Introduction

In this chapter we deal with exponents of rational numbers.

Positive integral exponent of a rational number

Let $\frac{a}{b}$ be any rational number and n be a positive integer. Then,

$(\frac{a}{b})^n = \frac{a^n}{b^n}$

Negative integral exponent of a rational number

Let $\frac{a}{b}$ be any rational number and n be a positive integer. Then,

$(\frac{a}{b})^{-n} = (\frac{b}{a})^n$

Laws of Exponents:

Let $\frac{a}{b}$ be any rational number and m and n be any integers. Then we have:

1) $(\frac{a}{b})^m * (\frac{a}{b})^n = (\frac{a}{b})^{m+n}$

2) $(\frac{a}{b})^m \div (\frac{a}{b})^n = (\frac{a}{b})^{m-n}$

3) $\{(\frac{a}{b})^m\}^n = (\frac{a}{b})^{mn}$

4) $(\frac{a}{b} * \frac{c}{d})^n = (\frac{a}{b})^n * (\frac{c}{d})^n$

5) $(\frac{a}{b} \div \frac{c}{d})^n = (\frac{a}{b})^n \div (\frac{c}{d})^n$

6) $(\frac{a}{b})^{-n} = (\frac{b}{a})^n$

7) $(\frac{a}{b})^0 = 1$

Examples:

Example 1 - Evaluate

(a) 5^{-3}

Solution - Since $(\frac{a}{b})^{-n} = (\frac{b}{a})^{n}$

Thus, $(\frac{5}{1})^{-3} = (\frac{1}{5})^{3} = \frac{1}{5^3} = \frac{1}{5*5*5} = \frac{1}{125}$

(b) $(\frac{1}{3})^{-4}$

Solution - Since $(\frac{a}{b})^{-n} = (\frac{b}{a})^{n}$

Thus, $(\frac{1}{3})^{-4} = (\frac{3}{1})^{4} = 3^4 = 81$

(c) $(\frac{5}{2})^{-3}$

Solution - Since $(\frac{a}{b})^{-n} = (\frac{b}{a})^{n}$

Thus, $(\frac{5}{2})^{-3} = (\frac{2}{5})^{3} = \frac{2^3}{5^3} = \frac{8}{125}$

(d) $(-2)^{-5}$

Solution - Since $(\frac{a}{b})^{-n} = (\frac{b}{a})^{n}$

Thus, $(\frac{-2}{1})^{-5} = (\frac{1}{-2})^{5} = \frac{1}{(-2)^5} = \frac{1}{-32} = \frac{-1}{32}$ (as $(-a)^{odd}$ = negative number)

(e) $(\frac{-3}{4})^{-4}$

Solution - Since $(\frac{a}{b})^{-n} = (\frac{b}{a})^{n}$

Thus, $(\frac{-3}{4})^{-4} = (\frac{4}{-3})^{4} = \frac{4^4}{(-3)^4} = \frac{256}{81}$ (as $(-a)^{even}$ = positive number)

Example 2 - Evaluate

(a) $(\frac{2}{3})^{3} * (\frac{2}{3})^{2}$

Solution - Since $(\frac{a}{b})^{m} * (\frac{a}{b})^{n} = (\frac{a}{b})^{m+n}$

Thus, $\left(\frac{2}{3}\right)^{3} * (\frac{2}{3})^{2} = (\frac{2}{3})^{3+2} = (\frac{2}{3})^{5} = \frac{2^5}{3^5} = \frac{32}{243}$

(b) $(\frac{4}{7})^{5} * (\frac{4}{7})^{-3}$

Solution - Since $(\frac{a}{b})^{m} * (\frac{a}{b})^{n} = (\frac{a}{b})^{m+n}$

Thus, $\left(\frac{4}{7}\right)^{5} * (\frac{4}{7})^{-3} = (\frac{4}{7})^{5+(-3)} = (\frac{4}{7})^{5-3} = (\frac{4}{7})^{2} = \frac{4^2}{7^2} = \frac{16}{49}$

(c) $(\frac{3}{2})^{-3} * (\frac{3}{2})^{-2}$

Solution - Since $(\frac{a}{b})^{m} * (\frac{a}{b})^{n} = (\frac{a}{b})^{m+n}$

Thus, $(\frac{3}{2})^{-3} * (\frac{3}{2})^{-2} = (\frac{3}{2})^{-3+(-2)} = (\frac{3}{2})^{-3-2} = (\frac{3}{2})^{-5}$

Since $(\frac{a}{b})^{-n} = (\frac{b}{a})^{n}$

Thus, $(\frac{3}{2})^{-5} = (\frac{2}{3})^{5} = \frac{2^5}{3^5} = \frac{32}{243}$

(d) $(\frac{8}{5})^{-3} * (\frac{8}{5})^{2}$

Solution - Since $(\frac{a}{b})^{m} * (\frac{a}{b})^{n} = (\frac{a}{b})^{m+n}$

Thus, $(\frac{8}{5})^{-3} * (\frac{8}{5})^{2} = (\frac{8}{5})^{-3+2} = (\frac{8}{5})^{-1}$

Since $(\frac{a}{b})^{-n} = (\frac{b}{a})^{n}$

Thus, $(\frac{8}{5})^{-1} = (\frac{5}{8})^{1} = \frac{5}{8}$

Example 3 - Evaluate $(\frac{3}{8})^{-2} * (\frac{4}{5})^{-3}$

Solution - Since $(\frac{a}{b})^{-n} = (\frac{b}{a})^{n}$

Thus, $(\frac{3}{8})^{-2} * (\frac{4}{5})^{-3}$

$= (\frac{8}{3})^{2} * (\frac{5}{4})^{3}$

$= \frac{8^2}{3^2} * \frac{5^3}{4^3}$

$= \frac{64}{9} * \frac{125}{64} = \frac{125}{9}$

Example 4 - Evaluate $(\frac{-2}{7})^{-4} * (\frac{-5}{7})^2$

Solution - Since $(\frac{a}{b})^{-n} = (\frac{b}{a})^n$

Thus, $(\frac{-2}{7})^{-4} * (\frac{-5}{7})^2$

$= (\frac{7}{-2})^4 * (\frac{-5}{7})^2$

$= \frac{7^4}{(-2)^4} * \frac{(-5)^2}{7^2}$

$= \frac{7^2 * (-5)^2}{(-2)^4}$ (since $\frac{7^4}{7^2} = 7^{4-2} = 7^2$)

$= \frac{49*25}{16} = \frac{1225}{16}$ (since $(-a)^{even}$ = positive number)

Example 6 - Evaluate $\{(\frac{-3}{2})^2\}^{-3}$

Solution - Since $\{(\frac{a}{b})^m\}^n = (\frac{a}{b})^{mn}$

Thus, $\{(\frac{-3}{2})^2\}^{-3} = (\frac{-3}{2})^{-6}$

Since $(\frac{a}{b})^{-n} = (\frac{b}{a})^n$

Thus, $(\frac{-3}{2})^{-6} = (\frac{2}{-3})^6$

$= \frac{2^6}{(-3)^6} = \frac{64}{729}$ (since $(-a)^{even}$ = positive number)

Example 7 - Simplify

(a) $(2^{-1} * 5^{-1})^{-1} \div 4^{-1}$

Solution - We first solve brackets

$(\frac{1}{2} * \frac{1}{5})^{-1} \div 4^{-1}$

$= (\frac{1}{10})^{-1} \div 4^{-1}$ (using $(\frac{a}{b})^{-n} = (\frac{b}{a})^n$)

$= 10 \div 4^{-1}$

$= 10 \div \frac{1}{4} = 10 * 4 = 40$

(b) $\mathbf{(4^{-1} + 8^{-1})^{\ 1} \div (\frac{2}{3})^{-1}}$

Solution - We first solve bracket

$= (\frac{1}{4} + \frac{1}{8}) \ \div (\frac{3}{2})^1$ (using $(\frac{a}{b})^{-n} = (\frac{b}{a})^n$)

$= (\frac{2+1}{8}) \div \frac{3}{2} = \frac{3}{8} * \frac{2}{3} = \frac{6}{24} = \frac{1}{4}$

Example 8 - Simplify $\mathbf{(\frac{1}{2})^{-2} + (\frac{1}{3})^{-2} + (\frac{1}{4})^{-2}}$

Solution $(\frac{1}{2})^{-2} + (\frac{1}{3})^{-2} + (\frac{1}{4})^{-2}$

$= 2^2 + 3^2 + 4^2$ (using $(\frac{a}{b})^{-n} = (\frac{b}{a})^n$)

= 4 + 9 + 16

= 29

Example 9 - By what number should $\mathbf{(\frac{1}{2})^{-1}}$ **be multiplied so that the product is** $\mathbf{(\frac{-5}{4})^{-1}}$**?**

Solution - Let the required number be x

According to given question,

$(\frac{1}{2})^{-1} \times x = (\frac{-5}{4})^{-1}$

=> $2 \times x = \frac{4}{-5}$ (using $(\frac{a}{b})^{-n} = (\frac{b}{a})^n$)

=> $x = \frac{4}{-5} \div 2 = \frac{4}{-5} \times \frac{1}{2} = \frac{2}{-5} = \frac{-2}{5}$

Example 10 - By what number should $\mathbf{(\frac{-3}{2})^{-3}}$ **be divided so that the quotient is** $\mathbf{(\frac{9}{4})^{-2}}$**?**

Solution - Let the required number be x

According to given question,

$(\frac{-3}{2})^{-3} \div x = (\frac{9}{4})^{-2}$

=> $(\frac{-2}{3})^3 * \frac{1}{x} = (\frac{4}{9})^2$

=> $\frac{(-2)^3}{(3)^3} * \frac{1}{x} = \frac{(4)^2}{(9)^2}$

=> $\frac{-8}{27} = \frac{16}{81} * x$ (as $(-a)^{odd}$ = negative number)

=> $x = \frac{-8}{27} * \frac{81}{16}$

=> $x = \frac{-3}{2}$ (as 8 is common divisor of 8 & 16 and 27 is common divisor of 81 & 27)

Exercise 2A

Question 1 - Evaluate

(a) 4^{-3}

Solution - Since $(\frac{a}{b})^{-n} = (\frac{b}{a})^{n}$

Thus, $4^{-3} = (\frac{1}{4})^{3}$

$= \frac{1}{4^3} = \frac{1}{64}$

(b) $(\frac{1}{2})^{-5}$

Solution - Since $(\frac{a}{b})^{-n} = (\frac{b}{a})^{n}$

Thus, $(\frac{1}{2})^{-5} = 2^5 = 32$

(c) $(\frac{4}{3})^{-3}$

Solution - Since $(\frac{a}{b})^{-n} = (\frac{b}{a})^{n}$

Thus, $(\frac{4}{3})^{-3} = (\frac{3}{4})^{3}$

$= \frac{3^3}{4^3} = \frac{27}{64}$

(d) $(-3)^{-4}$

Solution - Since $(\frac{a}{b})^{-n} = (\frac{b}{a})^{n}$

Thus, $(-3)^{-4} = (\frac{-1}{3})^{4}$

$= \frac{(-1)^4}{3^4} = \frac{1}{81}$

(e) $\mathbf{(\frac{-2}{3})^{-5}}$

Solution - Since $(\frac{a}{b})^{-n} = (\frac{b}{a})^{n}$

Thus, $(\frac{-2}{3})^{-5} = (\frac{-3}{2})^{5}$

$= \frac{(-3)^5}{2^5} = \frac{-243}{32}$

Question 2 - Evaluate

(a) $\mathbf{(\frac{5}{3})^{2} * (\frac{5}{3})^{2}}$

Solution - Since $(\frac{a}{b})^{m} * (\frac{a}{b})^{n} = (\frac{a}{b})^{m+n}$

Thus, $(\frac{5}{3})^{2} * (\frac{5}{3})^{2} = (\frac{5}{3})^{2+2}$

$= (\frac{5}{3})^{4} = \frac{5^4}{3^4} = \frac{625}{81}$

(b) $\mathbf{(\frac{5}{6})^{6} * (\frac{5}{6})^{-4}}$

Solution - Since $(\frac{a}{b})^{m} * (\frac{a}{b})^{n} = (\frac{a}{b})^{m+n}$

Thus, $(\frac{5}{6})^{6} * (\frac{5}{6})^{-4} = (\frac{5}{6})^{6+(-4)}$

$= (\frac{5}{6})^{6-4} = (\frac{5}{6})^{2} = \frac{5^2}{6^2} = \frac{25}{36}$

(c) $\mathbf{(\frac{2}{3})^{-3} * (\frac{2}{3})^{-2}}$

Solution - Since $(\frac{a}{b})^{m} * (\frac{a}{b})^{n} = (\frac{a}{b})^{m+n}$

Thus, $(\frac{2}{3})^{-3} * (\frac{2}{3})^{-2} = (\frac{2}{3})^{-3+(-2)}$

$= (\frac{2}{3})^{-3+(-2)} = (\frac{2}{3})^{-3-2} = (\frac{2}{3})^{-5}$

$= (\frac{3}{2})^{5} = \frac{3^5}{2^5} = \frac{243}{32}$

(d) $\mathbf{(\frac{9}{8})^{-3} * (\frac{9}{8})^{2}}$

Solution - Since $(\frac{a}{b})^{m} * (\frac{a}{b})^{n} = (\frac{a}{b})^{m+n}$

Thus, $(\frac{9}{8})^{-3} * (\frac{9}{8})^{2} = (\frac{9}{8})^{-3+2}$

$= (\frac{9}{8})^{-1} = (\frac{8}{9})^{1} = \frac{8}{9}$ (as $(\frac{a}{b})^{-n} = (\frac{b}{a})^{n}$)

Question 3 - Evaluate

(a) $(\frac{5}{9})^{-2} * (\frac{3}{5})^{-3} * (\frac{3}{5})^{0}$

Solution $(\frac{9}{5})^{2} * (\frac{5}{3})^{3} * (\frac{3}{5})^{0}$ (as $(\frac{a}{b})^{-n} = (\frac{b}{a})^{n}$)

$= \frac{9^2}{5^2} * \frac{5^3}{3^3} * 1$ (as $(\frac{a}{b})^{0} = 1$)

$= \frac{81}{25} * \frac{125}{27}$

= 3 * 5 = 15 (as 27 is common divisor of 81 & 27 and 25 is common divisor of 125 & 25)

(b) $(\frac{-3}{5})^{-4} * (\frac{-2}{5})^{2}$

Solution $(\frac{-5}{3})^{4} * (\frac{-2}{5})^{2}$ (as $(\frac{a}{b})^{-n} = (\frac{b}{a})^{n}$)

$= \frac{(-5)^4}{3^4} * \frac{(-2)^2}{5^2} = \frac{(5)^4}{3^4} * \frac{(2)^2}{5^2}$

$= \frac{(5)^{4-2} * 4}{81} = \frac{(5)^2 * 4}{81} = \frac{25*4}{81} = \frac{100}{81}$ $(as (\frac{a}{b})^{m} \div (\frac{a}{b})^{n} = (\frac{a}{b})^{m-n})$

(c) $(\frac{-2}{3})^{-3} * (\frac{-2}{3})^{-2}$

Solution - Since $(\frac{a}{b})^{m} * (\frac{a}{b})^{n} = (\frac{a}{b})^{m+n}$

Thus, $(\frac{-2}{3})^{-3} * (\frac{-2}{3})^{-2} = (\frac{-2}{3})^{-3+(-2)}$

$= (\frac{-2}{3})^{-3-2} = (\frac{-2}{3})^{-5} = (\frac{-3}{2})^{5}$

$= \frac{(-3)^5}{2^5} = \frac{-243}{32}$

Question 4 - Evaluate

(a) $\{(\frac{-2}{3})^{2}\}^{-2}$

Solution - Since $\{(\frac{a}{b})^{m}\}^{n} = (\frac{a}{b})^{mn}$

Thus, $\{(\frac{-2}{3})^2\}^{-2} = (\frac{-2}{3})^{-4}$

$= \left(\frac{-3}{2}\right)^4 = \frac{3^4}{2^4} = \frac{81}{16}$

(b) $[\{(\frac{-1}{3})^2\}^{-2}]^{-1}$

Solution - Since $\{(\frac{a}{b})^m\}^n = (\frac{a}{b})^{mn}$

Thus, $[\{(\frac{-1}{3})^2\}^{-2}]^{-1} = \{(\frac{-1}{3})^2\}^2$

$= (\frac{-1}{3})^4 = \frac{1}{3^4} = \frac{1}{81}$ $\quad (as\ (-1)^4 = 1)$

(c) $\{(\frac{3}{2})^{-2}\}^2$

Solution - Since $\{(\frac{a}{b})^m\}^n = (\frac{a}{b})^{mn}$

Thus, $\{(\frac{3}{2})^{-2}\}^2 = (\frac{3}{2})^{-4}$

$= (\frac{2}{3})^4 = \frac{2^4}{3^4} = \frac{16}{81}$

Question 5 - Evaluate $\{(\frac{1}{3})^{-3} - (\frac{1}{2})^{-3}\} \div (\frac{1}{4})^{-3}$

Solution - We first solve curly brackets

$= \{(\frac{1}{3})^{-3} - (\frac{1}{2})^{-3}\} \div (\frac{1}{4})^{-3}$

$= \{3^3 - 2^3\} \div 4^3$

$= \{27 - 8\} \div 64$

$= 19 \div 64$

$= \frac{19}{64}$

Question 6 - Evaluate $\{(\frac{4}{3})^{-1} - (\frac{1}{4})^{-1}\}^{-1}$

Solution $\{(\frac{3}{4})^1 - (\frac{4}{1})^1\}^{-1}$

$= \{\frac{3}{4} - \frac{4}{1}\}^{-1}$

$= \left\{\frac{3-16}{4}\right\}^{-1} = \left\{\frac{-13}{4}\right\}^{-1} = \frac{-4}{13}$

Question 7 - Evaluate $[(\mathbf{5^{-1} * 3^{-1})^{-1} \div 6^{-1}}]$

Solution $[(\frac{1}{5} * \frac{1}{3})^{-1} \div \frac{1}{6}]$

$= [\left(\frac{1}{15}\right)^{-1} \div \frac{1}{6}]$

$= [15 \div \frac{1}{6}]$

$= [15 * 6]$

= 90

Question 8 - Find the value of

(a) $\mathbf{(2^0 + 3^{-1}) * 3^2}$

Solution $(1 + \frac{1}{3}) * 9$

$= (\frac{3+1}{3}) * 9$

$= \frac{4}{3} * 9 = 12$

(b) $\mathbf{(2^{-1} * 3^{-1}) \div 2^{-3}}$

Solution $(\frac{1}{2} * \frac{1}{3}) \div 2^{-3}$

$= \frac{1}{6} \div \frac{1}{2^3} = \frac{1}{6} * 2^3$

$= \frac{8}{6} = \frac{4}{3}$

(c) $\mathbf{(\frac{1}{2})^{-2} + (\frac{1}{3})^{-2} + (\frac{1}{4})^{-2}}$

Solution $2^2 + 3^2 + 4^2$

= 4 + 9 + 16

= 29

Question 9 - Find the value of x for which $(\frac{5}{3})^{-4} * (\frac{5}{3})^{-5} = (\frac{5}{3})^{3x}$

Solution - Since $(\frac{a}{b})^m * (\frac{a}{b})^n = (\frac{a}{b})^{m+n}$

Thus, $(\frac{5}{3})^{-4} * (\frac{5}{3})^{-5} = (\frac{5}{3})^{3x}$

=> $(\frac{5}{3})^{-4-5} = (\frac{5}{3})^{3x}$

=> $(\frac{5}{3})^{-9} = (\frac{5}{3})^{3x}$

Now as base is same thus we can equate powers

=> -9 = 3x

=> x = $\frac{-9}{3}$ = -3

Question 10 - Find the value of x for which $(\frac{4}{9})^{4} * (\frac{4}{9})^{-7} = (\frac{4}{9})^{2x-1}$

Solution - Since $(\frac{a}{b})^m * (\frac{a}{b})^n = (\frac{a}{b})^{m+n}$

Thus, $(\frac{4}{9})^{4} * (\frac{4}{9})^{-7} = (\frac{4}{9})^{2x-1}$

=> $(\frac{4}{9})^{4-7} = (\frac{4}{9})^{2x-1}$

=> $(\frac{4}{9})^{-3} = (\frac{4}{9})^{2x-1}$

Now as base is same thus we can equate powers

=> -3 = 2x – 1

=> 2x = -3 + 1

=> 2x = -2

=> x = $\frac{-2}{2}$ = -1

Question 11 - By what number should $(-6)^{-1}$ be multiplied so that the product becomes 9^{-1}?

Solution - Let the required number be x

Then according to given question,

$(-6)^{-1} * x = 9^{-1}$

=> $\frac{-1}{6} * x = \frac{1}{9}$

=> $-x = \frac{1}{9} * 6$

=> $x = \frac{-2}{3}$

Question 12 - By what number should $(\frac{-2}{3})^{-3}$ be divided so that the quotient may be $(\frac{4}{27})^{-2}$?

Solution - Let the required number be x

Then according to given question,

$(\frac{-2}{3})^{-3} \div x = (\frac{4}{27})^{-2}$

=> $(\frac{-3}{2})^{3} \div x = (\frac{27}{4})^{2}$

=> $\frac{(-3)^3}{2^3} * \frac{1}{x} = \frac{27^2}{4^2}$

=> $\frac{-27}{8} * \frac{1}{x} = \frac{729}{16}$

=> $\frac{-27}{8} * \frac{16}{729} = x$

=> $\frac{-2}{27} = x$ (as 8 is common divisor of 16 & 8 and 27 is common divisor of 27 & 729)

Question 13 - If $5^{2x+1} \div 25 = 125$, find the value of x.

Solution - Since $25 = 5^2$ and $125 = 5^3$

Thus, we can write given equation as follows

$5^{2x+1} \div 25 = 125$

=> $5^{2x+1} \div 5^2 = 5^3$

=> $\frac{5^{2x+1}}{5^2} = 5^3$ $((\frac{a}{b})^m \div (\frac{a}{b})^n = (\frac{a}{b})^{m-n})$

=> $(5)^{2x+1-2} = 5^3$

=> $(5)^{2x-1} = 5^3$

Now as base is same so we can equate the powers

=> 2x-1 = 3

=> 2x = 3+1

=> 2x = 4

=> x = 2

Numbers in standard form: A number ($m \times 10^n$) is said to be in standard form if m is a decimal number such that $1 \leq m < 10$ and n is either a positive or a negative integer.

Examples:

Example 1 - Express each of the following numbers in standard form:

(a) 6872

Solution - For standard form, we write it in the form ($m \times 10^n$)

$6872 = 6.872 \times 1000 = 6.872 \times 10^3$

(b) 140000

Solution - For standard form, we write it in the form ($m \times 10^n$)

$140000 = 14 \times 10000 = 14 \times 10^4 = 1.4 \times 10 \times 10^4 = 1.4 \times 10^5$

(c) 15360000000

Solution - For standard form, we write it in the form ($m \times 10^n$)

$15360000000 = 1536 \times 10000000 = 1536 \times 10^7 = 1.536 \times 10^3 \times 10^7$

$= 1.536 \times 10^{10}$

Example 2 - The diameter of the sun is (1.4×10^9)m and the diameter of the earth is (1.2756×10^7)m. Show that the diameter of the sun is nearly 100 times the diameter of the earth.

Solution - We are given that diameter of sun = (1.4×10^9)m

Diameter of earth = 1.2756×10^7m

To prove: diameter of earth = 100 × (diameter of sun)

Proof: We find the ratio of diameters as follows:

=> $\frac{\text{diameter of sun}}{\text{diameter of earth}} = \frac{1.4 \times 10^9}{1.2756 \times 10^7}$

We first solve the powers of 10

$\frac{\text{diameter of sun}}{\text{diameter of earth}} = \frac{1.4 \times 10^{9-7}}{1.2756} = \frac{1.4 \times 10^{2}}{1.2756}$

We now remove decimal from the figures and convert in powers of 10

$\frac{\text{diameter of sun}}{\text{diameter of earth}} = \frac{14 \times 10^{2} \times 10^{4}}{12756 \times 10} = \frac{14 \times 10^{2+4-1}}{12756} = \frac{14 \times 10^{5}}{12756} = 100$ (nearly)

=> Diameter of earth = 100 × (diameter of sun)

Hence proved

Example 3 - In a stack there are 4 books each of thickness 24 mm and 6 paper sheets each of thickness 0.015 mm. What is the total thickness of the stack in standard form?

Solution - Thickness of 1 book = 24 mm

Thickness of 4 books = 24×4 = 96 mm

Thickness of 1 paper sheet = 0.015 mm

Thickness of 6 paper sheets = 0.015 × 6 = 0.09 mm

Thus total thickness of stack = (96+0.09) mm = 96.09 mm

In standard form,

We first remove decimal from figure

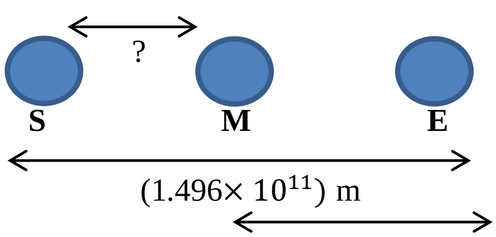
96.09 mm = $\frac{9609}{100}$ mm = $\frac{9.609 \times 1000}{100}$ = 9.609× 10 mm

Example 4 - The distance between sun and earth is (1.496×10^{11}) m and the distance between earth and moon is (3.84×10^{8}) m. During solar eclipse moon comes in between earth and sun. At that time what is the distance between moon and sun?

Solution - Given that distance between sun and earth = (1.496×10^{11})m

Distance between earth and moon = (3.84×10^{8})m

Distance between moon and sun =?



(3.84×10^{8})m

So we can see from the figure that, distance between moon and sun is obtained by subtracting distance between sun and earth & distance between moon and earth.

Thus, Distance between moon and sun = (1.496×10^{11}) - (3.84×10^{8})

= $(\frac{1496\times 10^{11}}{1000}) - (\frac{384\times 10^{8}}{100})$

= $(1496 \times 10^{11-3}) - (384 \times 10^{8-2})$

= $(1496 \times 10^{8}) - (384 \times 10^{6})$

We can take 10^{6} as common value

= $10^{6}\,((1496 \times 10^{2}) - 384)$

= 10^{6}(149600 - 384)

= 149216×10^{6}

In standard form,

= $1.49216 \times 10^{5} \times 10^{6}$

= 1.49216×10^{11} m

Example 5 - Write each of the following numbers in usual form:

(a) $\mathbf{4.61 \times 10^{5}}$

Solution - $4.61 \times 10^{5} = \frac{461\times 10^{5}}{10^{2}}$

= $461 \times 10^{5-2} = 461 \times 10^{3}$

= 461000

(b) $\mathbf{2.514 \times 10^{7}}$

Solution - $2.514 \times 10^{7} = \frac{2514\times 10^{7}}{10^{3}}$

= $2514 \times 10^{7-3} = 2514 \times 10^{4}$

= 25140000

(c) 2.0001×10^8

Solution - $2.0001 \times 10^8 = \frac{20001 \times 10^8}{10^4}$

$= 20001 \times 10^{8-4} = 20001 \times 10^4$

=200010000

Example 6 - Express each of the following numbers in standard form:

(a) 0.00002

Solution - Firstly we remove decimal,

$0.00002 = \frac{2}{10^5} = 2 \times 10^{-5}$

(b) 0.000000061

Solution - Firstly we remove decimal,

$0.000000061 = \frac{61}{10^9} = \frac{6.1 \times 10}{10^9} = 6.1 \times 10^{(1-9)}$

$= 6.1 \times 10^{-8}$

(c) 0.00000000837

Solution - Firstly we remove decimal,

$0.00000000837 = \frac{837}{10^{11}} = \frac{8.37 \times 10^2}{10^{11}}$

$= 8.37 \times 10^{(2-11)} = 8.37 \times 10^{-9}$

Example 7 -the size of a red blood cell is 0.000007 m and that of a plant cell is 0.00001275 m. Show that a red blood cell is half of plant cell in size.

Solution - Given that size of red blood cell = 0.000007 m = $\frac{7}{10^6} = 7 \times 10^{-6}$m

Size of plant cell = 0.00001275 m = $\frac{1275}{10^8} = 1275 \times 10^{-8} = 1.275 \times 10^3 \times 10^{-8}$

$= 1.275 \times 10^{3-8} = 1.275 \times 10^{-5}$m

Now to prove: size of red blood cell = $\frac{1}{2}$ (size of plant cell)

We find the ratio between the two:

$\frac{\text{size of red blood cell}}{\text{size of plant cell}} = \frac{7\times10^{-6}}{1.275\times10^{-5}} = \frac{7\times10^{-6+5}}{1.275} = \frac{7\times10^{-1}}{1.275}$

$= \frac{7\times10^{-1}\times10^{3}}{1275} = \frac{7\times10^{-1+3}}{1275} = \frac{7\times10^{2}}{1275} = \frac{700}{1275} = 0.5$ (approx.) $= \frac{1}{2}$ (approx.)

Thus size of red blood cell = $\frac{1}{2}$ (size of plant cell)

Example 8 - Express the following numbers in usual form:

(a) 2×10^{-5}

Solution $2 \times 10^{-5} = \frac{2}{10^5} = \frac{2}{100000}$

= 0.00002

(b) 6.32×10^{-4}

Solution $6.32 \times 10^{-4} = \frac{6.32}{10^4}$

$= \frac{6.32}{10000} = 0.000632$

(c) 1.596×10^{-6}

Solution $1.596 \times 10^{-6} = \frac{1.596}{10^6}$

$= \frac{1.596}{1000000} = 0.000001596$

Exercise 2B

Question 1 - Write each of the following numbers in standard form:

(a) 57.36

Solution**:** $57.36 = \frac{5736}{100} = \frac{5736}{10^2} = \frac{5.736\times10^3}{10^2}$

$= 5.736 \times 10^{3-2} = 5.736 \times 10^1$

(b) 3500000

Solution: $3500000 = 35 \times 10^5 = 3.5 \times 10 \times 10^5$

$= 3.5 \times 10^6$

(c) 273000

Solution: 273000 = $273 \times 10^3 = 2.73 \times 100 \times 10^3$

= 2.73×10^5

(d) 168000000

Solution: 168000000 = 168×10^6

= $1.68 \times 100 \times 10^6$ =1.68×10^8

(e) 4630000000000

Solution**:** 4630000000000 = 463×10^{10}

= $4.63 \times 100 \times 10^{10}$ = 4.63×10^{12}

(f) $\mathbf{345 \times 10^5}$

Solution: 345×10^5 = $3.45 \times 100 \times 10^5$

= 3.45×10^7

Question 2 - Write each of the following numbers in usual form:

(a) $\mathbf{3.74 \times 10^5}$

Solution: $\frac{374\times10^5}{100}$ =374×10^3 = 374000

(b) $\mathbf{6.912 \times 10^8}$

Solution: $\frac{6912\times10^8}{1000}$ = 6912×10^5

= 691200000

(c) $\mathbf{4.1253 \times 10^7}$

Solution: $\frac{41253\times10^7}{10^4}$ = 41253×10^3

= 41253000

(d) $\mathbf{2.5 \times 10^4}$

Solution: $\frac{25\times10^4}{10}$ = 25×10^3

= 25000

(e) 5.17×10^6

Solution: $\frac{517\times10^6}{100}$ = 517×10^4

= 5170000

(f) 1.679×10^9

Solution: $\frac{1679\times10^9}{10^3}$ = 1679×10^6

= 1679000000

Question 3 (a) -The height of Mount Everest is 8848 m. Write it in standard form

Solution: In standard form,

Height of Mount Everest = 8848 = 8.848 $\times$ 10^3 m

(b) The speed of light is 300000000 m /sec .Express it in standard form.

Solution: In standard form,

Speed of light = 300000000 = 3 $\times$ 10^8 m/sec

(c) The distance from the earth to the sun is 149600000000 m .Write it in standard form.

Solution: In standard form,

Distance from earth to sun = 149600000000 = 1496×10^8

= $1.496 \times 10^3 \times 10^8 = 1.496 \times 10^{11}$ m

Question 4 - Mass of earth is (5.97×10^{24})kg and mass of moon is (7.35×10^{22})kg. What is the total mass of the two?

Solution: Total mass = Mass of earth + Mass of moon

= (5.97×10^{24})+ (7.35×10^{22})

= 10^{22}((5.97×10^2)+7.35)

= $10^{22}((\frac{597\times100}{100}) + 7.35)$

= 10^{22}(597+7.35) = $10^{22}(604.35)$

$$= \frac{60435 \times 10^{22}}{100} = 60435 \times 10^{20} = 60435 \times 10^{4} \times 10^{22} = 6.0435 \times 10^{26} \text{ kg}$$

Question 5 - Write each of the following numbers in standard form:

(a) 0.0006

Solution: $\frac{6}{10000} = \frac{6}{10^4} = 6 \times 10^{-4}$

(b) 0.00000083

Solution: $\frac{83}{100000000} = \frac{83}{10^8} = 83 \times 10^{-8} = 8.3 \times 10 \times 10^{-8} = 8.3 \times 10^{-7}$

(c) 0.0000000534

Solution: $\frac{534}{10000000000} = \frac{534}{10^{10}} = 534 \times 10^{-10} = 5.34 \times 100 \times 10^{-10} = 5.34 \times 10^{-8}$

(d) 0.0027

Solution: $\frac{27}{10000} = \frac{27}{10^4} = 27 \times 10^{-4} = 2.7 \times 10 \times 10^{-4} = 2.7 \times 10^{-3}$

(e) 0.00000165

Solution: $\frac{165}{100000000} = \frac{165}{10^8} = 165 \times 10^{-8} = 1.65 \times 100 \times 10^{-8} = 1.65 \times 10^{-6}$

(f) 0.00000000689

Solution: $\frac{689}{100000000000} = \frac{689}{10^{11}} = 689 \times 10^{-11} = 6.89 \times 100 \times 10^{-11} = 6.89 \times 10^{-9}$

Question 6 (a) - 1 micron = $\frac{1}{1000000}$m. Express it in standard form.

Solution: $\frac{1}{1000000} = \frac{1}{10^6} = 1 \times 10^{-6} = 10^{-6}$m

(b) Size of a bacteria = 0.0000004 m. Express it in standard form.

Solution: $0.0000004 = \frac{4}{10000000} = 4 \times 10^{-7}$m

(c) Thickness of a paper = 0.03 mm. Express it in standard form.

Solution: $0.03 = \frac{3}{100} = 3 \times 10^{-2}$ mm

Question 7 - Write each of the following numbers in usual form:

(a) 2.06×10^{-5}

Solution: $\frac{206\times10^{-5}}{10^2}$ = $206 \times 10^{-5-2}$ = 206×10^{-7} = $\frac{206}{10^7}$ = $\frac{206}{10000000}$ = 0.0000206

(b) 5×10^{-7}

Solution: $\frac{5}{10^7}$ = $\frac{5}{10000000}$ = 0.0000005

(c) 6.82×10^{-6}

Solution: $\frac{682\times10^{-6}}{100}$ = $\frac{682}{10^{2+6}}$ = $\frac{682}{10^8}$ = $\frac{682}{100000000}$ = 0.00000682

(d) 5.673×10^{-4}

Solution: $\frac{5673\times10^{-4}}{10^3}$ = $\frac{5673}{10^7}$ = $\frac{5673}{10000000}$ = 0.0005673

(e) 1.8×10^{-2}

Solution: $\frac{18\times10^{-2}}{10}$ = $\frac{18}{10^3}$ = $\frac{18}{1000}$ = 0.018

(f) 4.129×10^{-3}

Solution: $\frac{4129\times10^{-3}}{10^3}$ = $\frac{4129}{10^6}$ = $\frac{4129}{1000000}$ = 0.004129

Exercise 2C

Question 1 - The value of $(\frac{2}{5})^{-3}$ is …

Solution: $\left(\frac{2}{5}\right)^{-3}$ = $\left(\frac{5}{2}\right)^{3}$ = $\frac{5^3}{2^3}$ = $\frac{125}{8}$

Question 2 - The value of $(-3)^{-4}$ is ….

Solution: $(-3)^{-4}$ = $\left(\frac{-1}{3}\right)^{4}$ = $\frac{1}{3^4}$ = $\frac{1}{81}$

Question 3 - The value of $(-2)^{-5}$ is ….

Solution: $(-2)^{-5}$ = $\left(\frac{-1}{2}\right)^{5}$ = $\frac{-1}{2^5}$ = $\frac{-1}{32}$

Question 4 - ($2^{-5} \div 2^{-2}$) =?

Solution: ($2^{-5} \div 2^{-2}$) = ($\frac{1}{2^5} \div \frac{1}{2^2}$)

$= \frac{1 \times 2^2}{2^5} = \frac{1}{2^3} = \frac{1}{8}$

Question 5 - The value of $(3^{-1} + 4^{-1})^{-1} \div 5^{-1}$

Solution: $\left(\frac{1}{3} + \frac{1}{4}\right)^{-1} \div \frac{1}{5}$

$= (\frac{4+3}{12})^{-1} \div \frac{1}{5}$

$= (\frac{7}{12})^{-1} \div \frac{1}{5}$

$= \frac{12}{7} \times 5$

$= \frac{60}{7}$

Question 6 - $(\frac{1}{2})^{-2}$+$(\frac{1}{3})^{-2}$+$(\frac{1}{4})^{-2}$

Solution: $2^2 + 3^2 + 4^2$

= 4+9+16

=29

Question 7 - $\{(\frac{1}{3})^{-3} - (\frac{1}{2})^{-3}\} \div (\frac{1}{4})^{-3}$

Solution: $\{ 3^3 - 2^3\} \div 4^3$

= {27-8}÷ 64

$= \frac{19}{64}$

Question 8 - $[\{(\frac{-1}{2})^2\}^{-2}]^{-1}$ =?

Solution: $[\{\frac{1}{2^2}\}^{-2}]^{-1}$

$= [\{\frac{1}{4}\}^{-2}]^{-1} = [\{4^2\}]^{-1}$

$= [16]^{-1} = \frac{1}{16}$

Question 9 - The value of x for which $(\frac{7}{12})^{-4} \times (\frac{7}{12})^{3x} = (\frac{7}{12})^5$ is

Solution: $(\frac{7}{12})^{-4+3x} = (\frac{7}{12})^5$

Since base is same, we can equate the powers

=> $-4 + 3x = 5$

=> 3x = 5+4

=> 3x = 9

=> x = 3

Question 10 - If $(2^{3x-1} + 10) \div 7 = 6$, then x is equal to

Solution: $(2^{3x-1} + 10) \div 7 = 6$

=> $\frac{2^{3x-1}+10}{7}$ = 6

=> $2^{3x-1} + 10$ = 42

=>2^{3x-1} = 42-10 = 32

=>2^{3x-1} = 32

=>2^{3x-1} = 2^5

Since base is same, we can equate powers

=> $3x - 1 = 5$

=> 3x = 5+1

=> 3x = 6

=> x = 2

Question 11 - $(\frac{2}{3})^0$

Solution: Since $(\frac{a}{b})^0$ = 1

Thus, $(\frac{2}{3})^0$ = 1

Question 12 - $(\frac{-5}{3})^{-1}$ =?

Solution: $(\frac{-5}{3})^{-1}$ = $(\frac{-3}{5})^1$ = $\frac{-3}{5}$

Question 13 - $(\frac{-1}{2})^3$ =?

Solution: $(\frac{-1}{2})^3 = -\frac{1}{2^3} = \frac{-1}{8}$

Question 14 - $(\frac{-3}{4})^2$ **=?**

Solution: $\frac{(-3)^2}{4^2} = \frac{9}{16}$

Question 15 - 3670000 in standard form is ….

Solution: 3670000 = $367 \times 10^4 = 3.67 \times 100 \times 10^4$

= 3.67×10^6

Question 16 - 0.0000463 in standard forms is ….

Solution: 0.0000463 = $\frac{463}{10000000} = \frac{463}{10^7}$

= 463×10^{-7} = $4.63 \times 100 \times 10^{-7}$

= 4.63×10^{-5}

Question 17 - 0.000367×10^4 **in usual form is …**

Solution: $0.000367 \times 10^4 = \frac{367\times10^4}{1000000} = \frac{367\times10^4}{10^6}$

= $\frac{367}{10^2} = \frac{367}{100}$ = 3.67