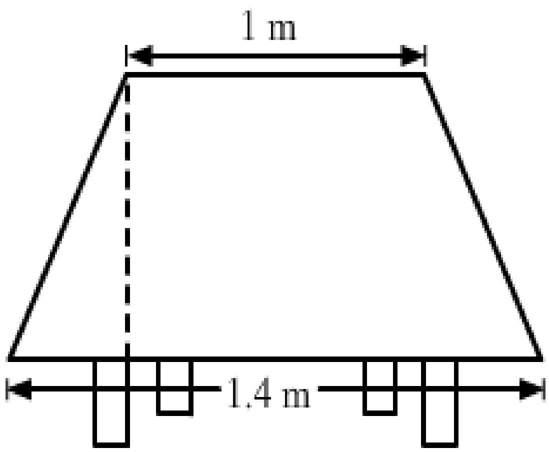
Solution **-** It is given that length of two parallel sides is 38.7 cm and 22.3 cm respectively

Distance between them (h) = 16 cm

Area of trapezium = $\frac{1}{2}$ × Sum of parallel sides × Distance between them

Area of trapezium = $\frac{1}{2} \times (38.7+22.3) \times 16 = \frac{61\times16}{2} = 488cm^2$

Question 3 – The shape of the top surface of a table is trapezium. Its parallel sides are 1 m and 1.4 m and the perpendicular distance between them is 0.9 m. Find its area.



Solution - It is given that length of two parallel sides is 1 m and 1.4 m respectively

Distance between them (h) = 0.9 m

Area of trapezium = $\frac{1}{2}$ × Sum of parallel sides × Distance between them

Area of trapezium = $\frac{1}{2}$ × (1+1.4) × 0.9 = $\frac{2.4 \times 0.9}{2} = 1.08m^2$

Question 4 – The area of a trapezium is 1080 cm^2. If the lengths of its parallel sides be 55 cm and 35 cm, find the distance between them.

Solution - Let distance between parallel sides be x cm

It is given that Area of a trapezium = 1080 cm^2

One parallel side = 55 cm

Other parallel side = 35 cm

We know that Area of trapezium = $\frac{1}{2}$ × Sum of parallel sides × Distance between them

1080 = $\frac{1}{2}$ × (55 + 35) × x

=> 2160 = 90 x

=> x = 2160/90 = 24

Thus, distance between parallel sides = 24 cm

Question 5 – A field is in the form of a trapezium. Its area is 1586 m^2 and the distance between its parallel sides is 26 m. If one of the parallel sides is 84 m, find the other.

Solution - Let the length of required side be x cm

It is given that Area of a trapezium = 1586 m^2

One parallel side = 84 m

Distance between two parallel sides = 26 m

We know that Area of trapezium = $\frac{1}{2}$ × Sum of parallel sides × Distance between them

1586 = $\frac{1}{2}$ × (84 + x) × 26

=> 3172/26 = 84+x

=> 122 = 84+x

=> 122-84 = x

=> x = 38 m

Question 6 – The area of a trapezium is 405cm^2. Its parallel sides are in the ratio 4:5 and the distance between them is 18 cm. Find the length of each of the parallel sides.

Solution - Let length of one parallel side be 4x cm

Then, length of other parallel side = 5x cm

Distance between them = 18 cm

Area of trapezium = 405 cm^2

We know that Area of trapezium = $\frac{1}{2}$ × Sum of parallel sides × Distance between them

$405 = \frac{1}{2} \times (4x + 5x) \times 18$

=> 810/18 = 9x

=> 45 = 9x

=> x = 45/9 = 5 cm

Thus, lengths of parallel sides are 4(5) =20 cm and 5(5) = 25 cm

Question 7 – The area of a trapezium is 180 cm^2 and its height is 9 cm. If one of the parallel sides is longer than the other by 6 cm, find the two parallel sides.

Solution **-** Let length of one parallel side be x cm

Then, length of other parallel side = (x+6) cm

Height = 9 cm

Area of trapezium = 180 cm^2

We know that Area of trapezium = $\frac{1}{2}$ × Sum of parallel sides × Distance between them

$180 = \frac{1}{2} \times (x + x + 6) \times 9$

=> 360/9 = 6+2x

=> 40 = 6+2x

=> 40-6 = 2x

=> x = 34/2 = 17 cm

Thus, lengths of parallel sides are 17 cm and 23 cm

Question 8 – In a trapezium- shaped field, one of the parallel sides is twice the other. If the area of the field is 9450 m^2 and the perpendicular distance between the two parallel sides is 84 m, find the length of the longer of the parallel sides.

Solution **-** Let length of one parallel side be x cm

Then, length of other parallel side = 2x cm

Distance between them = 84 m

Area of trapezium = 9450 m^2

We know that Area of trapezium = $\frac{1}{2}$ × Sum of parallel sides × Distance between them

9450 = $\frac{1}{2}$ × (x+ 2x) × 84

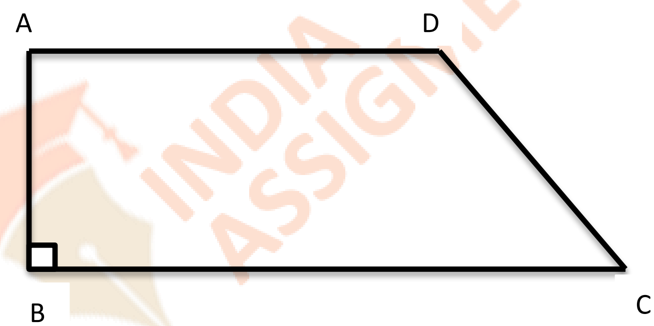
=> 18900/84 = 3x

=> 225 = 3x

=> x = 225/3 = 75 m

Thus, the length of longer parallel side is 2(75) = 150m

Question 9 – The length of the fence of a trapezium-shaped field ABCD is 130 m and side AB is perpendicular to each of the parallel sides AD and BC. If BC = 54 m, CD = 19 m and AD = 42 m, find the area of the field.



Solution - It is given that ABCD is a trapezium and length of fence = 130 m

BC = 54 m, CD = 19 m and AD = 42 m

We know that Length of fence = 130 m

=> Perimeter = 130 m

=> AB+BC+CD+AD = 130

=> AB+54+19+42 = 130

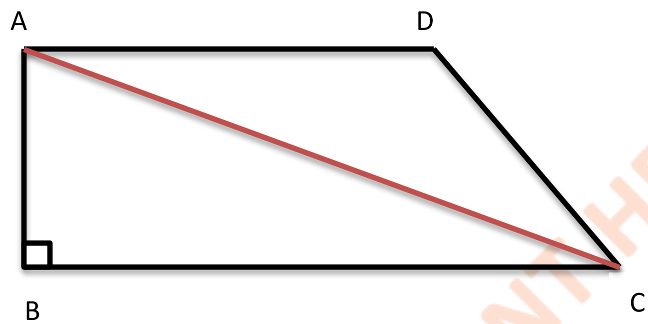
=> AB + 115 = 130

=> AB = 130-115 = 15 m

Area of trapezium = $\frac{1}{2}$ × Sum of parallel sides × Distance between them

Area of trapezium = $\frac{1}{2}$ × (54+42) × 15 = $\frac{96\times15}{2} = 720\ m^2$

Question 10 – In the given figure, ABCD is a trapezium in which AD//BC, $\angle$ ABC = 90^o, AD = 16 cm, AC = 41 cm and BC = 40 cm. Find the area of the trapezium.



Solution - It is given that ABCD is a trapezium with AD//BC

$\angle$ ABC = 90^o, AD = 16 cm, AC = 41 cm, BC = 40 cm

In triangle ABC, by Pythagoras theorem,

$AC^2 = AB^2 + BC^{\ 2}$

=> $41^2 = AB^2 + 40^{\ 2}$

=> 1681 = AB^2 + 1600

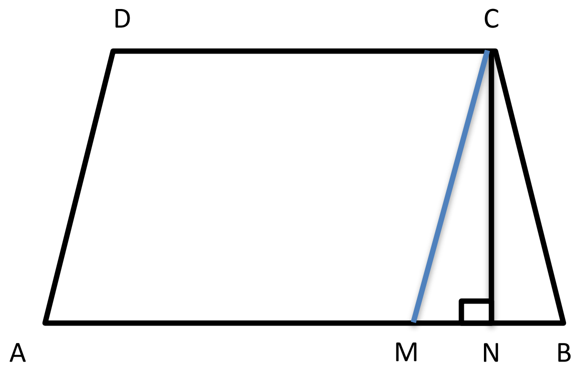
=> 1681-1600 = AB^2

=> AB^2 = 81 cm

=> AB = 9 cm

Question 11 – The parallel sides of a trapezium are 20 cm and 10 cm. Its nonparallel sides are both equal, each being 13 cm. Find the area of the trapezium.

Solution -



ABCD is a trapezium with AB//CD

It is given that AB = 20 cm, CD = 10 cm, AD = BC = 13 cm

We construct CM//AD and CN perpendicular to AB

So, we see that AMCD is a parallelogram.

Since, opposite sides of parallelogram are equal

Thus, AM = CD = 10 cm

MB = AB – AM = 20-10 = 10 cm

Also, AD = CM = 13 cm

Now, we have CM = 13 cm, BC = 13 cm

So, Δ CMB is an isosceles triangle and CN is perpendicular on MB

Thus, N is the midpoint of MB

=> MN = NB = MB/2 = 10/2 = 5 cm

Now, in Δ CMN, by Pythagoras theorem,

$CM^2 = CN^2 + MN^2$

=> $13^2 = CN^2 + 5^2$

=> 169 = CN^2 + 25

=> 144 = CN^2

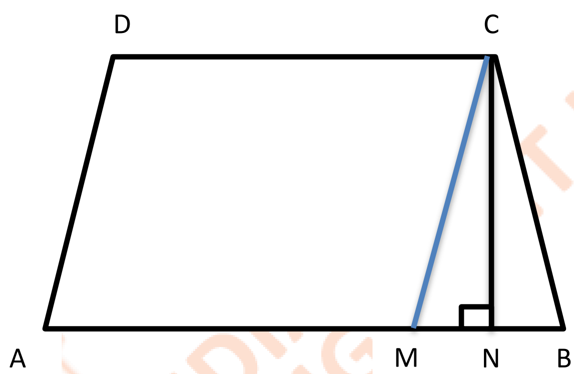
=> CN = 12 cm

Area of trapezium = $\frac{1}{2}$ × Sum of parallel sides × Distance between them

Area of trapezium = $\frac{1}{2}$ × (20+10) × 12 = $\frac{30\times12}{2} = 180cm^2$

Question 12 – The parallel sides of a trapezium are 25 cm and 11 cm, while its nonparallel sides are 15 cm and 13 cm. Find the area of the trapezium.

Solution -



ABCD is a trapezium with AB//CD

It is given that AB = 25 cm, CD = 11 cm, AD = 15 cm and BC = 13 cm

We construct CM//AD and CN perpendicular to AB

So, we see that AMCD is a parallelogram.

Since, opposite sides of parallelogram are equal

Thus, AM = CD = 11 cm

MB = AB – AM = 25-11 = 14 cm

Also, AD = CM = 15 cm

Now, in Δ CMB, we have CM = 15 cm, BC = 13 cm and MB = 14 cm

Area of ΔCMB = $\sqrt{s(s-a)(s-b)(s-c)}$

Where s = (15+13+14)/2 = 42/2 = 21 cm

Area of ΔCMB = $\sqrt{21(21-15)(21-14)(21-13)}$

Area of ΔCMB = $\sqrt{21 \times 6 \times 7 \times 8}$

= $\sqrt{7 \times 3 \times 2 \times 3 \times 7 \times 2 \times 2 \times 2}$

= 2×2×3×7 = 84 cm^2

Also, Area of ΔCMB = ½ × MB ×CN

=> 84 = ½ × 14 ×CN

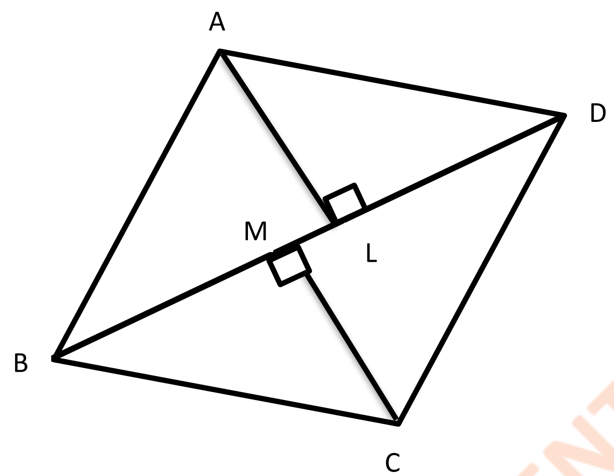
=> CN = 84/7 = 12 cm

Area of trapezium = $\frac{1}{2}$ × Sum of parallel sides × Distance between them

Area of trapezium = $\frac{1}{2}$ × (25+11) × 12 = $\frac{36\times12}{2} = 216cm^2$

Area of a polygon:

For a polygon whether regular or irregular, we calculate their areas by dividing them into rectangles, parallelograms, triangles, and trapezium.



Suppose ABCD is a quadrilateral and BD is a diagonal

We draw perpendicular AL and CM on BD

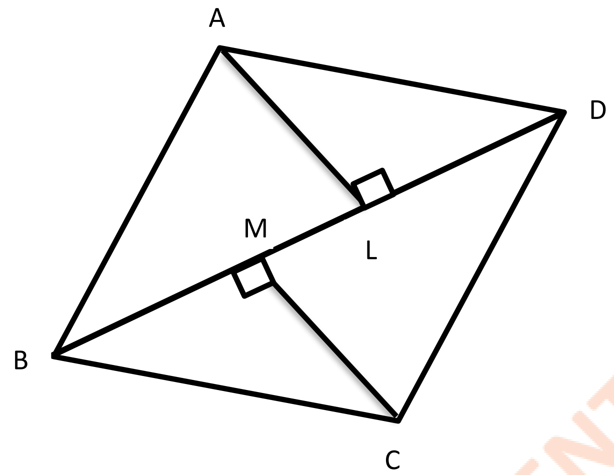
Area of Quadrilateral ABCD = Area of ΔABD + Area of ΔBCD

$= (\frac{1}{2} \times BD \times AL) + (\frac{1}{2} \times BD \times CM)$

$= (\frac{1}{2} \times BD)(AL+CM)$

Examples:

Example 1 – In the given figure, ABCD is a quadrilateral in which BD = 14 cm, AL ⊥ BD, CM ⊥ BD such that AL = 6 cm and CM = 8 cm. Find the area of quad. ABCD



Solution - Given that ABCD is a quadrilateral, AL and CM are perpendiculars on BD

BD = 14 cm, AL = 6 cm and CM = 8 cm

Since Area of Quadrilateral ABCD = Area of ∆ABD + Area of ∆BCD

= (½ × BD × AL) + (½ × BD × CM)

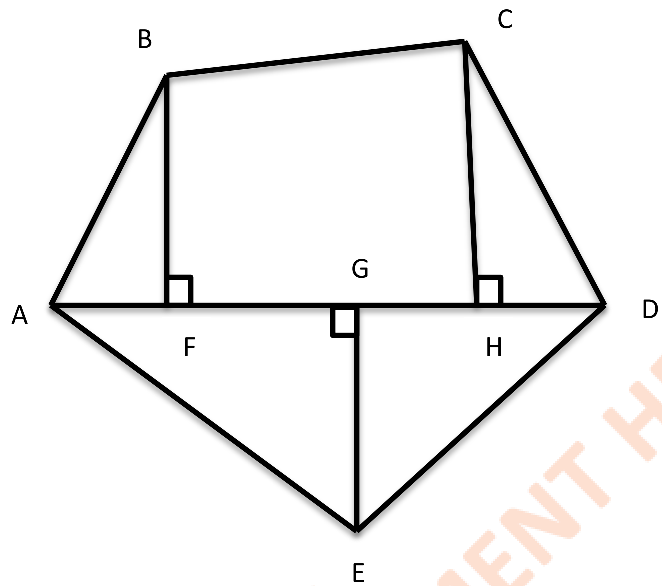
= (½ × BD) (AL+CM)

= (½ × 14) (6+8)

= ½ × 14× 14

= 98 cm^2

Example 2 – Find the area of the given pentagon ABCDE in which each one of BF, CH and EG is perpendicular to AD such that AF = 9 cm, AG = 13 cm, AH = 19 cm, AD = 24 cm, BF = 6 cm, CH = 8 cm and EG = 9 cm.



Solution - Given that ABCDE is a pentagon in which BF, CH and EG are perpendicular to AD

AF = 9 cm, AG = 13cm, AH = 19 cm, AD = 24 cm, BF = 6 cm, CH = 8 cm and EG = 9 cm

HD = AD – AH = 24 – 19 = 5 cm

FH = AH – AF = 19 – 9 = 10 cm

We can see that pentagon ABCDE is divided into 4 parts

Thus, Area of given pentagon ABCDE = area (ΔABF) + area (ΔCHD) + area (ΔAED) + area (trap. BCHF)

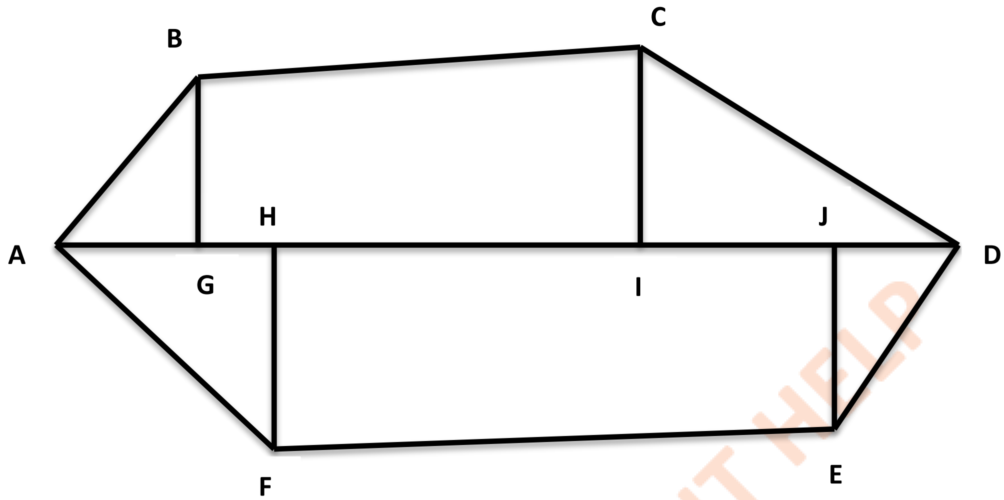
= (½ ×AF × BF) + (½ ×CH × HD) + (½ ×AD × EG) + (½ × (BF+CH) × FH)

= (½ ×9 × 6) + (½ ×8 × 5) + (½ ×24× 9) + (½ × (6+8) × 10)

= (27 + 20 + 108 +70)

= 225 cm^2

Example 3 – Find the area of the given hexagon ABCDEF in which each one of BG, CI, EJ and FH is perpendicular to AD and it is being given that AG = 6 cm, AH = 10 cm, AI = 18 cm, AJ = 21 cm, AD = 27 cm, BG = 5 cm, CI = 6 cm, EJ = 4 cm and FH = 6 cm.



Solution - Given that ABCDEF is a hexagon in which BG, CI, EJ and FH are perpendicular to AD

AG = 6 cm, AH = 10 cm, AI = 18 cm, AJ = 21 cm, AD = 27 cm, BG = 5 cm, CI = 6 cm, EJ = 4 cm and FH = 6 cm

DI = AD – AI = 27 – 18 = 9 cm

DJ = AD – AJ = 27 – 21 = 6 cm

GI = AI – AG = 18 - 6 = 12 cm

HJ = AJ – AH = 21 – 10 = 11 cm

We can see that hexagon ABCDEF is divided into 6 parts

Thus, Area of given hexagon ABCDEF = area (ΔAGB) + area (ΔCID) + area (ΔJDE) + area (ΔAHF) + area (trap. BCIG) + area (trap. HJEF)

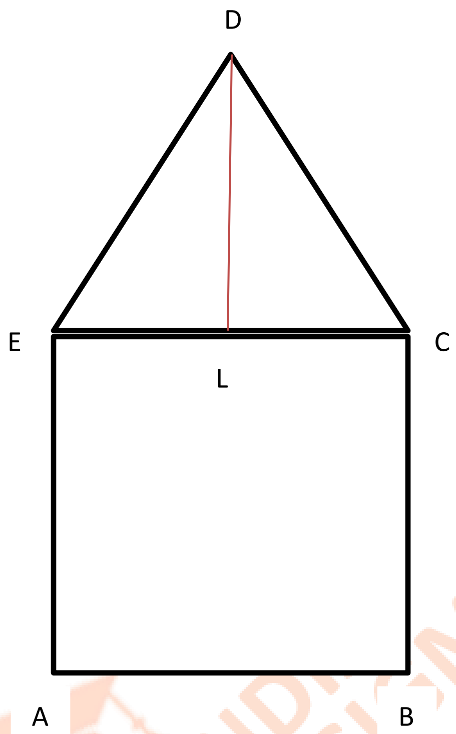
= (½ ×AG × BG) + (½ ×ID × CI) + (½ ×DJ × EJ) + (½ ×AH× HF) + (½ × (BG+CI) × GI) + (½ × (HF+JE) × HJ)

= (½ ×6 × 5) + (½ ×9 × 6) + (½ ×6 × 4) + (½ ×10× 6) + (½ × (5+6) × 12) + (½ × (6+4) × 11)

= (15+27+12+30+66+55)

= 205cm^2

Example 4 – In the given figure ABCDE is a pentagonal park in which DE = DC, AB = BC = CE = EA = 25 m and its total height is 41 m. Find the area of the park.



Solution - It is given that ABCDE is a pentagonal park

DE = DC, AB = BC =CE = EA = 25 m

Total height = 41 m

DL = total height – BC = 41 – 25 = 16 m

We draw DL perpendicular on EC

Area of pentagonal park = area (ΔDCE) + area (square ABCE)

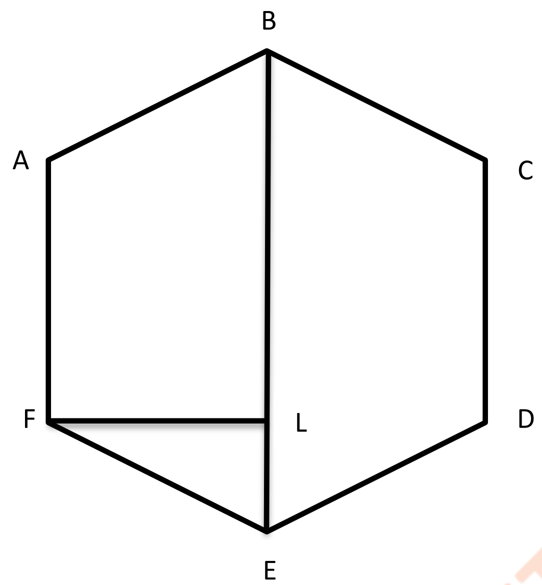
= (½ ×CE × DL) + (AB×AB)

= (½ ×25 × 16) + (25×25)

= 200 + 625

= 825 m^2

Example 5 – Find the area of the given hexagon ABCDEF in which each side measures 5 cm, height BE = 11 cm and width FD = 8 cm



Solution - It is given that ABCDEF is a hexagon in which each side is 5 cm

BE = 11 cm and FD = 8 cm

FL = 8/2 = 4 cm

We join BE

Area of hexagon ABCDEF = area (trap. ABEF) + area (trap. BCDE)

= 2× area (trap. ABEF)

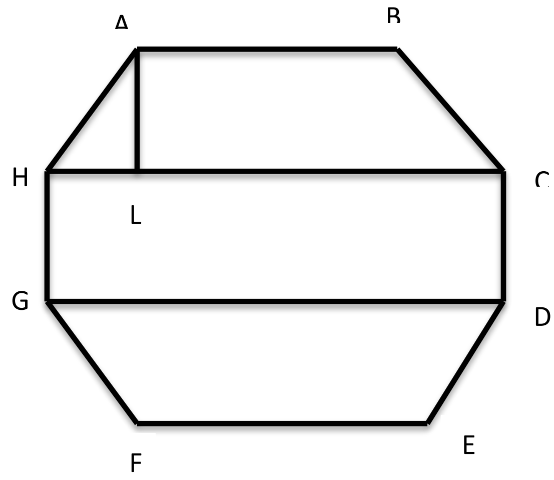
= 2× (½ × (AF+BE) × FL)

= 2× (½ × (5+11) ×4)

= 16×4

= 64 cm^2

Example 6 – Find the area of an octagon ABCDEFGH having each side equal to 5 cm, HC = 11 cm and AL ⊥ HC such that AL = 4 cm



Solution - It is given that ABCDEFGH is an octagon

Each side = 5 cm, HC = 11 cm, AL = 4 cm

We draw AL ⊥ HC

Area of octagon ABCDEFGH = area (trap. ABCD) + area (trap. DEFG) + area (rect. HCDG)

= 2× area (trap. ABCD) + area (rect. HCDG)

= 2× (½ × (AB+HC) × AL) + (GD×CD)

= ((AB+HC) × AL) + (GD×CD)

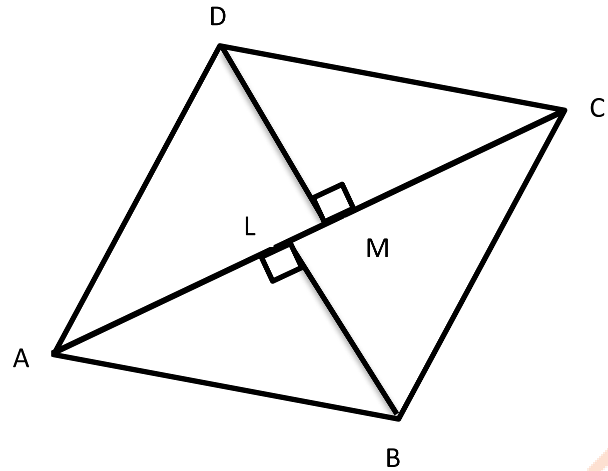
= ((5+11) × 4) + (11×5)

= (16× 4) + (55)

= 64+55 = 119 cm^2

Exercise 18B

Question 1 – In the given figure, ABCD is a quadrilateral in which AC = 24 cm, BL ⊥ AC and DM ⊥ AC such that BL = 8 cm and DM = 7 cm. Find the area of quad. ABCD



Solution - Given that ABCD is a quadrilateral, BL and DM are perpendiculars on AC

AC = 24 cm, BL = 8 cm and DM = 7 cm

Since, Area of Quadrilateral ABCD = Area of ΔADC + Area of ΔABC

= (½ × AC × DM) + (½ × AC × BL)

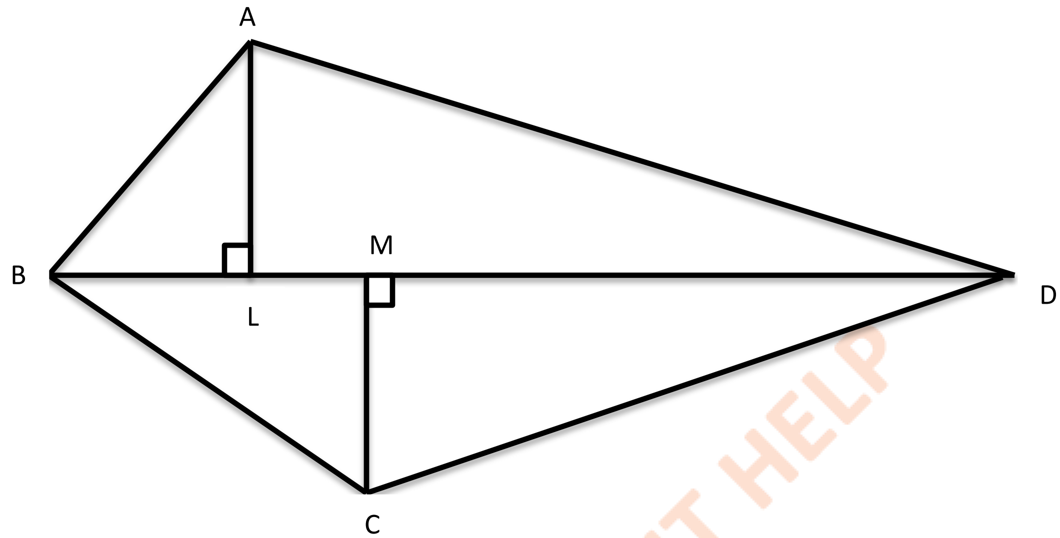
= (½ × AC) (DM+BL)

= (½ × 24) (7+8)

= ½ × 24× 15

= 180 cm^2

Question 2 – In the given figure, ABCD is a quadrilateral-shaped field in which diagonal BD is 36 m, AL ⊥ BD and CM ⊥ BD such that AL = 19 m and CM = 11 m. Find the area of the field.



Solution - It is given that ABCD is a quadrilateral

AL and CM are perpendiculars on BD

BD = 36 m, AL = 19 m and CM = 11 m

Since, Area of Quadrilateral ABCD = Area of ∆ABD + Area of ∆BCD

= (½ × BD × AL) + (½ × BD × CM)

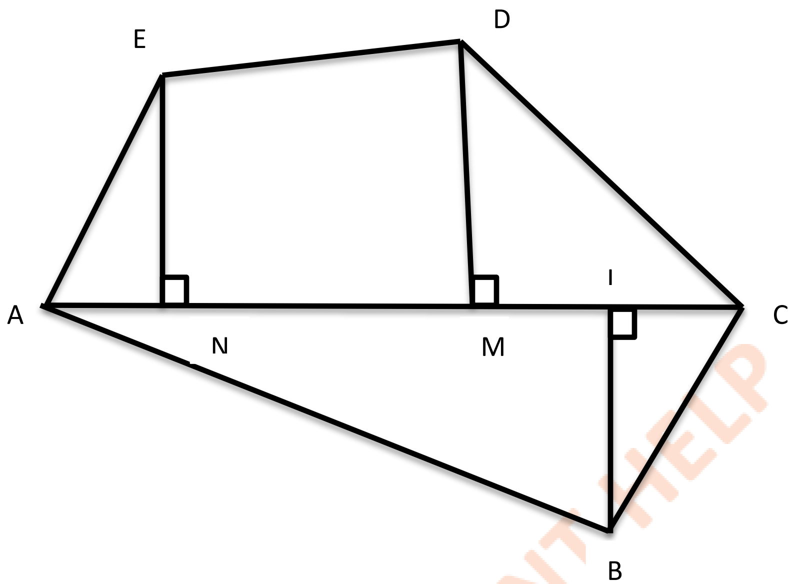
= (½ × BD) (AL+CM)

= (½ × 36) (19+11)

= ½ × 36× 30

= 540 m^2

Question 3 – Find the area of pentagon ABCDE in which BL ⊥ AC, DM ⊥ AC and EN ⊥ AC such that AC = 18 cm, AM = 14 cm, AN = 6 cm, BL = 4 cm, DM = 12 cm and EN = 9 cm.



Solution - Given that ABCDE is a pentagon in which BL, DM and EN are perpendicular on AC

AC = 18 cm, AM = 14 cm, AN = 6 cm, BL = 4 cm, DM = 12 cm and EN = 9 cm

CM = AC – AM = 18 – 14 = 4 cm

NM = AM – AN = 14 – 6 = 8 cm

We can see that pentagon ABCDE is divided into 4 parts

Thus, Area of given pentagon ABCDE = area (ΔAEN) + area (ΔDMC) + area (ΔABC) + area (trap. EDMN)

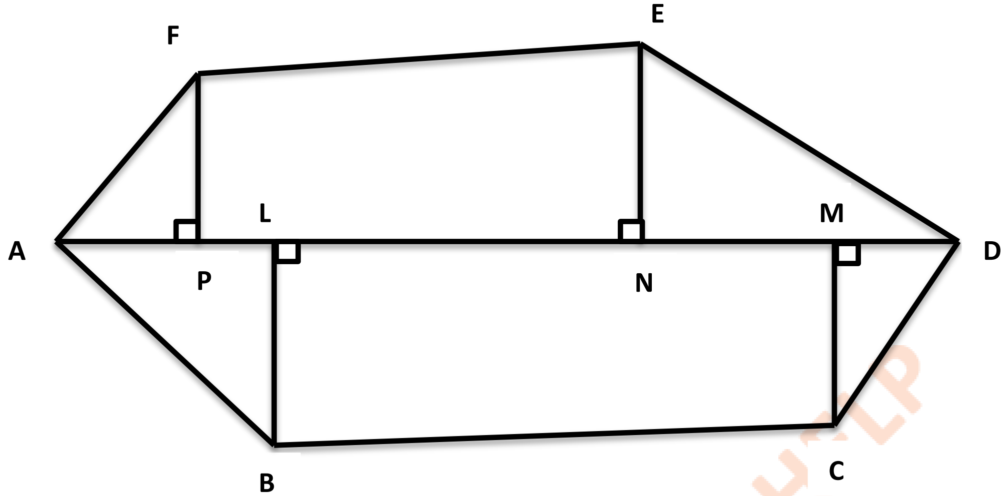
= (½ ×AN × EN) + (½ ×CM × DM) + (½ ×AC × BL) + (½ × (EN+DM) × NM)

= (½ ×6 × 9) + (½ ×4 × 12) + (½ ×18× 4) + (½ × (9+12) × 8)

= (27 + 24 + 36 +84)

= 171 cm^2

Question 4 – Find the area of hexagon ABCDEF in which BL ⊥ AD, CM ⊥ AD, EN ⊥ AD and FP ⊥ AD such that AP = 6 cm, PL = 2 cm, LN = 8 cm, NM = 2 cm, MD = 3 cm, FP = 8 cm, EN = 12 cm, BL = 8 cm and CM = 6 cm.



Solution - Given that ABCDEF is a hexagon in which BL, CM, EN and FP are perpendicular on AD

AP = 6 cm, PL = 2 cm, LN = 8 cm, NM = 2 cm, MD = 3 cm, FP = 8 cm, EN = 12 cm, BL = 8cm and CM = 6 cm

DN = NM+MD = 2+3 = 5 cm

AL = AP+PL = 6+2 = 8 cm

PN = PL+LN = 2+8 = 10 cm

LM = LN+NM = 8+2 = 10 cm

We can see that hexagon ABCDEF is divided into 6 parts

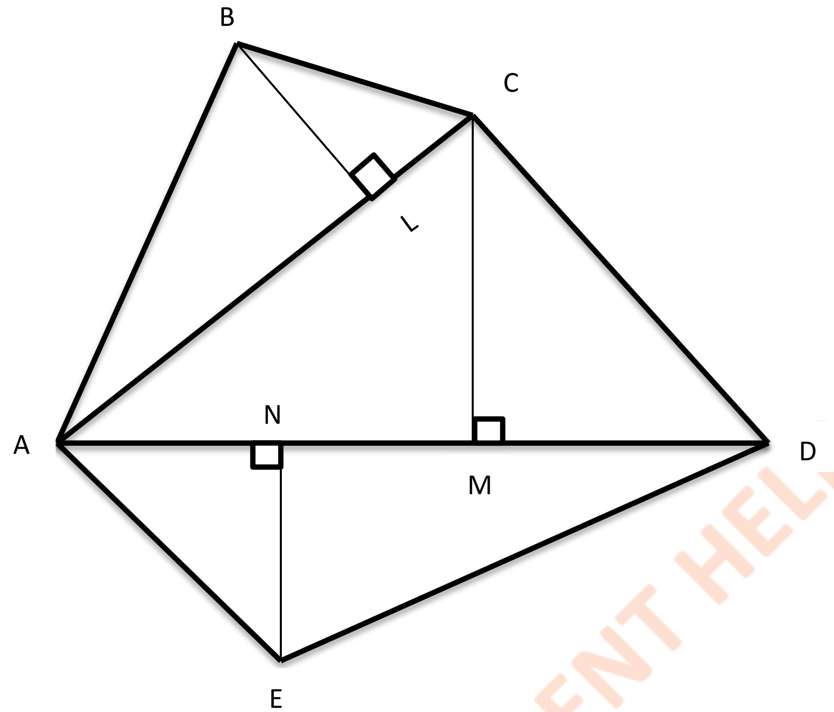
Thus, Area of given hexagon ABCDEF = area (ΔAFP) + area (ΔFND) + area (ΔDMC) + area (ΔABL) + area (trap. FENP) + area (trap. LMCB)

= (½ ×AP × FP) + (½ ×ND × EN) + (½ ×DM × CM) + (½ ×AL× BL) + (½ × (FP+EN) × PN) + (½ × (BL+CM) × LM)

= (½ ×6 × 8) + (½ ×5 × 12) + (½ ×3 × 6) + (½ ×8× 8) + (½ × (8+12) × 10) + (½ × (8+6) × 10)

= (24+30+9+32+100+70) = 256cm^2

Question 5 – Find the area of pentagon ABCDE in which BL ⊥ AC, CM ⊥ AD and EN ⊥ AD such that AC = 10 cm, AD = 12 cm, BL = 3 cm, CM = 7 cm and EN = 5 cm.



Solution - It is given that ABCDE is a pentagon in which BL, CM and EN are perpendicular on AD

AC = 10 cm, AD = 12 cm, BL = 3 cm, CM = 7 cm and EN = 5 cm

We can see that pentagon ABCDE is divided into 3 triangles

Area of pentagon ABCDE = area (ΔABC) + area (ΔADC) + area (ΔAED)

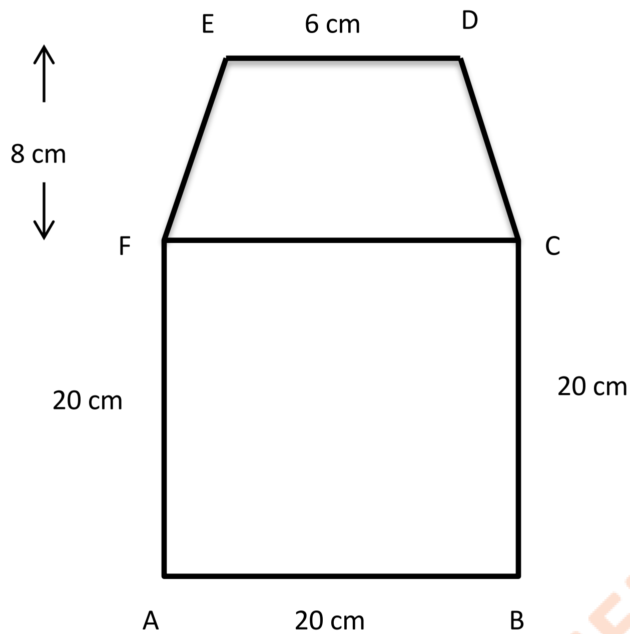
= (½ ×AC × BL) + (½ ×AD× CM) + (½ ×AD × EN)

= (½ ×10 × 3) + (½ ×12× 7) + (½ ×12 × 5)

= 15+42+30

= 87 cm^2

Question 6 – Find the area enclosed by the given figure ABCDEF as per dimensions given herewith.



Solution - From the figure we can see that side of square is 20 cm

And, parallel sides of trapezium are 6 cm and 20 cm respectively and height is 8 cm

Thus, Area of given figure ABCDEF = area (square ABCD) + area (trap. EDCF)

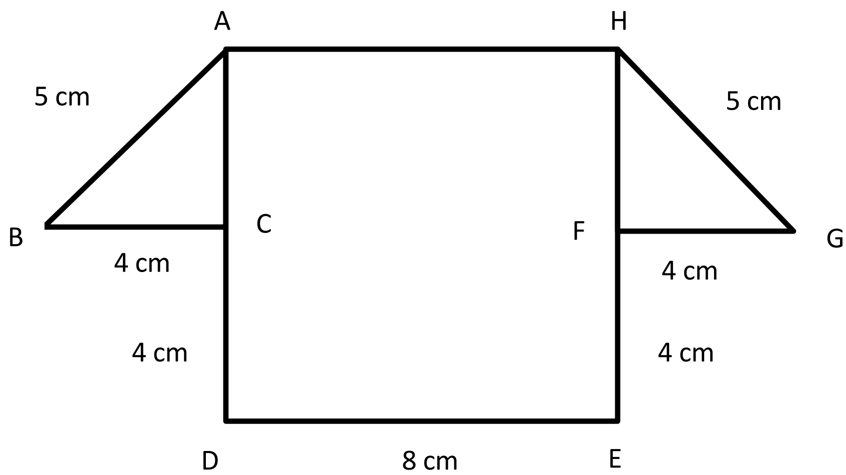
= (AB×AB) + ((½ × (ED+FC) × height)

= (20×20) + ((½ × (6+20) × 8)

= 400 + 104

= 504 cm^2

Question 7 – Find the area of given figure ABCDEFGH as per dimensions given in it.



Solution - From the given figure, we can see that figure ABCDEFGH is divided into 2 triangles and a rectangle

In ΔABC, by Pythagoras theorem,

$AB^2 = AC^2 + BC^2$

$5^2 = AC^2 + 4^2$

25 = AC^2 + 16

25 – 16 = AC^2

AC^2 = 9

AC = 3 cm

So, AC = HF = 3 cm

AD = AC+CD = 3+4 = 7 cm

Thus, area of given figure ABCDEFGH = 2× area (ΔABC) + area (rectangle AHED)

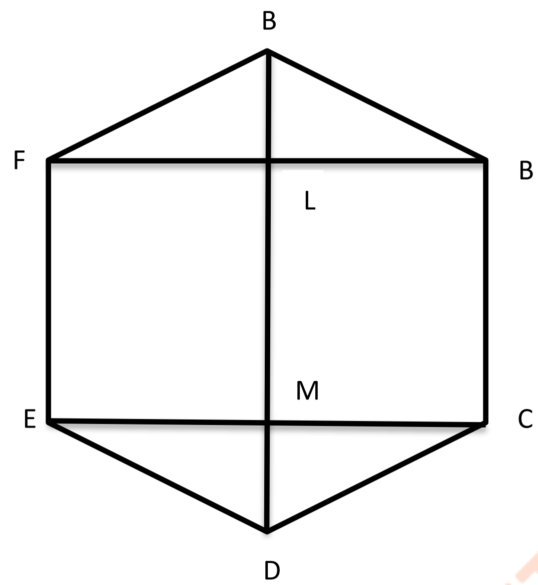
= 2× (½×BC×AC) + (DE×AD)

= (4×3) + (8×7)

= 12+56

= 68 cm^2

Question 8 – Find the area of a regular hexagon ABCDEF in which each side measures 13 cm and whose height is 23 cm, as shown in the given figure.



Solution - It is given that ABCDEF is a hexagon in which each side is 13 cm

Height = 23 cm

Let us take BL = DM = x cm

LM = 13 cm

Since, height = 23 cm

BD = 23 cm

x+x+13 = 23

2x + 13 = 23

2x = 23-13 = 10

x = 5

Thus, BL = DM = 5 cm

Now, in ΔBFL, by Pythagoras theorem,

$BF^2 = BL^2 + FL^2$

$13^2 = 5^2 + FL^{\ 2}$

169 = 25 + $FL^{\ 2}$

169 – 25 = FL^2

FL^2 = 144

FL = 12 cm

Area of hexagon ABCDEF = 2 × Area (trap. AFED)

= 2× (½ × (FE+BD) × FL)

= (13+23) ×12

= 36×12

= 432 cm^2

Exercise 18C

Question 1 – The parallel sides of a trapezium measure 14 cm and 18 cm and the distance between them is 9 cm. The area of the trapezium is?

Solution - It is given that length of two parallel sides is 14 cm and 18 cm respectively

Distance between them (h) = 9 cm

Area of trapezium = $\frac{1}{2}$ × Sum of parallel sides × Distance between them

Area of trapezium = $\frac{1}{2}$ × (14+18) × 9 = $\frac{32\times9}{2} = 144cm^2$

Question 2 – The lengths of the parallel sides of a trapezium are 19 cm and 13 cm and its area is $128cm^2$. The distance between the parallel sides is?

Solution - It is given that length of two parallel sides is 19 cm and 13 cm respectively

Distance between them (h) =?

Area of trapezium = 128 cm^2

Area of trapezium = $\frac{1}{2}$ × Sum of parallel sides × Distance between them

128 = $\frac{1}{2}$ × (19+13) × h

256 = 32h

h = 256/32 = 8 cm

Question 3 – The parallel sides of a trapezium are in the ratio 3:4 and the perpendicular distance between them is 12 cm. If the area of the trapezium is 630cm^2, then its shorter of the parallel sides is?

Solution **-** Let length of one parallel side be 3x cm

Then, length of other parallel side = 4x cm

Distance between them = 12 cm

Area of trapezium = 630 cm^2

We know that Area of trapezium = $\frac{1}{2}$ × Sum of parallel sides × Distance between them

630 = $\frac{1}{2}$ × (3x + 4x) × 12

=> 1260/12 = 7x

=> 105 = 7x

=> x = 105/7 = 15 cm

Thus, lengths of shorter parallel side is 3(15) =45

Question 4 – The area of a trapezium is 180cm^2 and its height is 9 cm. If one of the parallel sides is longer than the other by 6 cm, the length of the longer of the parallel sides is?

Solution **-** Let length of one parallel side be x cm

Then, length of other parallel side = (x+6) cm

Height = 9 cm

Area of trapezium = 180 cm^2

We know that Area of trapezium = $\frac{1}{2}$ × Sum of parallel sides × Distance between them

180 = $\frac{1}{2}$ × (x+ x+6) × 9

=> 360/9 = 6+2x

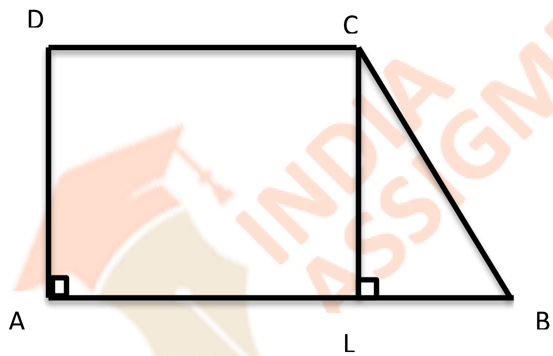
=> 40 = 6+2x

=> 40-6 = 2x

=> x = 34/2 = 17 cm

Thus, lengths of longer parallel side (17+6) = 23 cm

Question 5 – In the given figure, AB//DC and DA ⊥ AB. If DC = 7 cm, BC = 10 cm, AB = 13 cm and CL ⊥ AB, the area of trap. ABCD is?



Solution - It is given that AB//DC and DA and CL are perpendicular on AB

DC = 7 cm, BC = 10 cm, AB = 13 cm

Now, CD = AL = 7 cm

Thus, LB = AB – AL = 13 – 7 = 6 cm

In ΔCLB, by Pythagoras theorem,

$CB^2 = CL^2 + BL^2$

=> $10^2 = CL^2 + 6^2$

=> 100 = CL^2 + 36

=> 64 = CL^2

=> CL = 8 cm

Area of trapezium ABCD = 1/2 × Sum of parallel sides × Distance between them

= ½ × (CD + AB) ×CL

= ½× (7+13) ×8

= ½ × 20×8

= 80 cm^2