Introduction

There are two surface areas:

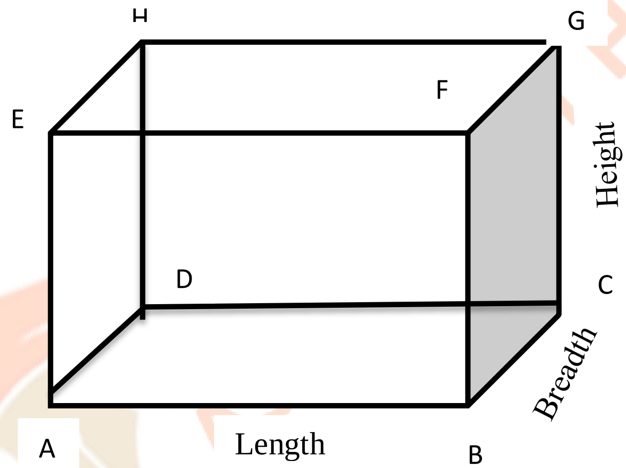
1) Curved surface area: It is the area of all curved region of a solid excluding the top and base area. It is also known as lateral surface area. It is also area of four walls.

2) Total surface area: It includes the curved surface area and area of top and base of solid.

Volume: It is the total space occupied by a solid. It is measured in cubic units.

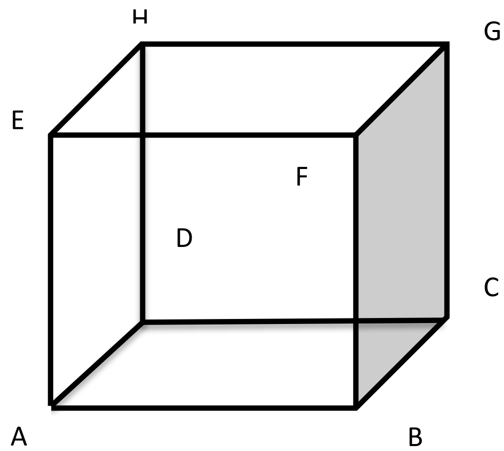
Formulas to be used in this chapter:

(1) Cuboid: It is a solid that have six rectangular plane faces.



(a) Volume of Cuboid = Length × breadth × height = (l×b×h) cubic units
(b) Curved surface area of cuboid = 2h (l + b) sq. units
(c) Total surface area of cuboid = 2(l b + b h + h l) sq. units
(d) Diagonal of a cuboid = $\sqrt{l^2 + b^2 + h^2}$ units

(1I) Cube: It is a solid in which length, breadth end height are equal. It is formed by six identical or congruent squares. Let 'a' be the edge of cube.



(a) Volume of Cube = $(edge)^3 = a^3$ cubic units

(b) Curved surface area of cube = 4 a^2sq. units

(c) Total surface area of cuboid = $6a^2$sq. units

(d) Diagonal of a cube = $\sqrt{3}$ a units

Examples:

Example 1 – Find the volume, the total surface area and the lateral surface area of a cuboid which is 8 m long, 6 m broad and 3.5 m high.

Solution **-** It is given that Length of cuboid (l) = 8 m

Breadth of cuboid (b) = 6 m

Height of cuboid (h) = 3.5 m

Volume of cuboid = (l×b×h) cubic meter

= (8 × 6 × 3.5) = 168 m^3

Total surface area of cuboid = 2(l b + b h + h l) sq. meter

= 2(8×6 + 6×3.5 + 8×3.5)

= 2(48 + 21 + 28) = 2(97) = 194 m^2

Curved surface area of cuboid = 2h (l + b) sq. meter

= 2(3.5) (8+6) = 7(14) = 98 m^2

Example 2 – How many bricks will be required for a wall which is 8 m long, 6 m high and 22.5 cm thick if each brick measures 25 cm × 11.25 cm × 6 cm?

Solution **-** It is given that length of the wall = 8 m = 800 cm

Breadth of the wall = 22.5 cm

Height of the wall = 6 m = 600 cm

Dimension of a brick = 25×11.25×6

Volume of the wall = (l×b×h) = 800×22.5×600 cubic cm

We need to find number of bricks:

Number of bricks = $\frac{Volume\ of\ the\ wall}{volume\ of\ a\ brick}$

= $\frac{800\times22.5\times600}{25\times11.25\times6}$ = 800×8 = 6400

Thus, number of bricks = 6400

Example 3 – Find the length of the longest pole that can be put in a room of dimensions10 m by 10 m by 5 m.

Solution **-** It is given that length of room = 10 m

Breadth of room = 10 m

Height of room = 5 m

Length of longest pole = length of the diagonal = $\sqrt{l^2 + b^2 + h^2}$ units

= $\sqrt{10^2 + 10^2 + 5^2}$ m

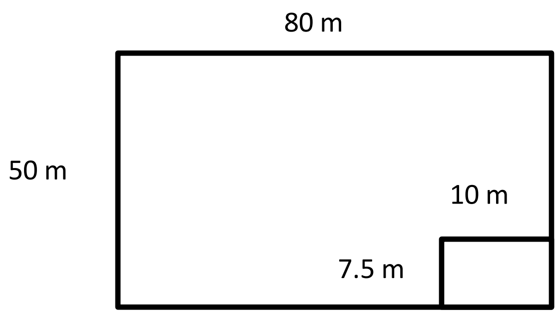
= $\sqrt{100 + 100 + 25}$ m

= $\sqrt{225}$ m

= 15 m

Example 4 – A field is 80 m long and 50 m broad. In one corner of the field, a pit which is 10 m long, 7.5 m broad and 8 m deep has been dug out. The earth taken out of it is evenly spread over the remaining part of the field. Find the rise in the level of the field.

Solution -



Given that length of field = 80 m

Breadth of field = 50 m

Area of field = 80×50 = 4000 m^2

Length of pit = 10 m

Breadth of pit = 7.5 m

Area of pit = 10×7.5 = 75 m^2

Thus, area over which earth is spread out = 4000 – 75 = 3925 m^2

Now, the volume of earth dug out = (10×7.5×8) m^2

= 600 m^2

Therefore, the rise in level = $\frac{volume\ of\ earth\ dug\ out}{area\ over\ which\ earth\ is\ spread\ out}$

= $\frac{600}{3925}$ m = $\frac{600\times100}{3925}$ cm = 15.28 = 15.3 cm (approx.)

Example 5 – The volume of a rectangular tank is 182m^3. If its length and breadth be 8 m and 6.5 m respectively, find its depth.

Solution **-** It is given that volume of a rectangular tank = 182 m^3

Length of park = 8 m

Breadth of park = 6.5 m

Depth of park =?

Let the depth of tank be h meter

We know that Volume of rectangular tank = (l×b×h) cubic meter

=> 182 = (8×6.5×h)

=> 182 = 52 h

=> h = 182/52 = 3.5 meters

Thus, the depth of rectangular tank = 3.5 meters

Example 6 – The volume of a reservoir is 108m^3. Water is poured into it at the rate of 60 litres per minute. How many hours will it take to fill the reservoir?

Solution **-** It is given that volume of a reservoir = 108 m^3

= 108×1000 litres (as 1m^3 = 1000 l)

Also, given that rate of flow of water = 60 litres per minute

Since, Time = Volume / Rate of flow

Thus, time taken to fill reservoir = (vol. of reservoir in litres) / (rate of flow in litres per min.)

= (108×1000) / (60) minutes

= 1800 minutes

=1800/60 hours = 30 hours

Example 7 – Find the volume of wood used to make a closed rectangular box of outer dimensions 60 cm × 45 cm × 32 cm, the thickness of wood being 2.5 cm all around. Also find the capacity of the box.

Solution **-** Given that outer dimensions of rectangular box = 60 cm × 45 cm × 32 cm

Also, given that thickness of wood = 2.5 cm

Thus, internal length = 60 - (2.5+2.5) = 60 – 5 = 55 cm

Internal breadth = 45 - (2.5+2.5) = 45 – 5 = 40 cm

Internal height = 32 - (2.5+2.5) = 32 – 5 = 27 cm

So, external volume = 60×45×32 = 86400 cm^3

And, internal volume = 55×40×27 = 59400 cm^3

Therefore, capacity of box = internal volume = 59400 cm^3

Volume of wood used = external volume – internal volume

= 86400 – 59400 = 27000 cm^3

Example 8 – An open rectangular cistern when measured from outside is 1.35 m long, 1.08 m broad and 90 cm deep, and is made of iron which is 2.5 cm thick. Find the capacity of the cistern and the volume of the iron used.

Solution - Given that outer dimensions of rectangular cistern = 1.35 m × 1.08 m × 90 cm

= 135 cm × 108 cm × 90 cm

Also, given that thickness of iron = 2.5 cm

Thus, internal length = 135 - (2.5+2.5) = 135 – 5 = 130 cm

Internal breadth = 108 - (2.5+2.5) = 108 – 5 = 103 cm

Internal height = 90 – 2.5 = 87.5 cm (since cistern is open)

So, external volume =135×108×90 = 1312200 cm^2

And, internal volume = 130×103×85 = 1171625 cm^2

Therefore, capacity of cistern = internal volume = 1171625 cm^2

Volume of iron used = external volume – internal volume

= 1312200 - 1171625 = 140575 cm^2

Example 9 – Find the volume, lateral surface area and the total surface area of a cube each of whose sides measures 8 cm.

Solution - It is given that length of each edge of cube (a) = 8 cm

Volume of cube = a^3 cubic units

= $8^3 = 8 \times 8 \times 8 = 512\ cm^3$

Lateral surface area of cube = 4 a^2sq. units

= $4 \times (8)^2 = 4(64) = 256\ cm^2$

Total surface area of cube = 6 a^2sq. units

$= 6 \times (8)^2 = 6(64) = 384\ cm^2$

Example 10 – Find the volume of a cube whose total surface area is 486 cm^2.

Solution - It is given that total surface area of cube = 486 cm^2

We know that TSA of cube = 6 a^2sq. units, where 'a' is the edge of cube

=> 486 = 6 a^2

=>486/6 = a^2

=> 81 = a^2

=> a = 9 cm

Therefore, volume of cube = a^3 cubic units

= $9^3 = 9 \times 9 \times 9 = 729\ cm^3$

Example 11 – Find the total surface area of the cube whose volume is 343 cm^3.

Solution - It is given that volume of cube = 343 cm^3

We know that volume of cube = a^3cubic units, where 'a' is the edge of cube

=> 343 = a^3

=> 7×7×7 = a^3

=> 7^3 = a^3

=> a = 7 cm

Therefore, total surface area of a cube = 6 a^2sq. units

$= 6 \times (7)^2 = 6(49) = 294\ cm^2$

Exercise 20A

Question 1 – Find the volume, lateral surface area and the total surface area of the cuboid whose dimensions are:

(a) Length = 22 cm, breadth = 12 cm and height = 7.56 cm

Solution - It is given that Length of cuboid (l) = 22 cm

Breadth of cuboid (b) = 12 cm

Height of cuboid (h) = 7.5 cm

Volume of cuboid = (l×b×h) cubic cm

= (22 ×12 × 7.5) = 1980 cm^3

Total surface area of cuboid = 2(l b + b h + h l) sq. cm

= 2(22×12 + 12×7.5 + 22×7.5)

= 2(264 + 90 + 165) = 2(519) = 1038 cm^2

Curved surface area of cuboid = 2h (l + b) sq. cm

= 2(7.5) (22+12) = 15(34) = 510 cm^2

(b) Length = 15 m, breadth = 6 m and height = 9 dm

Solution - It is given that Length of cuboid (l) = 15 m

Breadth of cuboid (b) = 6 m

Height of cuboid (h) = 9 dm = 9×10 cm = 90 cm = 0.9 m (as 1 dm = 10 cm)

Volume of cuboid = (l×b×h) cubic m

= (15 ×6 × 0.9) = 81 m^3

Total surface area of cuboid = 2(l b + b h + h l) sq. meter

= 2(15×6 + 6×0.9 + 15×0.9)

= 2(90 + 5.4 + 13.5) = 2(108.9) = 217.8 m^2

Curved surface area of cuboid = 2h (l + b) sq. meter

= 2(0.9) (15+6) = 1.8(21) = 37.8 m^2

(c) Length = 24 m, breadth = 25 cm and height = 6 m

Solution - It is given that Length of cuboid (l) = 24 m

Breadth of cuboid (b) = 25 cm = 0.25 m

Height of cuboid (h) = 6 m

Volume of cuboid = (l×b×h) cubic m

= (24 ×0.25 × 6) = 36 m^3

Total surface area of cuboid = 2(l b + b h + h l) sq. meter

= 2(24×0.25 + 0.25×6 + 24×6)

= 2(6 + 1.5 + 144) = 2(151.5) = 303 m^2

Curved surface area of cuboid = 2h (l + b) sq. meter

= 2(6) (24+0.25) = 12(24.25) = 291 m^2

(d) Length = 48 cm, breadth = 6 dm and height = 1 m

Solution - It is given that Length of cuboid (l) = 48 cm

Breadth of cuboid (b) = 6 dm = 6×10 = 60 cm (as 1 dm = 10 cm)

Height of cuboid (h) = 1 m = 100 cm

Volume of cuboid = (l×b×h) cubic cm

= (48 ×60 × 100) = 288000 cm^3

Total surface area of cuboid = 2(l b + b h + h l) sq. cm

= 2(48×60 + 60×100 + 48×100)

= 2(2880 + 6000 + 4800) = 2(13680) = 27360 cm^2

Curved surface area of cuboid = 2h (l + b) sq. cm

= 2(100) (48+60) = 200(108) = 21600 cm^2

Question 2 – The dimensions of a rectangular water tank are 2 m 75 cm by 1 m 80 cm by 1 m 40 cm. How many litres of water does it hold when filled to the brim?

Solution - It is given that length of rectangular water tank = 2m 75 cm = 2 +0.75 = 2.75 m

Breadth of tank = 1m 80 cm = 1 + 0.8 = 1.80 m

Height of tank = 1m 40cm = 1 + 0.40 = 1.40 m

Volume of water it can hold = l × b × h

= 2.75 × 1.80 × 1.40 = 6.93 m^3

Now, 1 m^3 = 1000 litres

Thus, volume = 6.93 × 1000 = 6930 litres

Question 3 – A solid rectangular piece of iron measures 1.05 m × 70 cm × 1.5 cm. Find the weight of this piece in kilograms if 1 cm^3 of iron weighs 8 grams.

Solution - It is given that length of iron piece = 1.05 m = 105 cm

Breadth of iron piece = 70 cm

Height of iron piece = 1.5 cm

Thus, volume of iron piece = l × b × h

= 105 × 70 × 1.5 = 11025 cm^3

Now given that, weight of 1 cm^3 of iron = 8 grams

Weight of 11025 cm^3 of iron = 11025×8 = 88200 grams

Also, we know that 1 kg = 1000 grams

Weight of iron in kg = 88.2 kg

Question 4 – The area of a courtyard is 3750m^2. Find the cost of covering it with gravel to a height of 1 cm if the gravel costs Rs 6.40 per cubic metre.

Solution - It is given that area of courtyard = 3750 m^2

And, height of courtyard = 1 cm = 1/100 m = 0.01 m

We know that volume = area × height

=> Volume = 3750 × 0.01 = 37.5 m^3

Cost of 1 m^3 gravel = Rs 6.40

Cost of 37.5 m^3 gravel = 6.40 × 37.5 = Rs 240

Question 5 – How many person can be accommodated in a hall of length 16 m, breadth 12.5 m and height 4.5 m, assuming that 3.6 m^3 of air is required for each person?

Solution **-** Given that length of hall = 16 m

Breadth of hall = 12.5 m

Height of hall = 4.5 m

Volume of hall = l × b × h

= 16 × 12.5 × 4.5 = 900 m^3

Now, given that 3.6 m^3 of air is required for each person

No of persons accommodated in hall = (volume of hall) / (volume required by each person)

= 900/3.6 = 250 persons

Question 6 – A cardboard box is 1.2 m long, 72 cm wide and 54 cm high. How many bars of soap can be put into it if each bar measures 6 cm × 4.5 cm × 4 cm?

Solution **-** Length of cardboard box = 1.2 m = 120 cm

Breadth of box = 72 cm

Height of box = 54 cm

Thus, volume of box **=** l × b × h

= 120 × 72 × 54 cm^3

Length of soap bar = 6 cm

Breadth of soap bar = 4.5 cm

Height of soap bar = 4 cm

Volume of soap bar = l × b × h

= 6 × 4.5 × 4 cm^3

= 108 cm^3

Number of soap bars = (volume of cardboard box) / (volume of 1 soap bar)

= (120 × 72 × 54) / 108

= 4320 bars

Question 7 – The size of a matchbox is 4 cm × 2.5 cm × 1.5 cm. What is the volume of a packet containing 144 matchboxes? How many such packets can be placed in a carton of size 1.5 m × 84 cm × 60 cm?

Solution **-** Length of matchbox = 4 cm

Breadth of matchbox = 2.5 cm

Height of matchbox = 1.5 cm

Volume of matchbox = l × b × h

= 4 × 2.5 × 1.5 cm^3

= 15 cm^3

Total matchbox = 144

Thus, volume of a packet containing 144 matchbox = 15 × 144 = 2160 cm^3

Dimensions of a carton = 1.5 m × 84 cm ×60 cm = 150 cm × 84 cm ×60 cm

Volume of a carton = 150 × 84 × 60 cm^3

Number of packets = (volume of carton) / (volume of a packet)

= (150 × 84 × 60) / 2160

= 350 packets

Question 8 – How many planks of size 2 m × 25 cm × 8 cm can be prepared from a wooden block 5 m long, 70 cm broad and 32 cm thick, assuming that there is no wastage?

Solution **-** Length of wooden block = 5 m = 500 cm

Breadth of block = 70 cm

Height of block = 32 cm

Volume of wooden block = l × b × h

= 500 × 70 × 32 cm^3

Length of plank = 2 m = 200 cm

Breadth of plank = 25 cm

Height of plank = 8 cm

Volume of plank = l × b × h

= 200 × 25 × 8 cm^3

Number of planks = (volume of wooden block) / (volume of a plank)

= (500 × 70 × 32) / (200 × 25 × 8)

= 7 × 4 = 28 planks

Question 9 – How many bricks, each of size 25 cm × 13.5 cm × 6 cm, will be required to build a wall 8 m long, 5.4 m high and 33 cm thick?

Solution **-** Length of wall = 8 m = 800 cm

Breadth of wall = 5.4 m = 540 cm

Height of wall = 33 cm

Volume of wall = l × b × h

= 800 × 540 × 33 cm^3

Length of a brick = 25 cm

Breadth of a brick = 13.5 cm

Height of brick = 6 cm

Volume of brick = l × b × h

= 25 × 13.5 × 6 cm^3

Number of bricks = (volume of wall) / (volume of a brick)

= (800 × 540 × 33) / (25 × 13.5 × 6)

= 32 × 22 × 10 = 7040 bricks

Question 10 – A wall 15 m long, 30 cm wide and 4 m high is made of bricks, each measuring 22 cm × 12.5 cm × 7.5 cm. If $\frac{1}{12}$ of the total volume of the wall consists of mortat, how many bricks are there in the wall?

Solution **-** Length of wall = 15 m = 1500 cm

Breadth of wall = 30 cm

Height of wall = 4m = 400 cm

Volume of wall = l × b × h

= 1500 × 30 × 400 cm^3

Length of a brick = 22 cm

Breadth of a brick = 12.5 cm

Height of brick = 7.5 cm

Volume of brick = l × b × h

= 22 × 12.5 × 7.5 cm^3

Given that 1/12 of the total volume consists of mortar

Thus, volume of bricks in wall = total volume – 1/12 (total volume)

= (1500 × 30 × 400) – $\frac{1}{12}$(1500 × 30 × 400)

= (1500 × 30 × 400) $(1 - \frac{1}{12})$

= (1500 × 30 × 400) ×$\frac{11}{12}$

= 125× 30 × 400× 11 cm^3

Number of bricks = (volume of bricks in wall) / (volume of a brick)

= (125× 30 × 400× 11) / (22 × 12.5 × 7.5)

= 10 × 400 × 2 = 8000 bricks

Question 11 – Find the capacity of a rectangular cistern in litres whose dimensions are 11.2 m × 6 m × 5.8 m. Find the area of the iron sheet required to make the cistern.

Solution **-** Length of rectangular cistern = 11.2 m

Breadth of cistern = 6 m

Height of cistern = 5.8 m

Volume of cistern = l × b × h

= 11.2 × 6 × 5.8 m^3

= 389.76 m^3

Now, 1 m^3 = 1000 litres

Thus, volume = 389.76 × 1000 = 389760 litres

Area of iron sheet required to make cistern = Total surface area

2(l b + b h + h l) sq. meter

= 2(11.2×6 + 6×5.8 + 11.2×5.8)

= 2(67.2 + 34.8 + 64.96) = 2(166.96) = 333.92 m^2

Question 12 – The volume of a block of gold is $0.5m^3$. If it is hammered into a sheet to cover an area of 1 hectare, find the thickness of the sheet.

Solution - Given that volume of block of gold = 0.5 m^3

Also, area = 1 hectare = 10000 m (as 1 hectare = 10000 m)

Thickness of the sheet = volume / area

= 0.5 / 10000 m = 1 / 20000 m

= $\frac{1}{20000}$ × 100 × 10 mm (as 1 m = 100 cm and 1 cm = 10 mm)

= 1/20 = 0.05 mm

Question 13 – The rainfall recorded on a certain day was 5 cm. Find the volume of water that fell on a 2-hectare field.

Solution - It is given that rainfall recorded (height) = 5 cm = 5/100 = 0.05 m

Also, area of field = 2 hectares = 20000 m (as 1 hectare = 10000 m)

Thus, volume of water fell on that field = area × height

= 20000 × 0.05 = 1000 m^3

Question 14 – A river 2 m deep and 45 m wide is flowing at the rate of 3 km/h. Find the quantity of water that runs into the sea per minute.

Solution - It is given that depth of river = 2 m

Width of river = 45 m

Thus, area of cross section of river = 45 × 2 = 90 m^2

Rate of flow = 3 km/h

= 3000/60 m/min

= 50 m/min

Volume of water flow in 1 minute = area × depth

= 90 × 50

= 4500 m^3

Question 15 – A pit 5 m long and 3.5 m wide is dug to a certain depth. If the volume of earth taken out of it is $14m^3$, what is the depth of the pit?

Solution - Let the required depth of pit be x m

Length of pit = 5 m

Width of pit = 3.5 m

Thus, volume of pit = l × b × h

= (5 × 3.5 × x) m^3

But, the volume of earth taken out is given to be 14 m^3

=> (5 × 3.5 × x) = 14

=> x = 14/17.5 m

=> x = 0.8 m

=> x = 0.8 × 100 = 80 cm

Question 16 – A rectangular water tank is 90 cm wide and 40 cm deep. If it can contain 576 litres of water, what is its length?

Solution - Let the required length of water tank be x m

Depth of pit = 40 cm = 0.4 m

Width of pit = 90 cm = 0.9 m

Thus, volume of water tank = l × b × h

= (0.4 × 0.9 × x) m^3

But, the volume of water in tank is given to be 576 litres = 0.576 m^3 (as $1m^3$= 1000 l)

=> (0.4 × 0.9 × x) = 0.576

=> x = 0.576/0.36 m

=> x = 1.6 m

Question 17 – A beam of wood is 5 m long and 36 cm thick. It is made of 1.35m^3 of wood. What is the width of the beam?

Solution - Let the required width of beam be x m

Thickness of beam = 36 cm = 0.36 m

Length of beam = 5 m

Thus, volume of beam = l × b × h

= (0.36 × 5 × x) m^3

But, the volume is given to be 1.35 m^3

=> (0.36 × 5 × x) = 1.35

=> x = 1.35/1.8 m

=> x = 0.75 m

=> x = 75 cm

Question 18 – The volume of a room is 378 m^3 and the area of its floor is 84m^2. Find the height of the room.

Solution - It is given that volume of a room = 378 m^3

Area of floor = 84 m^2

Let the required height of room be x m

We know that, Volume = Area × height

=> Height = volume / area

=> x = 378 / 84

=> x = 4.5 m

Question 19 – A swimming pool is 260 m long and 140 m wide. If 546000 cubic meters of water is pumped into it, find the height of the water level in it.

Solution - Let the required height of water level be x m

It is given that length of swimming pool = 260 m

Width of pool = 140 m

Volume of water = (260×140×x) m^3

Also, volume is given to be 54600 m^3

=> (260×140×x) = 54600

=> x = 54600/36400

=> x = 1.5 m

Question 20 – Find the volume of wood used to make a closed box of outer dimensions 60 cm × 45 cm × 32 cm, the thickness of wood being 2.5 cm all around.

Solution - Given that outer dimensions of closed box = 60 cm × 45 cm × 32 cm

Also, given that thickness of wood = 2.5 cm

Thus, internal length = 60 - (2.5+2.5) = 60 – 5 = 55 cm

Internal breadth = 45 - (2.5+2.5) = 45 – 5 = 40 cm

Internal height = 32 - (2.5+2.5) = 32 – 5 = 27 cm

So, external volume = 60×45×32 = 86400 cm^3

And, internal volume = 55×40×27 = 59400 cm^3

Therefore, capacity of box = internal volume = 59400 cm^3

Volume of wood used = external volume – internal volume

= 86400 – 59400 = 27000 cm^3

Question 21 – Find the volume of iron required to make an open box whose external dimensions are 36 cm × 25 cm × 16.5 cm, the box being 1.5 cm thick throughout. If 1 cm^3 of iron weighs 8.5 grams, find the weight of the empty box in kilograms.

Solution - Given that outer dimensions of open box = 36 cm × 25 cm × 16.5 cm

Also, given that thickness of wood = 1.5 cm

Thus, internal length = 36 - (1.5+1.5) = 36 – 3 = 33 cm

Internal breadth = 25 - (1.5+1.5) = 25 – 3 = 22 cm

Internal height = 16.5 - (1.5) = 15 cm

So, external volume = 36×25×16.5 = 14850 cm^2

And, internal volume = 33×22×15 = 10890 cm^2

Volume of iron required = external volume – internal volume

= 14850 – 10890 = 3960 cm^3

Now, weight of 1 cm^3 of iron = 8.5 grams

Weight of 3960 cm^3 of iron = 8.5 × 3960 = 33660 grams

= 33660/1000 kg = 33.66 kg (as 1kg = 1000 g)

Question 22 – A box with a lid is made which is 3 cm thick. Its external length, breadth and height are 56 cm, 39 cm and 30 cm respectively. Find the capacity of the box. Also, find the volume of wood used to make the box.

Solution - Given that outer dimensions of box = 56 cm × 39 cm × 30 cm

Also, given that thickness of wood = 3 cm

Thus, internal length = 56 - (3+3) = 56 – 6 = 50 cm

Internal breadth = 39 - (3+3) = 39 – 6 = 33 cm

Internal height = 30 - (3+3) = 30 – 6 = 24 cm

So, external volume = 56×39×30 = 65520 cm^3

And, internal volume = 50×33×24 = 39600 cm^3

Therefore, capacity of box = internal volume = 39600 cm^3

Volume of wood used = external volume – internal volume

= 65520 – 39600 = 25920 cm^3

Question 23 – The external dimensions of a closed wooden box are 62 cm, 30 cm and 18 cm. If the box is made of 2-cm-thick wood, find the capacity of the box.

Solution - Given that outer dimensions of closed box = 62 cm × 30 cm × 18 cm

Also, given that thickness of wood = 2 cm

Thus, internal length = 62 - (2+2) = 62 – 4 = 58 cm

Internal breadth = 30 - (2+2) = 30 – 4 = 26 cm

Internal height = 18 - (2+2) = 18 – 4 = 14 cm

So, external volume = 62×30×18 = 33480 cm^3

And, internal volume = 58×26×14 = 21112 cm^3

Therefore, capacity of box = internal volume = 21112 cm^3

Question 24 – A closed wooden box 80 cm long, 65 cm wide and 45 cm high, is made of 2.5-cm-thick wood. Find the capacity of the box and its weight if 100 cm^3 of wood weighs 8 g.

Solution - Given that outer dimensions of closed box = 80 cm × 65 cm × 45 cm

Also, given that thickness of wood = 2.5 cm

Thus, internal length = 80 - (2.5+2.5) = 80 – 5 = 75 cm

Internal breadth = 65 - (2.5+2.5) = 65 – 5 = 60 cm

Internal height = 45 - (2.5+2.5) = 45 - 5 = 40 cm

So, external volume = 80×65×45 = 234000 cm^2

And, internal volume = 75×60×40 = 180000 cm^2

Capacity of box = internal volume = 180000 cm^2

Volume of wood used = external volume – internal volume

= 234000 – 180000 = 54000 cm^3

Now, weight of 100 cm^3 of wood = 8 grams

Weight of 1cm^3 of wood = 8/100 grams = 0.08 grams

Weight of 54000 cm^3 of wood = 0.08 × 54000 = 4320 grams

= 4320/1000 kg = 4.320 kg (as 1kg = 1000 g)

Question 25 – Find the volume, lateral surface area and the total surface area of a cube each of whose edges measures:

Solution - (a) 7 m

It is given that length of each edge of cube (a) = 7 m

Volume of cube = a^3 cubic units

= $7^3 = 7 \times 7 \times 7 = 343\ m^3$

Lateral surface area of cube = 4 a^2sq. units

= $4 \times (7)^2 = 4(49) = 196\ m^2$

Total surface area of cube = 6 a^2sq. units

= $6 \times (7)^2 = 6(49) = 294\ m^2$

(b) 5.6 cm

It is given that length of each edge of cube (a) = 5.6 m

Volume of cube = a^3 cubic units

= $(5.6)^3 = 5.6 \times 5.6 \times 5.6 = 175.616\ cm^3$

Lateral surface area of cube = 4 a^2sq. units

= $4 \times (5.6)^2 = 4(31.36) = 125.44\ cm^2$

Total surface area of cube = 6 a^2sq. units

= $6 \times (5.6)^2 = 6(31.36) = 188.16\ cm^2$

(c) 8 dm 5 cm = 80+5 = 85 cm

Since 1 dm = 10 cm

It is given that length of each edge of cube (a) = 85 cm

Volume of cube = a^3 cubic units

= $(85)^3 = 85 \times 85 \times 85 = 614125\ cm^3$

Lateral surface area of cube = 4 a^2sq. units

= $4 \times (85)^2 = 4(7225) = 28900\ cm^2$

Total surface area of cube = 6 a^2sq. units

$= 6 \times (85)^2 = 6(7225) = 43350\ cm^2$

Question 26 – The surface area of a cube is 1176cm^2. Find its volume.

Solution - It is given that total surface area of cube = 1176 cm^2

We know that TSA of cube = 6 a^2sq. units, where ‘a’ is the edge of cube

=> 1176 = 6 a^2

=>1176/6 = a^2

=> 196 = a^2

=> a = 14 cm

Therefore, volume of cube = a^3 cubic units

$= 14^3 = 14 \times 14 \times 14 = 2744\ cm^3$

Question 27 – The volume of a cube is 729cm^3. Find its surface area.

Solution - It is given that volume of cube =729 cm^3

We know that volume of cube = a^3cubic units, where ‘a’ is the edge of cube

=> 729 = a^3

=> 9×9×9 = a^3

=> 9^3 = a^3

=> a = 9 cm

Therefore, total surface area of a cube = 6 a^2sq. units

$= 6 \times (9)^2 = 6(81) = 486\ cm^2$

Question 28 – The dimensions of a metal block are 2.25 m by 1.5 m by 27 cm. It is melted and recast into cubes, each of side 45 cm. How many cubes are formed?

(Note: when there is a case like a solid is melted and recast into another solid, and then the volumes of both solids are equal.)

Solution - It is given that length of metal block = 2.25 m = 225 cm

Breadth of block = 1.5 m = 150 cm

Height of block = 27 cm

Volume of metal block = l × b × h

= 225 × 150 × 27 cm^3

Given that metal block is melted and recast into cubes each of side 45 cm

Then, total volume of metal block = total volume of cubes

Volume of 1 cube = a^3 cubic units

= $(45)^3 = 45 \times 45 \times 45\ cm^3$

Let the number of cubes be x

Then, total volume of cube = no of cubes × volume of 1 cube

=> Total volume of cube = $(45 \times 45 \times 45)$ × x

Now, 225 × 150 × 27 =$(45 \times 45 \times 45)$ × x

=> x = (225 × 150 × 27) / $(45 \times 45 \times 45)$)

=> x = 10 cubes

Question 29 – If the length of each edge of a cube is doubled, how many times does its volume become? How many times does its surface area become?

Solution - Let the length of edge of cube be 'a'

Then, volume of cube = a^3 cubic units

And, surface area = 6 a^2sq. units

If length of each edge becomes doubled, then

New length of edge = '2a'

New volume = $(2a)^3$ cubic units

= $8a^3$ cubic units

Thus, new volume becomes 8 times the volume.

New surface area = 6 $(2a)^2$sq. units

= 6 $(4a^{2)}$sq. units

= 24 a^2sq. units

= 4 $(6a^2)$ sq. units

Thus, new surface area becomes 4 times the surface area.

Question 30 – A solid cubical block of fine wood costs Rs 256 at Rs 500 perm^3. Find its volume and the length of each side.

Solution **-** It is given that cost of 1 m^3 = Rs 500

Original cost = Rs 256

Now, volume of cubical block × cost of 1 m^3 = original cost

=> Volume × 500 =256

=> Volume = 256/500 m^3

=> Volume = 0.512 m^3

=> Volume = 0.512 × 100 ×100 ×100 cm^3

= 512000 cm^3

Now, volume of cube = a^3 cubic units

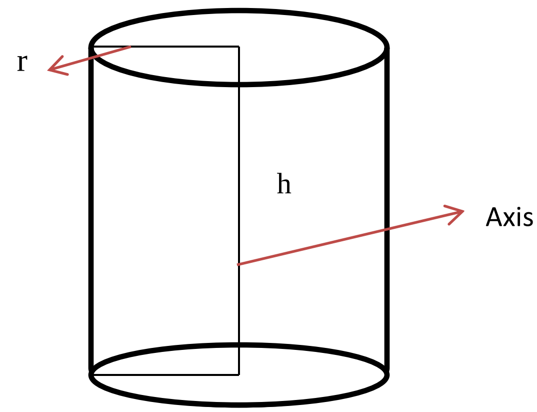
=> 512000 = a^3

=> 80 × 80 × 80 = a^3

=> a = 80 cm

Right Circular Cylinder

It is a solid whose base is circular in shape and the axis of the cylinder is perpendicular to its base**.**



Some characteristics are:

(a) A cylinder had 2 ends which are circular in shape.

(b) Height of cylinder is the length between the two ends.

(c) Axis: the line which joins the centers of these two ends.

(d) Radius of cylinder is the radius of cross section of a cylinder.

(e) It is generated by revolving rectangle about one of its sides.

Let us suppose a cylinder with height 'h' units and radius 'r' units

Volume of cylinder = $\pi r^2 h$ cubic units

Curved surface area of cylinder = $2\pi rh$ sq. units

Total surface area of cylinder = $2\pi r(r + h)$ sq. units

Examples

Example 1 – Find the volume, curved surface area and the total surface area of a cylinder having base radius 10.5 cm and height 18 cm.

Solution **-** It is given that base radius (r) = 10.5 cm

Height (h) = 18 cm

Volume of cylinder = $\pi r^2 h$ cubic units

= $\frac{22}{7} \times 10.5 \times 10.5 \times 18$

= $22 \times 1.5 \times 10.5 \times 18$

= 6237 cm^3

Curved surface area of cylinder = 2πrh sq. units

= 2× $\frac{22}{7} \times 10.5 \times 18$

= 2 × 22 × 1.5 × 18

= 1188 cm^2

Total surface area of cylinder = 2πr(r + h) sq. units

= 2× $\frac{22}{7} \times 10.5$ (10.5 + 18)

= 44 × 1.5 (28.5)

= 1881 cm^2

Example 2 – The circumference of the base of a cylinder is 176 cm and its height is 65 cm. Find the volume of the cylinder and its lateral surface area.

Solution **-** It is given that circumference of the base of cylinder = 176 cm

And, we know that circumference of base of cylinder = 2πr, where r is the radius.

=> 2πr = 176

=> 2× $\frac{22}{7} \times r$ = 176

=> r = 1232/44 = 28 cm

Height is given to be 65 cm

Volume of cylinder = $\pi r^2 h$ cubic units

= $\frac{22}{7} \times 28 \times 28 \times 65$

= $22 \times 4 \times 28 \times 65$

= 160160 cm^3

Lateral surface area = 2πrh sq. units

= 2× $\frac{22}{7} \times 28 \times 65$

= 2 × 22 × 4 × 65

= 11440 cm^2

Example 3 – A cylindrical tank has a capacity of 5632m^3. If the diameter of its base is 16 m, find its depth.

Solution **-** Let the required depth of cylindrical tank be h meters

It is given that capacity of cylindrical tank = 5632 m^3

Diameter of base = 16 m

Then, radius = 16/2 = 8 m

Now, Volume of cylinder = $\pi r^2 h$ cubic units

=> $\pi r^2 h$ = 5632

=> $\frac{22}{7} \times 8 \times 8 \times h$ = 5632

=> h = 39424/1408

=> h = 28 m

Therefore, depth of tank = 28 m

Example 4 – A rectangular paper of width 14 cm is rolled along its width and a cylinder of radius 20 cm is formed. Find the volume of the cylinder.

Solution **-** It is given that a cylinder is formed by rolling a rectangular paper of width 14 cm

Thus, Height of cylinder = 14 cm

Radius of cylinder is given to be 20 cm

Volume of cylinder = $\pi r^2 h$ cubic units

= $\frac{22}{7} \times 20 \times 20 \times 14$

= $22 \times 20 \times 20 \times 2$

= 17600 cm^3

Example 5 – A rectangular piece of paper 22 cm × 6 cm is folded without overlapping to make a cylinder of height 6 cm. Find the volume of the cylinder.

Solution **-** It is given that a cylinder is formed by folding a rectangular piece of paper 22cm×6cm

Thus, length of rectangle becomes circumference of circular base of cylinder and breadth of rectangle becomes height of cylinder

So, 2πr = 22

=> 2× $\frac{22}{7} \times r$ = 22

=> r = 154/44 = 3.5 cm

And, h = 6 cm

Therefore, Volume of cylinder = $\pi r^2 h$ cubic units

= $\frac{22}{7} \times 3.5 \times 3.5 \times 6$

= $22 \times 0.5 \times 3.5 \times 6$

= 231 cm^3

Example 6 – How many cubic meters of earth must be dug out to sink a well which is 16 m deep and which has a radius of 3.5 m? If the earth taken out is spread over a rectangular plot of dimensions 25 m × 16 m, what is the height of the platform so formed?

Solution **-** It is given that depth of well = 16 m

Radius = 3.5 m

Volume of earth dug out = $\pi r^2 h$ cubic units

= $\frac{22}{7} \times 3.5 \times 3.5 \times 16$

= $22 \times 0.5 \times 3.5 \times 16$

= 616 m^3

Now, given that earth taken out is spread over a rectangular plot of 25 m × 16 m

Let the height of plot be 'h' m

Then, Volume of plot = volume of earth dug out

=> l × b × h = 616

=> 25 × 16 × h = 616

=> h = 616/400

=> h = 1.54 m

Therefore, the height of the plot = 1.54 m

Example 7 – A closed metallic cylindrical box is 1.25 m high and it has a base whose radius is 35 cm. If the sheet of metal costs Rs 80 perm^2, then find the cost of the material used in the box. Also, find the capacity of the box in litres.

Solution **-** It is given that height of cylindrical box (h) = 1.25 m

Radius of base = 35 cm = 35/100 = 0.35 m

Area of metal required is the total surface area of box

Total surface area of cylinder = 2πr(r + h) sq. units

= $2\times \frac{22}{7} \times 0.35\ (0.35 + 1.25)$

= 44 × 0.05 (1.6)

= 3.52 m^2

Now, cost of 1 m^2 metal = Rs 80

Cost of 3.52 m^2 metal = 80 × 3.52 = Rs 281.6

Capacity of box = volume of box

= $\pi r^2 h$ cubic units

= $\frac{22}{7} \times 0.35 \times 0.35 \times 1.25$

= $22 \times 0.05 \times 0.35 \times 1.25$

= 0.48125 m^3

Now, 1 m^3 = 1000 litres

Therefore, Capacity = 481.25 litres

Example 8 – An iron pipe is 21 cm long and its external diameter is 8 cm. If the thickness of the pipe is 1 cm and iron weighs 8 g/cm^3, find the weight of the pipe .

Solution - It is given that height of iron pipe = 21 cm

External diameter of pipe = 8 cm

External radius (R) = 4 cm

Thickness = 1 cm

Internal radius (r) = 4 – 1 = 3 cm

Thus, volume of the pipe = External volume – Internal volume

= $\pi R^2 h - \pi r^2 h$

= $\pi h(R^2 - r^2)$

= $\pi h(\mathrm{R} + \mathrm{r})(R - r)$

= $\frac{22}{7} \times 21(4 + 3)(4 - 3)$

= 22 ×3 ×7

= 462 cm^3

Now, weight of 1 cm^3 iron = 8 grams

Weight of 462 cm^3 = 8 × 462 = 3696 grams

As 1 kg = 1000 g

Weight of pipe = 3696/1000 kg

= 3.696 kg

Exercise 20B

Question 1 – Find the volume, curved surface area and total surface area of each of the cylinders whose dimensions are:

(a) Radius of the base = 7 cm and height = 50 cm

Solution - It is given that base radius (r) = 7 cm

Height (h) = 50 cm

Volume of cylinder = $\pi r^2 h$ cubic units

= $\frac{22}{7} \times 7 \times 7 \times 50$

= $22 \times 7 \times 50$

= 7700 cm^3

Curved surface area of cylinder = $2\pi rh$ sq. units

= $2\times \frac{22}{7} \times 7 \times 50$

= $2 \times 22 \times 50$

= 2200 cm^2

Total surface area of cylinder = $2\pi r(r + h)$ sq. units

= $2\times \frac{22}{7} \times 7\ (7 + 50)$

= $44 \times (57)$

= 2508 cm^2

(b) Radius of the base = 5.6 m and height = 1.25 m

Solution - It is given that base radius (r) = 5.6 m

Height (h) = 1.25 m

Volume of cylinder = $\pi r^2 h$ cubic units

= $\frac{22}{7} \times 5.6 \times 5.6 \times 1.25$

= $22 \times 0.8 \times 5.6 \times 1.25$

= 123.2 m^3

Curved surface area of cylinder = 2πrh sq. units

= 2× $\frac{22}{7}$ × 5.6 × 1.25

= 2 × 22 × 0.8 × 1.25

= 44 m^2

Total surface area of cylinder = 2πr(r + h) sq. units

= 2× $\frac{22}{7}$ × 5.6 (5.6 + 1.25)

= 44 × 0.8 (6.85)

= 241.12 m^2

(c) Radius of the base = 14 dm and height = 15 m

Solution - It is given that base radius (r) = 14 dm = 140 cm = 1.4 m

Height (h) = 15 m

Volume of cylinder = $\pi r^2 h$ cubic units

= $\frac{22}{7}$ × 1.4 × 1.4 × 15

= 22 × 0.2 × 1.4 × 15

= 92.4 m^3

Curved surface area of cylinder = 2πrh sq. units

= 2× $\frac{22}{7}$ × 1.4 × 15

= 2 × 22 × 0.2 × 15

= 132 m^2

Total surface area of cylinder = 2πr(r + h) sq. units

= 2× $\frac{22}{7}$ × 1.4 (1.4 + 15)

= 44 × 0.2 (16.4)

= 144.32 m^2

Question 2 – A milk tank is in the form of a cylinder whose radius is 1.5 m and height is 10.5 m. find the quantity of milk in litres that can be stored in the tank.

Solution - Given that radius of milk tank = 1.5 m

Height of tank = 10.5 m

Quantity of milk stored = volume of tank

= $\pi r^2 h$ cubic units

= $\frac{22}{7} \times 1.5 \times 1.5 \times 10.5$

= $22 \times 1.5 \times 1.5 \times 1.5$

= 74.25 m^3

Question 3 – A wooden cylindrical pole is 7 m high and its base radius is 10 cm. Find its weight if the wood weighs 225 kg per cubic metre.

Solution - Given that height of cylindrical pole = 7 m

Radius of base = 10 cm = 10/100 = 0.1 m

Volume of pole = $\pi r^2 h$ cubic units

= $\frac{22}{7} \times 0.1 \times 0.1 \times 7$

= $22 \times 0.1 \times 0.1$

= 0.22 m^3

Weight of $1m^3$ wood = 225 kg

Weight of 0.22 m^3 wood = 225 × 0.22

= 49.5 kg

Question 4 – Find the height of the cylinder whose volume is 1.54m^3 and diameter of the base is 140 cm?

Solution - Let the required height of cylinder be h meters

It is given that volume of cylinder = 1.54 m^3

Diameter of base = 140 cm = 140/100 = 1.4 m

Radius of cylinder = Diameter ÷ 2

= 1.4 ÷ 2

= 0.7 m

Now, Volume of cylinder = $\pi r^2 h$ cubic units

1.54 = $\frac{22}{7} \times 0.7 \times 0.7 \times h$

1.54 = $22 \times 0.1 \times 0.7 \times h$

1.54 = $1.54 \times h$

=> h = 1 m

Question 5 – The volume of a circular iron rod of length 1 m is 3850cm^3. Find its diameter.

Solution - Let the radius of cylinder be r meters

It is given that volume of circular iron rod = 3850 cm^3

And, length of rod (h) = 1 m = 100 cm

Now, Volume of cylinder = $\pi r^2 h$ cubic units

3850 = $\frac{22}{7} \times r \times r \times 100$

3850×7 = $22 \times r \times r \times 100$

26950 = 2200$\times r \times r$

26950 / 2200 = $r \times r$

$r \times r$ = 12.25

$r \times r$ = 3.5 × 3.5

r = 3.5 cm

Therefore, diameter of cylinder = 3.5 × 2 = 7 cm

Question 6 – A closed cylindrical tank of diameter 14 m and height 5 m is made from a sheet of metal. How much sheet of metal will be required?

Solution - It is given that diameter of closed cylindrical tank = 14 m

Height of tank = 5 m

Radius of tank = Diameter ÷ 2

= 14 ÷ 2

= 7 m

Sheet of metal required = Total surface area of tank

= 2πr(r + h) sq. units

= 2× $\frac{22}{7} \times 7$ (7 + 5)

= 44 × 12

= 528 m^2

Question 7 – The circumference of the base of a cylinder is 88 cm and its height is 60 cm. Find the volume of the cylinder and its curved surface area.

Solution **-** It is given that circumference of base of cylinder = 88 cm

Height of cylinder = 60 cm

Since, circumference = 88 cm

2πr = 88

2× $\frac{22}{7} \times r$ = 88

r = 616/44 = 14 cm

Thus, Volume of cylinder = $\pi r^2 h$ cubic units

= $\frac{22}{7} \times 14 \times 14 \times 60$

= $22 \times 2 \times 14 \times 60$

= 36960 cm^3

Curved surface area = 2πrh sq. units

= 2× $\frac{22}{7} \times 14 \times 60$

= 2 × 22 × 2 × 60

= 5280 cm^2

Question 8 – The lateral surface area of a cylinder of length 14 m is 220m^2. Find the volume of the cylinder.

Solution - It is given that lateral surface of cylinder = 220 m^2

Length of cylinder (h) = 14 m

Since, lateral surface area = 220

2πrh = 220

2× $\frac{22}{7} \times r \times 14$ = 220

2 ×22 × r × 2 = 220

88 r = 220

r = 220/88

r = 2.5 m

Thus, Volume of cylinder = πr^2h cubic units

= $\frac{22}{7} \times 2.5 \times 2.5 \times 14$

= $22 \times 2.5 \times 2.5 \times 2$

= 275 m^3

Question 9 – The volume of a cylinder of height 8 cm is 1232cm^3. Find its curved surface area and the total surface area.

Solution - It is given that volume of cylinder = 1232 cm^3

Height of cylinder = 8 cm

Since, volume of cylinder = 1232 cm^3

πr^2h = 1232

$\frac{22}{7} \times r \times r \times 8$ = 1232

$176 \times r \times r$ = 1232 × 7

$176 \times r \times r$ = 8624

$r \times r$ = 8624/176

$r \times r$ = 49

$r \times r$ = 7 × 7

r = 7 cm

Thus, curved surface area = 2πrh sq. units

= 2× $\frac{22}{7}$ × 7 × 8

= 2 × 22 × 8

= 352 cm^2

Total surface area = 2πr(r + h) sq. units

= 2× $\frac{22}{7}$ × 7 (7 + 8)

= 44 × 15

= 660 cm^2

Question 10 – The radius and height of a cylinder are in the ratio 7:2. If the volume of the cylinder is 8316cm^3, find the total surface area of the cylinder.

Solution - Let the radius and height of cylinder be 7x and 2x respectively

Given that volume of cylinder = 8316 cm^3

=> $\pi r^2 h$ = 8316

$\frac{22}{7} \times 7x \times 7x \times 2x$ = 8316

$22 \times x \times 7x \times 2x$ = 8316

$308 \times x \times x \times x$ = 8316

$x \times x \times x$ = 8316 / 308

$x \times x \times x$ = 27

$x \times x \times x$ = 3 × 3 × 3

x = 3

Thus, radius = 7x = 7(3) = 21 cm

Height = 2x = 2(3) = 6 cm

Total surface area = 2πr(r + h) sq. units

= 2× $\frac{22}{7}$ × 21 (21+ 6)

= 44 × 3 × 27

= 3564 cm^2

Question 11 – The curved surface area of a cylinder is 4400 cm^2 and the circumference of its base is 110 cm. Find the volume of the cylinder.

Solution - It is given that curved surface area of cylinder = 4400 cm^2

Circumference of base = 110 cm

2πr = 110

2× $\frac{22}{7} \times r$ = 110

r = 770/44 = 17.5 cm

Now, curved surface area of cylinder = 4400 cm^2

2πrh = 220

2× $\frac{22}{7} \times 17.5 \times h$ = 4400

2 ×22 × 2.5 × h = 4400

110 h = 4400

h = 4400/110

h = 40 cm

Thus, Volume of cylinder = $\pi r^2 h$ cubic units

= $\frac{22}{7} \times 17.5 \times 17.5 \times 40$

= 22 × 2.5 × 17.5 × 40

= 38500 cm^3

Question 12 – A particular brand of talcum powder is available in two packs, a plastic can with a square base of side 5 cm and of height 14 cm, or one with a circular base of radius 3.5 cm and of height 12 cm. Which of them has greater capacity and by how much?

Solution - Case 1: Side of square base (a) = 5 cm

Height = 14 cm

Volume = a^2h cubic units

= 5 ×5 × 14

= 350 cm^3

Case 2: radius of circular base = 3.5 cm

Height = 12 cm

Volume = πr^2h cubic units

= $\frac{22}{7} \times 3.5 \times 3.5 \times 12$

= $22 \times 0.5 \times 3.5 \times 12$

= 462 cm^3

Since, volume in case 2 is more than volume in case 1

Thus, capacity of plastic can with circular base is more than that with square base

Difference = 462 – 350 = 112 cm^3

Question 13 – Find the cost of painting 15 cylindrical pillars of a building at Rs 2.50 per square metre if the diameter and height of each pillar are 48 cm and 7 metres respectively.

Solution - It is given that diameter of cylindrical pillar = 48 cm = 48/100 = 0.48 m

Height of pillar = 7 m

Radius of pillar = Diameter ÷ 2

= 0.48 ÷ 2

= 0.24 m

We need to paint the pillar, so we will find its curved surface area

Curved surface area of 1 pillar = 2πrh sq. units

$= 2\times \frac{22}{7} \times 0.24 \times 7$

= 2 × 22 × 0.24

= 10.56 m^2

Curved surface area of 15 pillars = 15 × 10.56

= 158.4 m^2

Cost of painting 1 m^2 = Rs 2.50

Cost of painting 158.4 m^2 = 2.50 × 158.4

= Rs 396

Question 14 – A rectangular vessel 22 cm by 16 cm by 14 cm is full of water. If the total water is poured into an empty cylindrical vessel of radius 8 cm, find the height of water in the cylindrical vessel.

Solution **-** It is given that water in rectangular vessel is poured into an empty cylinder vessel

Thus, Volume of rectangular vessel = Volume of cylindrical vessel

Now, it is given that length of rectangular vessel (l) = 22 cm

Breadth (b) = 16 cm

Height (h) = 14 cm

Thus, volume of rectangular vessel = l × b × h

= 22 × 16 × 14 = 4928 cm^3 $\longrightarrow$ 1

Radius of cylindrical vessel = 8 cm

Let the required height be h cm

Volume of cylindrical vessel **=** $\pi r^2 h$ cubic units

$= \frac{22}{7} \times 8 \times 8 \times h\ cm^3$ $\longrightarrow$ 2

Now, equating 1 and 2

$\frac{22}{7} \times 8 \times 8 \times h$ = 4928

1408 h = 4928 × 7

1408 h = 34496

h = 34496 / 1408

h = 24.5 cm

Question 15 – A piece of ductile metal is in the form of a cylinder of diameter 1 cm and length 11 cm. It is drawn out into a wire of diameter 1 mm. What will be the length of the wire so obtained?

Solution - It is given that diameter of cylindrical metal = 1 cm

Height of cylinder = 11 cm

Radius = Diameter ÷ 2

= 1 ÷ 2

= 0.5 cm

Thus, volume of metal = $\pi r^2 h$ cubic units

$= \frac{22}{7} \times 0.5 \times 0.5 \times 11\ cm^3 \longrightarrow 1$

Given that a piece of ductile metal is drawn out into a wire

Thus, volume of metal = volume of wire

Diameter of wire = 1 mm

= 1/10 = 0.1 cm (as 1 cm = 10 mm)

Radius of wire = Diameter ÷ 2

= 0.1 ÷ 2

= 0.05 cm

Let the required length of wire be h cm

Thus, volume of wire = $\pi r^2 h$ cubic units

$= \frac{22}{7} \times 0.05 \times 0.05 \times h\ cm^3 \longrightarrow 2$

Equating 1 and 2, we get

$\frac{22}{7} \times 0.05 \times 0.05 \times h = \frac{22}{7} \times 0.5 \times 0.5 \times 11$

25 h = 0.25 × 11

0.0025 h = 2.75

h = 2.75 / 0.0025

h = 1100 cm

h = 1100/100 = 11 m

Question 16 – A solid cube of metal each of whose sides measures 2.2 cm is melted to form a cylindrical wire of radius 1 mm. Find the length of the wire so obtained.

Solution **-** It is given that solid cube is melted to form cylindrical wire

Thus, volume of cube = volume of wire

Now, side of cube (a) = 2.2 cm

Volume of cube = a^3 cubic cm

= $(2.2)^3$ cm^3

= 10.648 cm^3 $\longrightarrow$ 1

Radius of cylinder = 1 mm

= 10/100 cm (as 1 cm = 10 mm)

= 0.1 cm

Let the required length of wire be h cm

Volume of wire = $\pi r^2 h$ cubic units

= $(\frac{22}{7} \times 0.1 \times 0.1 \times h)$ cm^3 $\longrightarrow$ 2

Equating 1 and 2, we get

$(\frac{22}{7} \times 0.1 \times 0.1 \times h)$ = 10.648

0.22 h = 10.648 × 7

0.22 h = 74.536

h = 74.536 / 0.22

h = 338.8 cm

Question 17 – How many cubic metres of earth must be dug out to sink a well which is 20 m deep and has a diameter of 7 metres? If the earth so dug out is spread over a rectangular plot 28 m by 11 m, what is the height of the platform so formed?

Solution - It is given that depth of well = 20 m

Diameter = 7 m

Radius = 7/2 = 3.5 m

Volume of earth dug out = $\pi r^2 h$ cubic units

= $\frac{22}{7} \times 3.5 \times 3.5 \times 20$

= $22 \times 0.5 \times 3.5 \times 20$

= 770 m^3

Now, given that earth taken out is spread over a rectangular plot of 28 m × 11 m

Let the height of plot be 'h' m

Then, Volume of plot = volume of earth dug out

=> l × b × h = 770

=> 28 × 11 × h = 770

=> h = 770/308

=> h = 2.5 m

Therefore, the height of the plot = 2.5 m

Question 18 – A well of inner diameter 14 m is dug to a depth of 12 m. Earth taken out of it has been evenly spread all around it to a width of 7 m to form an embankment. Find the height of the embankment so formed.

Solution **-** It is given that depth of well = 12 m

Diameter = 14m

Radius = 14/2 = 7 m

Volume of earth dug out = $\pi r^2 h$ cubic units

= $\frac{22}{7} \times 7 \times 7 \times 12$

$= 22 \times 7 \times 12$

= 1848 m^3

Now, given that earth taken out is spread over an embankment of well with width 7 m

Outer radius (R) = r + 7 = 7+7 = 14 m

Let the height of embankment be 'h' m

Then, Volume of embankment = $\pi R^2 h - \pi r^2 h$

$= \pi h(R^2 - r^2)$

$= \pi h(\mathrm{R} + \mathrm{r})(R - r)$

$= \frac{22}{7} \times h(14 + 7)(14 - 7)$

$= \frac{22}{7} \times h \times 21 \times 7$

= 462 h cm^3

Now, volume of earth dug out = volume of embankment

462 h = 1848

h = 1848 / 462

h = 4 cm

Question 19 – A road roller takes 750 complete revolutions to make once over to level a road. Find the area of the road if the diameter of the road roller is 84 cm and its length is 1 m.

Solution **-** It is given that diameter of road roller = 84 cm = 84/100 = 0.84 m

Radius of road roller = 0.84 / 2 = 0.42 m

Length of road roller = 1 m

Number of revolutions = 750

Area of road = Curved surface area of road roller × number of revolutions

= 2πrh sq. units × 750

$= 2\times \frac{22}{7} \times 0.42 \times 1 \times 750$

= 2 × 22 × 0.06 × 1× 750

= 1980 m^2

Question 20 – A cylinder is open at both ends and is made of 1.5-cm-thick metal. Its external diameter is 12 cm and height is 84 cm. What is the volume of metal used in making the cylinder? Also, find the weight of the cylinder if 1 cm^3 of the metal weighs 7.5 g.

Solution **-** It is given that height of cylindrical pipe = 84 cm

External diameter of pipe = 12 cm

External radius (R) = 6 cm

Thickness = 1.5 cm

Internal radius (r) = 6 – 1.5 = 4.5 cm

Thus, volume of the metal used = External volume – Internal volume

= $\pi R^2 h - \pi r^2 h$

= $\pi h(R^2 - r^2)$

= $\pi h(\mathrm{R} + \mathrm{r})(R - r)$

= $\frac{22}{7} \times 84(6 + 4.5)(6 - 4.5)$

= 22 ×12 ×10.5×1.5

= 4158 cm^3

Now, weight of 1 cm^3 iron = 7.5 grams

Weight of 4158 cm^3 = 7.5 × 4158 = 31185 grams

As 1 kg = 1000 g

Weight of cylinder = 31185/1000 kg

= 31.185 kg

Question 21 – The length of a metallic tube is 1 metre, its thickness is 1 cm and its inner diameter is 12 cm. Find the weight of the tube if the density of the metal is 7.7 grams per cubic centimeter.

Solution **-** It is given that length of metallic tube = 1 m = 100 cm

Thickness = 1 cm

Inner diameter = 12 cm

Inner radius (r) = 6 cm

Outer radius (R) = r + 1 = 6+1 = 7 cm

Thus, volume of the metallic tube = External volume – Internal volume

$= \pi R^2 h - \pi r^2 h$

$= \pi h(R^2 - r^2)$

$= \pi h(\mathrm{R} + \mathrm{r})(R - r)$

$= \frac{22}{7} \times 100(7 + 6)(7 - 6)$

$= \frac{22}{7} \times 100 \times 13 \;\; cm^3$

Now, density of 1 cm^3 metal = 7.7 grams

Weight = density × volume

Weight $= \frac{22}{7} \times 100 \times 13 \times 7.7$

$22 \times 100 \times 13 \times 1.1$

31460 grams

As 1 kg = 1000 g

Weight = 31460 / 1000 kg

= 31.46 kg

Exercise 20C

Question 1 – The maximum length of a pencil that can be kept in a rectangular box of dimensions 12 cm × 9 cm × 8 cm, is?

Solution - It is given that length of rectangular box = 12 cm

Breadth of box = 9 cm

Height of box = 8 cm

Maximum length of pencil that can be put in box = length of diagonal

$\sqrt{l^2 + b^2 + h^2}$ Units

= $\sqrt{12^2 + 9^2 + 8^2}$ cm

= $\sqrt{144 + 81 + 64}$ cm

= $\sqrt{289}$ cm

= 17 cm

Question 2 – The total surface area of a cube is 150 cm^2. Its volume is?

Solution - It is given that total surface area of cube = 150 cm^2

We know that total surface area of cube = $6a^2$sq. units where 'a' is the edge of cube

=> $6a^2$ = 150

=> a^2 = 150/6

=> a^2 = 25

=> a = 5 cm

Thus, volume of cube = a^3 cubic cm

= $(5)^3$ = 125 cubic cm

Question 3 – The volume of a cube is 343 cm^3. Its total surface area is?

Solution - It is given that volume of cube = 343 cm^3

We know that volume of cube = a^3sq. units where 'a' is the edge of cube

=> a^3 = 343

=> a^3 = 7×7×7

=> a = 7 cm

Thus, total surface area of cube = $6a^2$ square cm

= $6(7)^2 = 6(49) = 294\ cm^2$

Question4 – The cost of painting the whole surface area of a cube at the rate of 10 paise per cm^2 is Rs 264.60. Then, the volume of the cube is?

Solution **-** It is given that cost of 1 cm^2 =10 paise = Rs 0.1

Original cost = Rs 264.60

Now, Total surface area of cube × cost of 1 cm^2 = original cost

=> TSA × 0.1 = 264.60

=> TSA = 264.60/0.1 cm^2

=> TSA = 2646 cm^2

=> $6a^2$ = 2646

=> a^2 = 2646/6

=> a^2 = 441

=> a = 21 m

Thus, volume of cube = a^3 cubic cm

= $(21)^3$ = 9261 cubic cm

$6a^2$ = 150

=> a^2 = 150/6

=> a^2 = 25

=> a = 5 cm

Thus, volume of cube = a^3 cubic cm

= $(5)^3$ = 125 cubic cm

Question 5 – How many bricks, each measuring 25 cm × 11.25 cm × 6 cm, will be needed to build a wall 8 m long, 6 m high and 22.5 cm thick?

Solution - It is given that length of the wall = 8 m = 800 cm

Breadth of the wall = 22.5 cm

Height of the wall = 6 m = 600 cm

Dimension of a brick = 25×11.25×6

Volume of the wall = (l×b×h) = 800×22.5×600 cubic cm

We need to find number of bricks

Number of bricks = $\frac{Volume\ of\ the\ wall}{volume\ of\ a\ brick}$

= $\frac{800\times22.5\times600}{25\times11.25\times6}$ = 800×8 = 6400

Thus, number of bricks = 6400

Question 6 – How many cubes of 10 cm edge can be put in a cubical box of 1 m edge?

Solution - It is given that edge of cubical box (a) = 1 m = 100 cm

Edge of 1 cube = 10 cm

Volume of the box = a^3 cubic cm

= $(100)^3$ = 1000000 cubic cm

Volume of 1 cube = a^3 cubic cm

= $(10)^3$ = 1000 cubic cm

We need to find number of cubes

Number of cubes = $\frac{Volume\ of\ the\ box}{volume\ of\ a\ cube}$

= 1000000/1000 = 1000

Thus, number of cubes = 1000

Question 7 – The edges of a cuboid are in the ratio 1:2:3 and its surface area is 88 cm^2. The volume of the cuboid is?

Solution - Let the length, breadth and height of cuboid be x, 2x and 3x respectively

Given that surface area = 88 cm^2

=> 2(l b + b h + h l) = 88

=> 2(x×2x+ 2x×3x + x×3x) = 88

=> $2(2x^2 + 6x^2 + 3x^2)$ = 88

=> $2(11x^2)$ = 88

=> x^2 = 88/22

=> x^2 = 4

=> x = 2 cm

Thus, sides of cuboid are l = 2, b = 2(2) = 4 cm, h = 3(2) = 6 cm

Volume of cuboid = (l×b×h) = 2×4×6 = 48 cubic cm

Question 8 – Two cubes have their volumes in the ratio 1:27. The ratio of their surface areas is?

Solution - Let the edges of two cubes be 'a' and 'b' respectively

Given that, $\frac{volume\ of\ cube\ 1}{volume\ of\ cube\ 2} = \frac{1}{27}$

=> $\frac{a^3}{b^3} = \frac{1}{27}$

=> $(\frac{a}{b})^3 = (\frac{1}{3})^3$

=> $\frac{a}{b} = \frac{1}{3}$

$$\frac{Total\ surface\ area\ of\ cube\ 1}{Total\ surface\ area\ of\ cube\ 2} = \frac{6a^2}{6b^2}$$

=> $\frac{a^2}{b^2} = (\frac{a}{b})^2 = (\frac{1}{3})^2 = \frac{1}{9}$

Requires ratio = 1: 9

Question 9 – The surface area of a (10 cm × 4 cm × 3 cm) brick is?

Solution - Length of brick = 10 cm

Breadth of brick = 4 cm

Height of brick = 3 cm

Total surface area of brick = 2(l b + b h + h l) sq. cm

= 2(10×4 + 4×3 + 10×3)

= 2(40 + 12 + 30) = 2(82) = 164 cm^2

Question 10 – An iron beam is 9 m long, 40 cm wide and 20 cm high. If 1 cubic metre of iron weighs 50 kg, what is the weight of the beam?

Solution - It is given that length of iron beam = 9 m

Breadth of beam = 40 cm = 0.4 m

Height of beam = 20 cm = 0.2 m

Volume of beam = (l×b×h) = 9×0.4×0.2 = 0.72 cubic m

Weight of 1 cubic m iron = 50 kg

Weight of 0.72 cubic m = 50 × 0.72 = 36 kg

Question 11 – A rectangular water reservoir contains 42000 litres of water. If the length of reservoir is 6 m and its breadth is 3.5 m, the depth of the reservoir is?

Solution - Let the required depth of water reservoir be x m

Length of reservoir = 6 m

Breadth of reservoir = 3.5 m

Thus, volume of reservoir = l × b × h

= (6 × 3.5 × x) m^3

But the volume of water in reservoir is given to be 42000 litres = 42 m^3 (as 1m^3= 1000 l)

=> (6 × 3.5 × x) = 42

=> x = 42/21 m

=> x = 2 m

Therefore, depth of the reservoir = 2 m

Question 12 – The dimensions of a room are (10 m × 8 m × 3.3 m). How many men can be accommodated in this room if each man requires $3m^3$ of space?

Solution **-** It is given that length of room = 10 m

Breadth of room = 8 m

Height of room = 3.3 m

Volume of room **=** l × b × h

= (10 × 8 × 3.3) m^3

= 264 m^3

Now, it is given that each man requires 3 m^3 of space

Therefore, number of men = (Volume of room) / (each man space)

= 264 / 3 = 88 men

Question 13 – A rectangular water tank is 3 m long, 2 m wide and 5 m high. How many litres of water can it hold?

Solution **-** It is given that length of rectangular water tank = 3 m

Breadth of tank = 2 m

Height of tank = 5 m

Capacity of tank = l × b × h

= (3 × 2 × 5) m^3

= 30 m^3

Now, as 1 m^3 = 1000 l

Volume of tank = 30 × 1000 = 30000 litres

Question 14 – The area of the cardboard needed to make a box of size 25 cm × 15 cm × 8 cm will be?

Solution - It is given that length of box = 25 cm

Breadth of box = 15 cm

Height of box = 8 cm

Area of cardboard needed to make box = total surface area of box

= 2(l b + b h + h l) sq. cm

= 2(25×15 + 15×8 + 25×8)

= 2(375 + 120 + 200) = 2(695) = 1390 cm^2

Question 15 – The diagonal of a cube measures 4$\sqrt{3}$cm. Its volume is?

Solution - It is given that diagonal of cube = 4$\sqrt{3}$ cm

Diagonal of cube = $\sqrt{3}$ a units

$\sqrt{3}$ a = 4 $\sqrt{3}$

a = 4 cm

Volume of cube = a^3

= $(4)^3 = 64\ cm^3$

Question 16 – The diagonal of a cube is 9$\sqrt{3}$cm long. Its total surface area is?

Solution - It is given that diagonal of cube = 9$\sqrt{3}$ cm

Diagonal of cube = $\sqrt{3}$ a units

$\sqrt{3}$ a = 9 $\sqrt{3}$

a = 9 cm

Total surface area of cube = $6a^2$

= $6(9)^2 = 6(81) = 486\ cm^3$

Question 17 – If each side of a cube is doubled then its volume?

Solution - Let the length of edge of cube be ‘a’

Then, volume of cube = a^3 cubic units

If length of each edge becomes doubled, then

New length of edge = ‘2a’

New volume = $(2a)^3$ cubic units

= $8a^3$ cubic units

Thus, new volume becomes 8 times the volume.

Question 18 – If each side of a cube is doubled, its surface area is?

Solution - Let the length of edge of cube be ‘a’

Surface area = 6 a^2sq. units

If length of each edge becomes doubled, then

New length of edge = ‘2a’

New surface area = 6 $(2a)^2$sq. units

= 6 $(4a^{2)}$sq. units

= 24 a^2sq. units

= 4 $(6a^2)$ sq. units

Thus, new surface area becomes 4 times the surface area.

Question 19 – Three cubes of iron whose edges are 6 cm, 8 cm and 10 cm respectively are melted and formed into a single cube. The edge of the new cube formed is?

Solution - Let the edge of new cube be ‘a’ cm

It is given that three cubes of edges 6 cm, 8 cm and 10 cm respectively are melted to form single cube

It means that sum of volumes of three cubes is equal to volume of single new cube

Volume of cube of edge 6 cm = $(6)^3 = 216\ cm^2$

Volume of cube of edge 8 cm = $(8)^3 = 512\ cm^2$

Volume of cube of edge 10 cm = $(10)^3 = 1000\ cm^2$

Thus, volume of new cube = 216 + 512 + 1000 = 1728 cm^2

=> a^3= 1728

=> $a^3 = (12)^3$

=> a = 12 cm

Question 20 – Five equal cubes, each of edge 5 cm, are placed adjacent to each other. The volume of the cuboid so formed, is?

Solution - When 5 equal cubes of edge 5 cm are placed adjacent to each other, then sides of cuboid are as follows:

Length = 5+5+5+5+5 = 25 cm

Breadth = 5 cm

Height = 5 cm

Thus, volume of cuboid = l × b × h

= 25 × 5 × 5

= 625 cm^3

Question 21 – A circular well with a diameter of 2 metres, is dug to a depth of 14 metres. What is the volume of the earth dug out?

Solution - It is given that diameter of circular well = 2 m

Height = 14 m

Radius = 2/2 = 1 m

Volume of earth dug out = $\pi r^2 h$ cubic units

= $\frac{22}{7} \times 1 \times 1 \times 14$

= 22×2

= 44 m^3

Question 22 – If the capacity of a cylindrical tank is 1848m^3 and the diameter of its base is 14 m, the depth of the tank is?

Solution - It is given that capacity of cylindrical tank = 1848 m^3

Diameter of base = 14 m

Radius of base = 7 m

Let the required height of tank be 'h' m

Since Volume of tank = $\pi r^2 h$ cubic units

$\frac{22}{7} \times 7 \times 7 \times h = 1848$

154 h = 1848

h = 1848 / 154

h = 12 m

Question 23 – The ratio of the total surface area to the lateral surface area of a cylinder whose radius is 20 cm and height 60 cm, is?

Solution - It is given that radius of cylinder = 20 cm

Height = 60 cm

Total surface area of cylinder = 2πr(r + h) sq. units

And lateral surface area of cylinder = 2πrh sq. units

Now, $\frac{TSA}{CSA} = \frac{2\pi r(r+h)}{2\pi rh} = \frac{r+h}{h} = \frac{20+60}{60} = \frac{80}{60} = \frac{4}{3}$

Question 24 – The number of coins, each of radius 0.75 cm and thickness 0.2 cm, to be melted to make a right circular cylinder of height 8 cm and base radius 3 cm is?

Solution - It is given that height of cylinder = 8 cm

Radius of base = 3 cm

Volume of cylinder = $\pi r^2 h$ cubic units

= $\frac{22}{7} \times 3 \times 3 \times 8$

Radius of 1 coin = 0.75 cm

Thickness = 0.2 cm

Volume of 1 coin = $\pi r^2 h$ cubic units

= $\frac{22}{7} \times 0.75 \times 0.75 \times 0.2$

Now, given that coins are melted to form right circular cylinder

Thus, number of coins = (volume of cylinder) ÷ (volume of 1 coin)

= $\left(\frac{22}{7} \times 3 \times 3 \times 8\right) \div \left(\frac{22}{7} \times 0.75 \times 0.75 \times 0.2\right)$

= 72 ÷ 0.1125

= 640 coins

Question 25 – 66cm^3 of silver is drawn into a wire 1 mm in diameter. The length of the wire will be?

Solution **-** It is given that 66 $cm^3$of silver is drawn into a wire of diameter 1 mm

Thus, volume of silver = volume of wire

Diameter of wire = 1 mm

= 1/10 = 0.1 cm (as 1 cm = 10 mm)

Radius of wire = Diameter ÷ 2

= 0.1 ÷ 2

= 0.05 cm

Let the required length of wire be h cm

Thus, volume of wire = $\pi r^2 h$ cubic units

= $\frac{22}{7} \times 0.05 \times 0.05 \times h\ cm^3$

Now, $\frac{22}{7} \times 0.05 \times 0.05 \times h$ = 66

0.055 h = 66 × 7

0.055 h = 462

h = 462 / 0.055

h = 8400 cm

h = 8400/100 = 84 m

Question 26 – The height of a cylinder is 14 cm and its diameter is 10 cm. The volume of the cylinder is?

Solution - It is given that height of cylinder = 14 cm

Diameter = 10 cm

Radius = 10/2 = 5 cm

Volume of cylinder = $\pi r^2 h$ cubic units

= $\frac{22}{7} \times 5 \times 5 \times 14$

= $22 \times 5 \times 5 \times 2$

= 1100 cm^3

Question 27 – The height of a cylinder is 80 cm and the diameter of its base is 7 cm. The whole surface area of the cylinder is?

Solution - It is given that height of cylinder = 80 cm

Diameter of base = 7 cm

Radius of base = 7/2 cm = 3.5

Total surface area of cylinder = 2πr(r + h) sq. units

= 2× $\frac{22}{7} \times \frac{7}{2}$ (3.5 + 80)

= 22 × 83.5

= 1837 cm^2

Question 28 – The height of a cylinder is 14 cm and its curved surface area is 264cm^2. The volume of the cylinder is?

Solution - It is given that height of cylinder = 14 cm

Curved surface area = 264 cm^2

=> 2πrh = 264

$2\times \frac{22}{7} \times r \times 14$ = 264

2 ×22 × r × 2 = 264

88 r = 264

r= 264/88

r = 3 cm

Volume of cylinder = $\pi r^2 h$ cubic units

= $\frac{22}{7} \times 3 \times 3 \times 14$

= $22 \times 3 \times 3 \times 2$

= 396 cm^3

Question 29 – The diameter of a cylinder is 14 cm and its curved surface area is 220cm^2. The volume of the cylinder is?

Solution **-** It is given that diameter of cylinder = 14 cm

Radius of cylinder = 14/2 = 7 cm

Curved surface area = 220 cm^2

=> 2πrh = 220

$2\times \frac{22}{7} \times 7 \times h$ = 220

44 × h = 220

44 h = 220

h = 220/44

h = 5 cm

Volume of cylinder = $\pi r^2 h$ cubic units

= $\frac{22}{7} \times 7 \times 7 \times 10$

= $22 \times 7 \times 10 = 1540$ cm^3

Question 30 – The ratio of the radii of two cylinders is 2:3 and the ratio of their heights is 5:3. The ratio of their volumes will be?

Solution **-** Let r1 and r2 be the radius of two cylinders respectively

And, the height of two cylinders be h1 and h2 respectively

It is given that $\frac{r1}{r2} = \frac{2}{3}$ and $\frac{h1}{h2} = \frac{5}{3}$

We need to find $\frac{V1}{V2}$ =?

We know that Volume of cylinder = $\pi r^2 h$ cubic units

Thus, $\frac{V1}{V2} = \frac{\pi(r1)^2 h1}{\pi(r2)^2 h2} = \frac{2\times2\times5}{3\times3\times3} = \frac{20}{27}$