

Introduction

Some definitions:

Data: It is the systematic record of numerical facts of a specific type of information.

Raw data: Randomly collected information is called raw data. It is the collection of initially gathered observations

Range: It is the difference between the highest and the lowest value of the observation.

Statistics: It is the branch of mathematics which deals with the collection, presentation, analysis and interpretation of the numerical data.

Frequency of an observation: It means the number of times a particular observation occurs in a given data.

Frequency distribution: Representation of data in tabular form showing the frequency of each observation.

Tally marks: They are a form of numeral used for counting. It is a good way of keeping track of numbers in groups of five. We represent |||| for counting 4 numbers and ||||| for counting 5 numbers.

Examples

Example 1 – Given below are the marks (out of 100) in mathematics obtained by 20 students of a class in an annual examination.

23, 75, 56, 42, 70, 84, 92, 51, 40, 63, 87, 58, 35, 80, 14, 63, 49, 72, 66, 61

Arrange the above data in ascending order and find

(a) The lowest marks obtained

(b) The highest marks obtained

(c) The range of the given data

Solution - Given data in ascending order is as follows:

14, 23, 35, 40, 42, 49, 51, 56, 58, 61, 63, 63, 66, 70, 72, 75, 80, 84, 87, 92

(a) Lowest marks obtained = 14

(b) Highest marks obtained = 92

(c) Range of the given data = Maximum value – Minimum value
= 92 – 14 = 78

Example 2 – Given below are the heights (in cm) of 11 boys of a class:

146, 143, 148, 132, 128, 139, 140, 152, 154, 142, 149

Arrange the above data in ascending order and find

(a) The height of the tallest boy

(b) The height of the shortest boy

(c) The range of the given data

Solution - Given data in ascending order is as follows:

128, 132, 139, 140, 142, 143, 146, 148, 149, 152, 154

(a) Height of tallest boy = 154 cm

(b) Height of shortest boy = 128 cm

(c) Range of the given data = Maximum value – Minimum value
= 154 – 128 = 26 cm

Example 3 – Suppose we make a survey of 20 families of a locality and find out the number of children in each family. Let the observations be

2, 2, 3, 1, 1, 2, 3, 2, 2, 1, 2, 2, 3, 1, 2, 1, 1, 3, 2, 2

State the frequency of each observation.

Solution - Let us first arrange the data in ascending order as follows:

1, 1, 1, 1, 1, 1, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 3, 3, 3, 3

For the representation of the above data, we draw a frequency distribution table and for counting, we use tally marks.

Number of children	Tally Marks	Frequency
1		6
2		10
3		4
Total		20

Exercise 21A

Question 1 – The number of members in 20 families are given below:

4, 6, 5, 5, 4, 6, 3, 3, 5, 5, 3, 5, 4, 4, 6, 7, 3, 5, 5, 7

Prepare a frequency distribution of the data.

Solution - Let us first arrange the data in ascending order as follows:

3, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 5, 5, 5, 5, 6, 6, 6, 7, 7

For the representation of the above data, we draw a frequency distribution table and for counting, we use tally marks.

Number of members	Tally Marks	Frequency
3		4
4		4
5		7
6		3
7		2
Total		20

Question 2 – A dice was thrown 30 times and the following outcomes were noted:

2, 1, 2, 4, 6, 1, 2, 3, 6, 5, 4, 4, 3, 1, 1, 3, 1, 1, 5, 6, 6, 2, 2, 3, 4, 2, 5, 5, 6, 4

Solution - Let us first arrange the data in ascending order as follows:

1, 1, 1, 1, 1, 1, 2, 2, 2, 2, 2, 2, 3, 3, 3, 3, 4, 4, 4, 4, 4, 4, 5, 5, 5, 5, 6, 6, 6, 6, 6

For the representation of the above data, we draw a frequency distribution table and for counting, we use tally marks.

Number of outcomes	Tally Marks	Frequency
1		6
2		6
3		4
4		5
5		4
6		5
Total		30

Question 3 – The following data gives the number of children in 40 families:

1, 2, 6, 5, 1, 5, 1, 3, 2, 6, 2, 3, 4, 2, 0, 4, 4, 3, 2, 2, 0, 0, 1, 2, 2, 4, 3, 2, 1, 0, 5, 1, 2, 4, 3, 4, 1, 6, 2, 2

Solution - Let us first arrange the data in ascending order as follows:

0, 0, 0, 0, 1, 1, 1, 1, 1, 1, 1, 1, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 3, 3, 3, 3, 3, 3, 4, 4, 4, 4, 4, 4, 4, 5, 5, 5, 6, 6, 6

For the representation of the above data, we draw a frequency distribution table and for counting, we use tally marks.

Number of children	Tally Marks	Frequency
0		4
1		7
2		12
3		5
4		6
5		3
6		3
Total		40

Frequency distribution of grouped data

When the data is huge and it is difficult to create the frequency distribution table of individual data, we express the data by making groups of equal intervals. These groups are called class intervals. The lower value in the class interval is called lower limit and the highest value in the class interval is called the upper limit. For example 0-5 has its 0 as its lower limit and 5 as its upper limit. 0-5 will have all the data which is greater than or equal to 0 and less than 5. The difference between upper limit and lower limit is known as class size.

Examples

Example 1 – The marks obtained by 40 students of class VIII in an examination are given below:

16, 17, 18, 3, 7, 23, 18, 13, 10, 21, 7, 1, 13, 21, 13, 15, 19, 24, 16, 2, 23, 5, 12, 18, 8, 12, 6, 8, 16, 5, 3, 5, 0, 7, 9, 12, 20, 10, 2, 23

Solution - Firstly we will arrange the data in ascending order as follows:

0, 1, 2, 2, 3, 3, 5, 5, 5, 6, 7, 7, 7, 8, 8, 9, 10, 10, 12, 12, 12, 13, 13, 13, 15, 16, 16, 16, 17, 18, 18, 18, 19, 20, 21, 21, 23, 23, 23, 24

We make five groups of data 0 -5, 5 – 10, 10 – 15, 15 – 20 and 20 – 25

Marks	Tally Marks	Frequency
0-5		6
5-10		10
10-15		8
15-20		9
20-25		7
Total		40

Example 2 – The weights (in kg) of 35 persons are given below:

43, 51, 47, 62, 48, 40, 50, 62, 53, 56, 40, 48, 56, 53, 50, 42, 55, 52, 48, 46, 45, 54, 52, 50, 47, 44, 54, 55, 60, 63, 58, 55, 60, 58, 53

Solution - We will make five class intervals 40 -45, 45 –50, 50 – 55, 55 – 60 and 60 – 65

The frequency distribution table is as follows:

Weight (in kg)	Tally Marks	Frequency
40-45		5
45-50		7
50-55		11
55-60		7
60-65		5
Total		35

Exercise 21B

Question 1 – The marks obtained by 40 students of a class in an examination are given below:

8, 47, 22, 31, 17, 13, 38, 26, 3, 34, 29, 11, 22, 7, 15, 24, 38, 31, 21, 35, 42, 24, 45, 23, 21, 27, 29, 49, 25, 48, 21, 15, 18, 27, 19, 45, 14, 34, 37, 34

Prepare a frequency distribution table with equal class intervals, starting from 0-10 (where 10 is not included)

Solution - We will make five class intervals 0-10, 10-20, 20-30, 30-40 and 40-50

The frequency distribution table is as follows:

Marks	Tally Marks	Frequency
0-10		3
10-20	 /	8
20-30	 / / / / /	14
30-40	 / / / / /	9
40-50	 / /	6
Total		40

Question 2 – The electricity bills (in rupees) of 25 houses of certain locality for a month are given below:

324, 700, 617, 400, 356, 365, 435, 506, 548, 736, 780, 378, 570, 685, 312, 630, 584, 674, 754, 776, 596, 745, 565, 763, 472

Arrange the above data in increasing order and form a frequency table using equal class intervals, starting from 300-400, where 400 is not included.

Solution - We will arrange the given data in the increasing order as follows:

312, 324, 356, 365, 378, 400, 435, 472, 506, 548, 565, 570, 584, 596, 617, 630, 674, 685, 700, 736, 745, 754, 763, 776, 780

We will make five class intervals 300-400, 400-500, 500-600, 600-700 and 700-800

The frequency distribution table is as follows:

Amount of bill (in Rs)	Tally Marks	Frequency
300-400		5
400-500		3
500-600		6
600-700		4
700-800		7
Total		25

Question 3 – The weekly wages (in rupees) of 28 workers of a factory are given below:

668, 610, 642, 658, 668, 620, 719, 720, 700, 690, 710, 642, 672, 654, 692, 706, 718, 702, 704, 678, 615, 540, 680, 716, 705, 615, 636, 656

Construct a frequency table with equal class intervals; taking the first of the class intervals as 610-630, where 630 is not included.

Solution - We will make six class intervals 610-630, 630-650, 650-670, 670-690, 690-710 and 710-730

The frequency distribution table is as follows:

Wages (in Rs)	Tally Marks	Frequency
610-630		4
630-650		4
650-670		5
670-690		3
690-710		7
710-730		5
Total		28

Question 4 – The weekly pocket expenses (in rupees) of 30 students of a class are given below:

62, 80, 110, 75, 84, 73, 60, 62, 100, 87, 78, 94, 117, 86, 65, 68, 90, 80, 118, 72, 95, 72, 103, 96, 64, 94, 87, 85, 105, 115

Construct a frequency table with class intervals 60-70 (where 70 is not included), 70-80, 80-90, etc.

Solution - We will make six class intervals 60-70, 70-80, 80-90, 90-100, 100-110 and 110-120

The frequency distribution table is as follows:

Expenses (in Rs)	Tally Marks	Frequency
60-70		6
70-80		5
80-90		7
90-100		5
100-110		3
110-120		4
Total		30

Question 5 – The daily earnings (in rupees) of 24 stores in a market was recorded as under:

715, 650, 685, 550, 573, 530, 610, 525, 742, 680, 736, 524, 500, 585, 723, 545, 532, 560, 580, 545, 625, 630, 645, 700

Prepare a frequency distribution table, taking equal class intervals and starting from 125-130, where 130 is not included.

Solution - We will make five class intervals 500-550, 550-600, 600-650, 650-700 and 700-750

The frequency distribution table is as follows:

Daily Earning (in Rs)	Tally Marks	Frequency
500-550		7
550-600		5
600-650		4
650-700		3
700-750		5
Total		24

Question 6 – The heights (in cm) of 22 students were recorded as under:

125, 132, 138, 144, 142, 136, 134, 125, 135, 130, 126, 132, 135, 142, 143, 128, 126, 136, 135, 130, 130, 133

Solution - We will make six class intervals 125-130, 130-135, 135-140 and 140-145

The frequency distribution table is as follows:

Height (in cm)	Tally Marks	Frequency
125-130		5
130-135		7
135-140		6
140-145		4
Total		22

Histogram

A histogram is the graph that represents the discrete or continuous data. It is same as bar graph but the difference is that there are no gaps between the bars.

Method of drawing histogram:

- 1) Mark the class intervals on the x-axis.
- 2) Mark the respective frequencies on the y-axis
- 3) Construct rectangles with class intervals as bases and the respective frequencies as heights.

Examples:

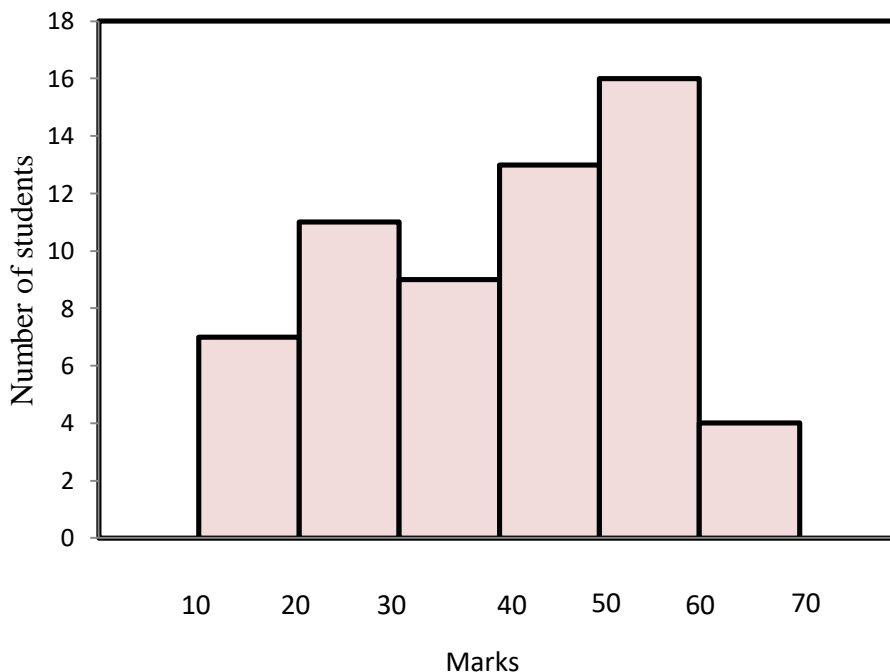
Example 1 – The marks obtained by 60 students of a class in a mathematics test are given below:

Marks	10-20	20-30	30-40	40-50	50-60	60-70
Number of students	7	11	9	13	16	4

Represent the above data by means of a histogram.

Solution - On the graph paper, we represent the class intervals along the x-axis and the number of students along the y-axis.

The histogram is shown as below:



Example 2 – The daily wages (in Rs) of 30 workers in a factory are:

365, 335, 315, 339, 308, 355, 341, 367, 322, 335, 347, 305, 332, 379, 386, 306, 312, 335, 345, 368, 370, 384, 343, 333, 369, 357, 335, 346, 336, 388

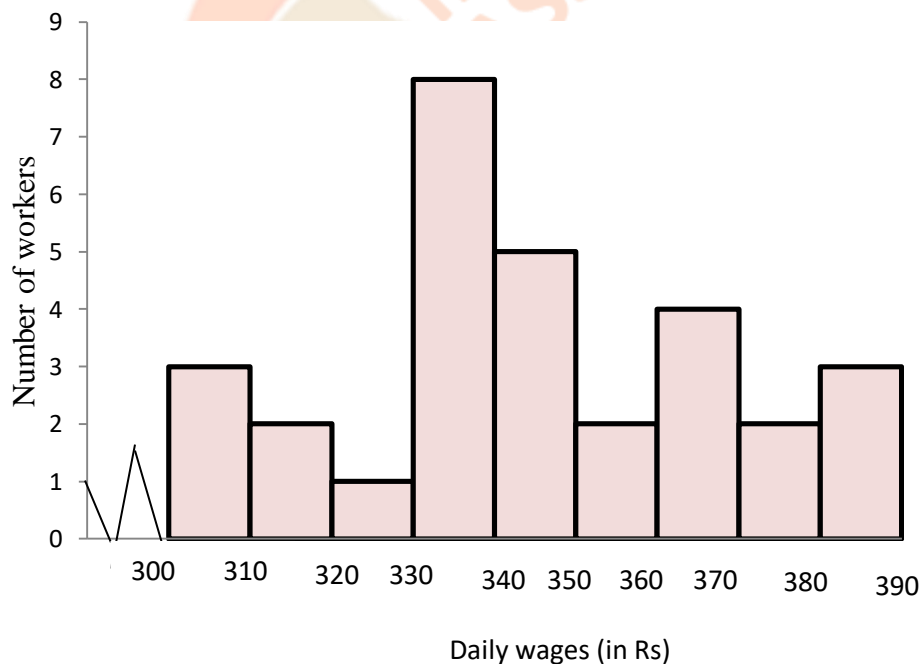
Solution - Firstly we will make frequency distribution table by using tally marks and taking the intervals as 300-310, 310-320, and so on.

The frequency distribution table of the following data is as follows:

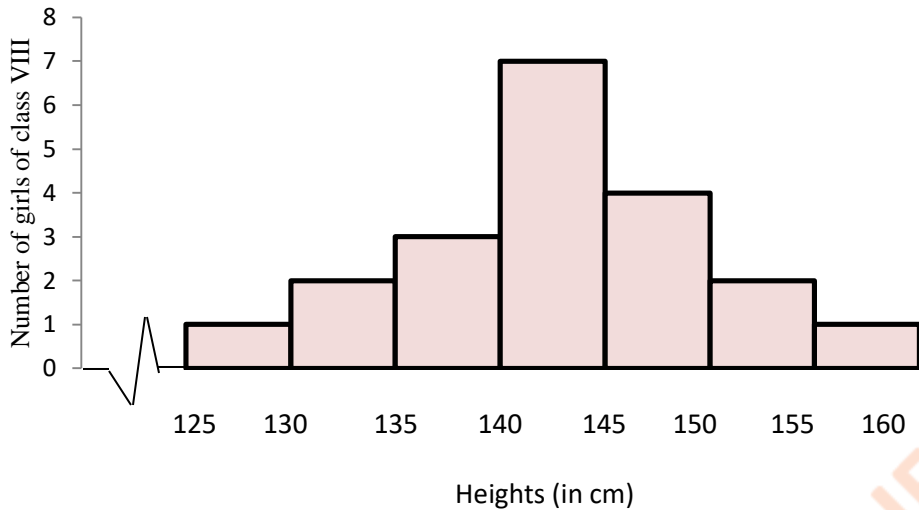
Class interval	Tally Marks	Frequency
300-310		3
310-320		2
320-330		1
330-340		8
340-350		5
350-360		2
360-370		4
370-380		2
380-390		3
Total		30

On the graph paper, we represent the class intervals along the x-axis and the daily wages along the y-axis. Here, we use kink and start with the interval 300-310.

The histogram is shown as below:



Example 3 – Look at the histogram given below and answer the following questions.



(i) What information is being given by the histogram?

(ii) Which group contains maximum girls?

(iii) How many girls have a height of 140 cm and more?

(iv) If we divide the girls into the following three categories, how many would there be in each?

150 cm and above – Group A

140 cm to less than 150 cm – Group B

Less than 140 cm – Group C

Solution – (i) It shows the heights of 20 girls of class VIII which are given in the groups as class intervals from 125-130 cm to 155-160 cm

(ii) It can be shown from the given graph that group 140-145 cm contains maximum girls.

(iii) Number of girls having height 140 cm or more = $7 + 4 + 2 + 1 = 14$ girls

(iv) Group A: 150 cm or above

Number of girls in group A = $2 + 1 = 3$

Group B: 140 cm to less than 150 cm

Number of girls in group B = $7 + 4 = 11$

Group C: Less than 140 cm

Number of girls in group C = $1 + 2 + 3 = 6$

Exercise 21 C

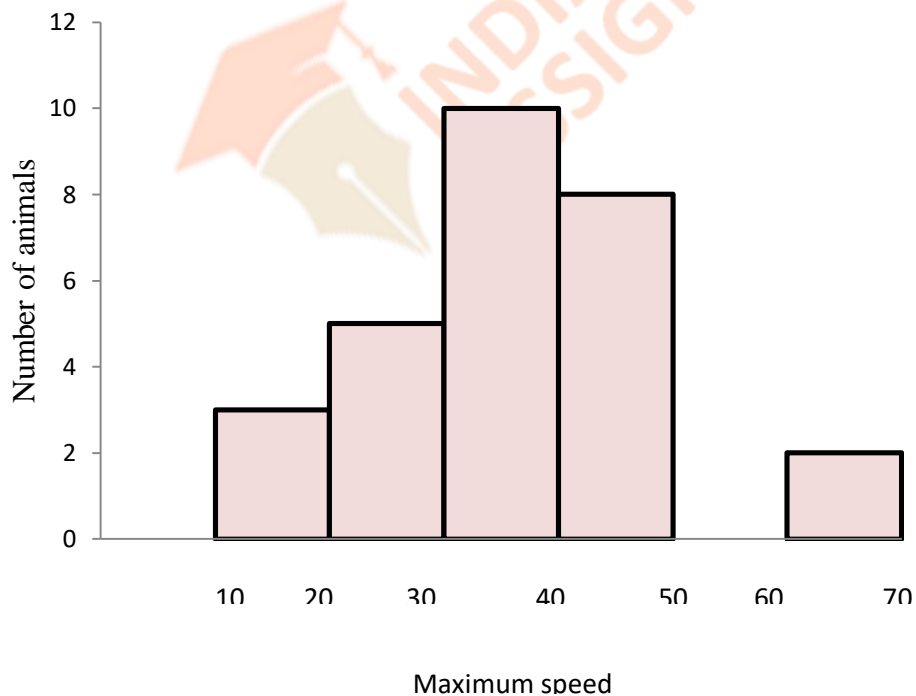
Question 1 – The top speeds of 30 different land animals have been organized into a frequency table given below:

Maximum speed (in km/h)	10-20	20-30	30-40	40-50	50-60	60-70
Number of animals	3	5	10	8	0	2

Draw a histogram for the given data.

Solution - On the graph paper, we represent the class intervals along the x-axis and the number of animals along the y-axis.

The histogram is shown as below:



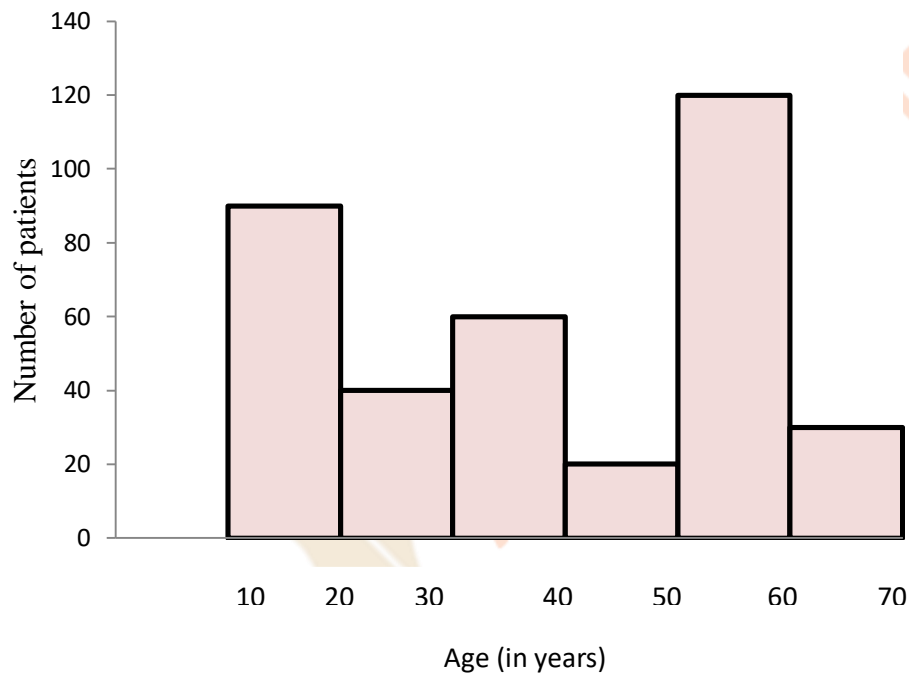
Question 2 – The ages (in years) of 360 patients treated in a hospital on a particular day are given below:

Age (in years)	10-20	20-30	30-40	40-50	50-60	60-70
Number of patients	90	40	60	20	12	30

Draw a histogram for the given data.

Solution - On the graph paper, we represent the class intervals along the x-axis and the number of patients along the y-axis.

The histogram is shown as below:

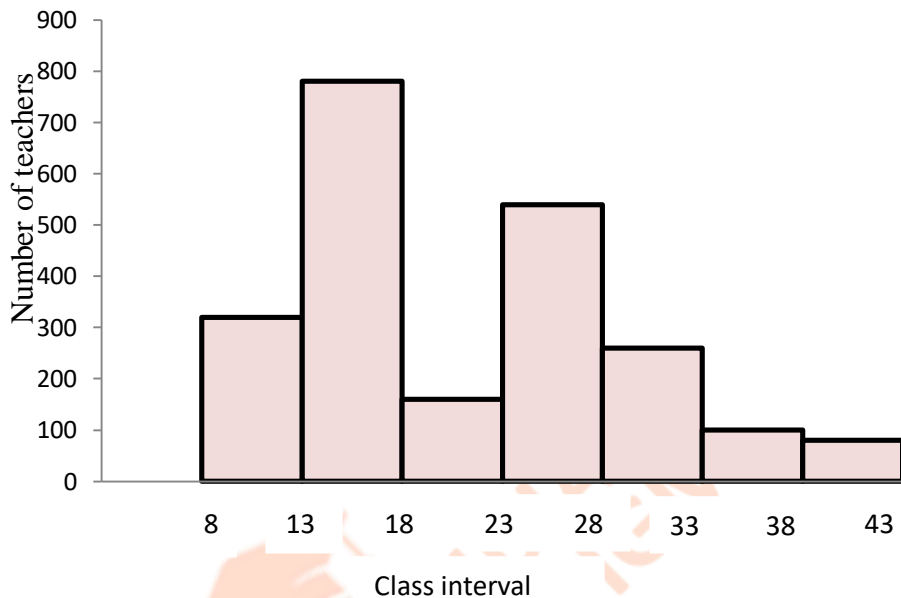


Question 3 – Draw a histogram for the frequency distribution of the following data:

Class interval	8-13	13-18	18-23	23-28	28-33	33-38	38-43
frequency	320	780	160	540	260	100	80

Solution - On the graph paper, we represent the class intervals along the x-axis and the frequency along the y-axis.

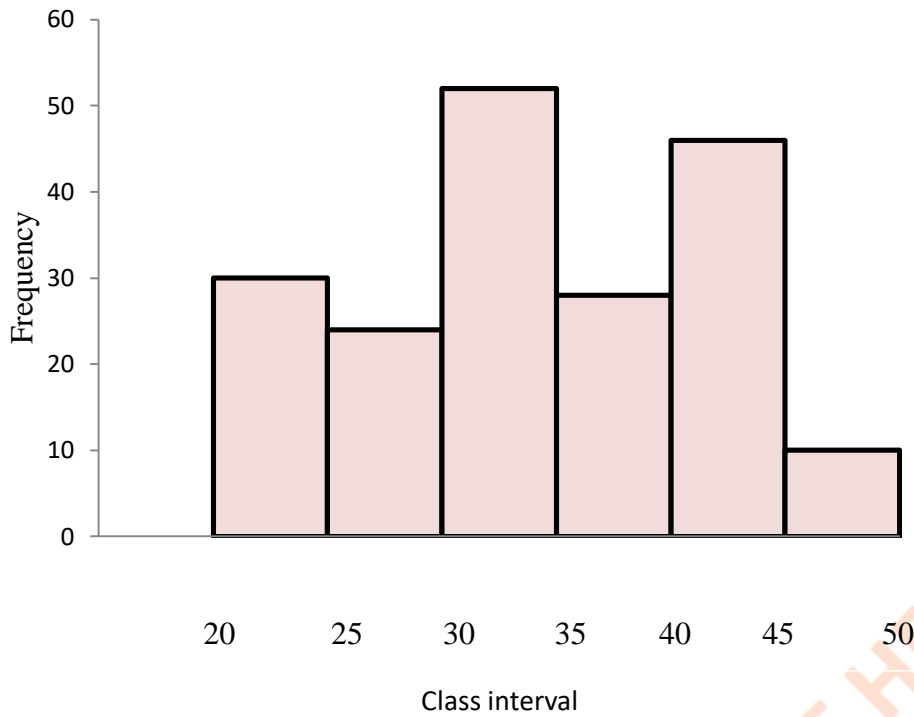
The histogram is shown as below:



Question 4 – Draw a histogram for the frequency distribution of the following data:

Class interval	20-25	25-30	30-35	35-40	40-45	45-50
frequency	30	24	52	28	46	10

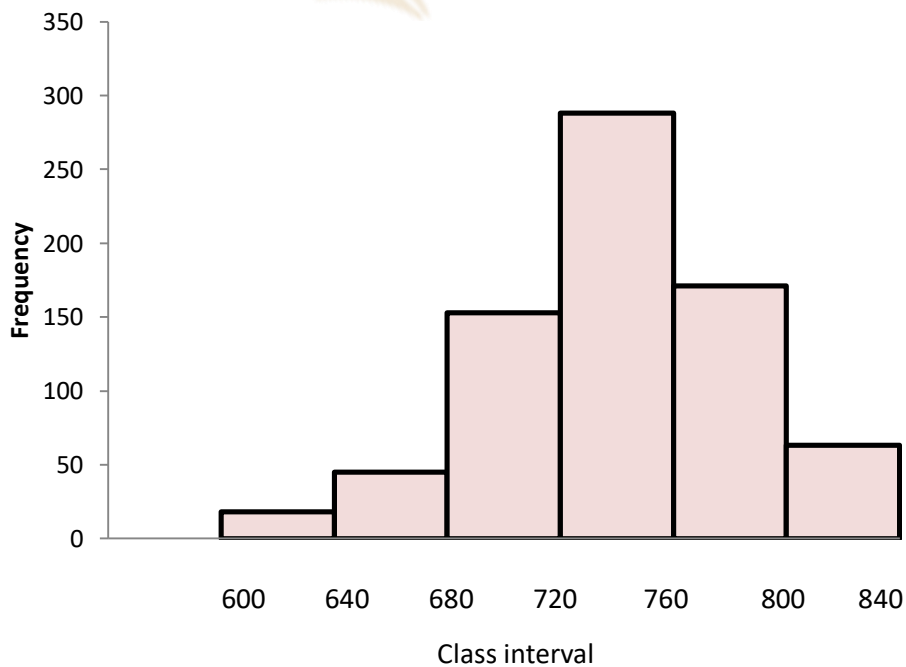
Solution - On the graph paper, we represent the class intervals along the x-axis and the frequency along the y-axis.



Question 5 – Draw a histogram for the following data:

Class interval	600-640	640-680	680-720	720-760	760-800	800-840
frequency	18	45	152	288	171	63

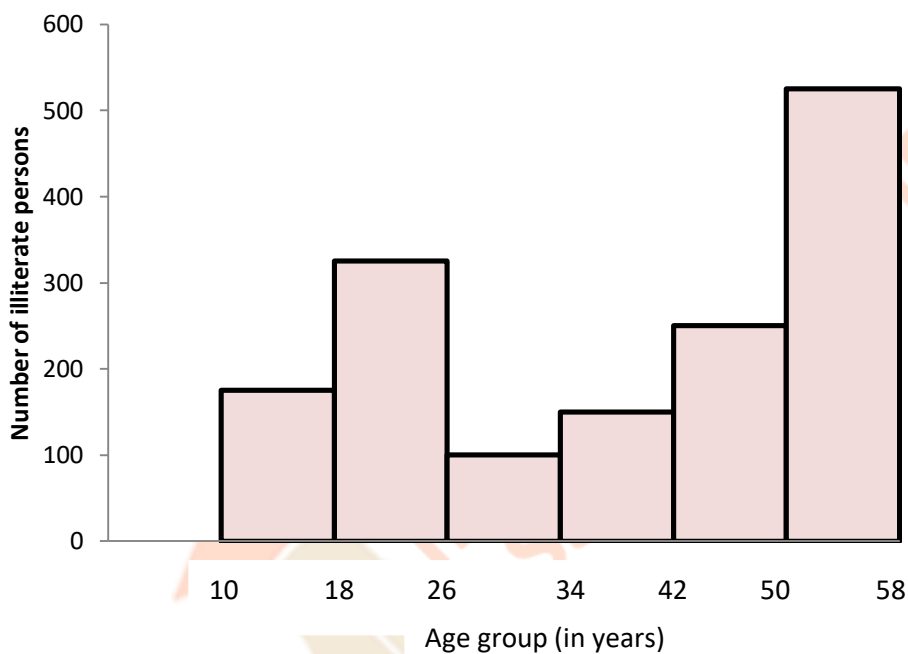
Solution - On the graph paper, we represent the class intervals along the x-axis and the frequency along the y-axis.



Question 6 – The following table shows the number of illiterate persons in the age group (10-58 years) in a town. Represent the given data by means of a histogram.

Age group (in years)	10-18	18-26	26-34	34-42	42-50	50-58
Number of illiterate persons	175	325	100	150	250	525

Solution - On the graph paper, we represent the age groups along the x-axis and the number of illiterate persons along the y-axis.



Question 7 – The marks obtained (out of 20) by 30 students of a class in a test are given below:

7, 10, 8, 16, 13, 14, 15, 11, 18, 11, 15, 10, 7, 14, 20, 19, 15, 16, 14, 20, 10, 11, 14, 17, 13, 12, 15, 14, 16, 17

Prepare a frequency distribution table for the above data using class intervals of equal width in which one class intervals is 3-8 (including 3 and excluding 8). From the frequency distribution table so obtained, draw a histogram.

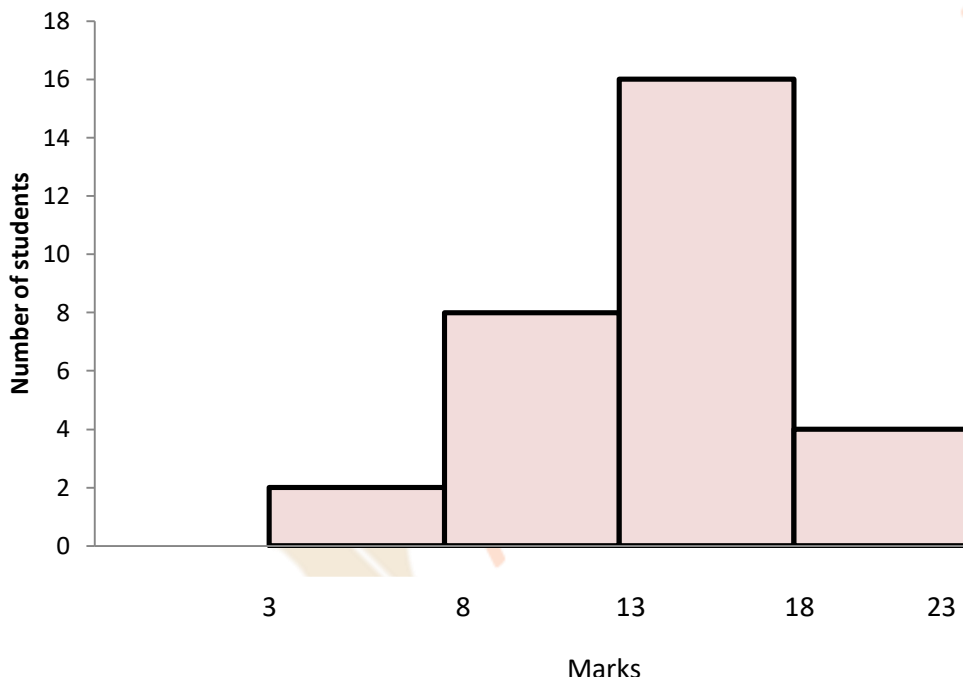
Solution - Firstly we will make frequency distribution table by using tally marks and taking the intervals as 3-8, 8-13, and so on.

The frequency distribution table of the following data is as follows:

Class interval	Tally Marks	Frequency
3-8		2
8-13		8
13-18		16
18-23		4
Total		30

On the graph paper, we represent the class intervals along the x-axis and the marks obtained along the y-axis.

The histogram is shown as below:



Question 8 – The weights (in kg) of 30 students of a class are

39, 43, 32, 37, 29, 26, 31, 45, 46, 31, 37, 38, 30, 39, 36, 41, 35, 34, 41, 46, 39, 38, 36, 38, 40, 42, 33, 43, 44, 33

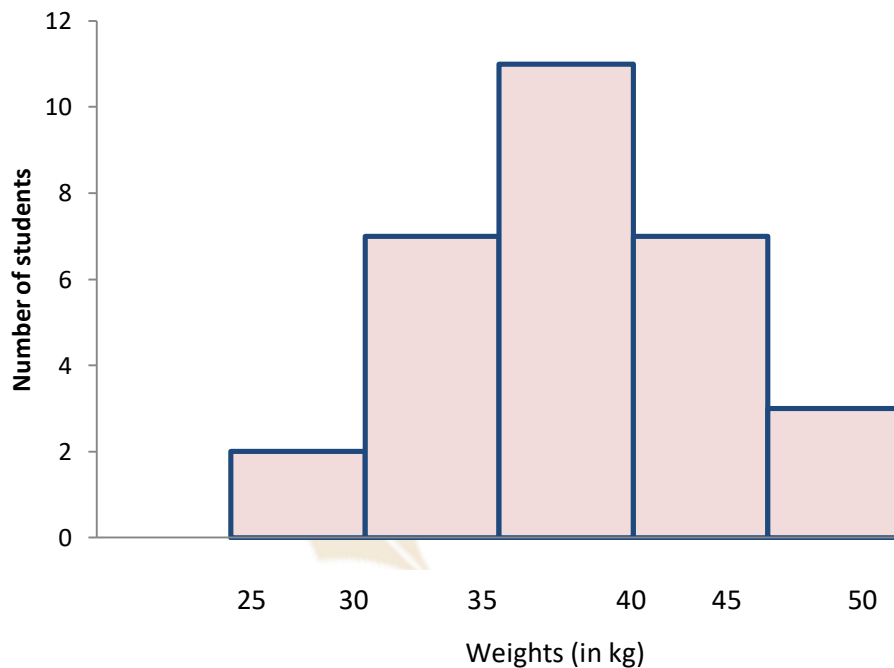
Solution - Firstly we will make frequency distribution table by using tally marks and taking the intervals as 30-35, 35-40, and so on.

The frequency distribution table of the following data is as follows:

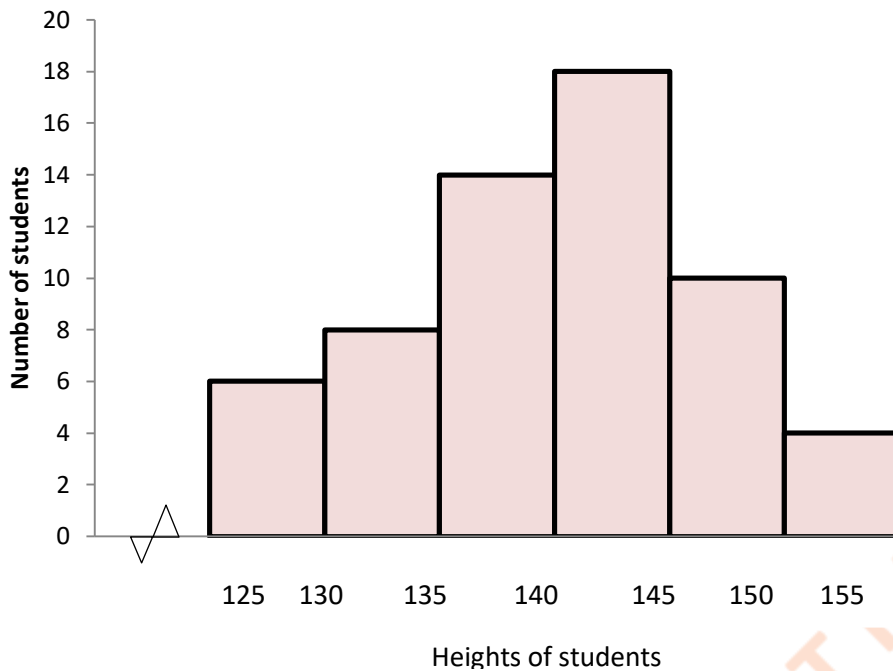
Class Interval	Tally Marks	Frequency
25-30		2
30-35		7
35-40		11
40-45		7
45-50		3
Total		30

On the graph paper, we represent the class intervals along the x-axis and the marks obtained along the y-axis.

The histogram is shown as below:



Question 9 – Look at the histogram given below and answer the questions that follow:



- (a) How many students have height more than or equal to 135 cm but less than 150 cm?
- (b) Which class interval has the least number of students?
- (c) What is the class size?
- (d) How many students have height less than 140 cm?

Solution: (i) From the above graph, it is clear that number of students having height more than or equal to 135 cm but less than 150 cm = $14 + 18 + 10 = 42$

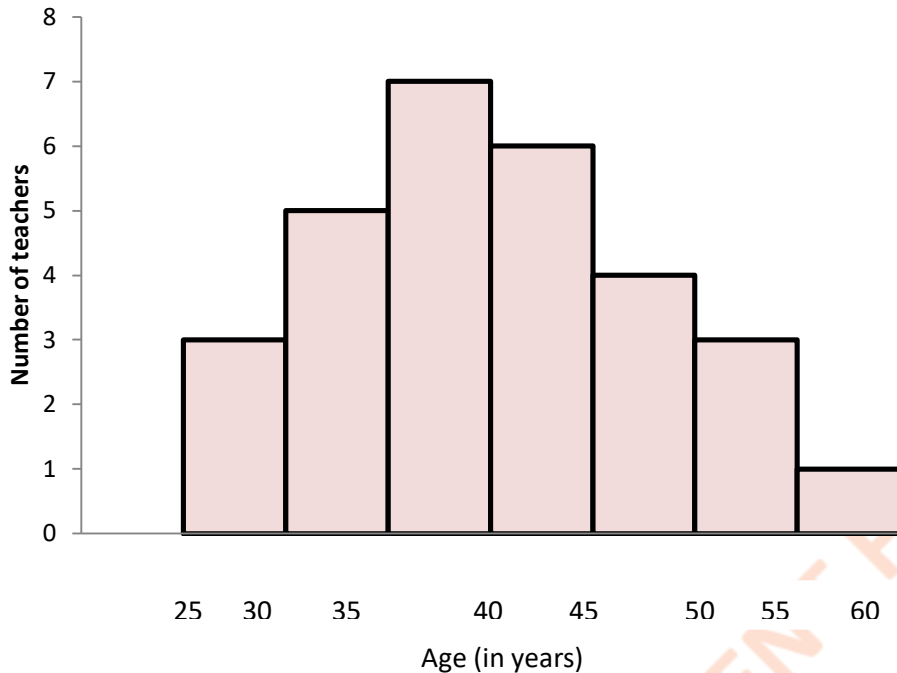
(ii) Class interval 150-155 cm has least number of students

(iii) Class size = Upper limit – lower limit

So, in each interval class size is 5

(iv) Number of students having height less than 140 cm = $6 + 8 + 14 = 28$

Question 10 – Look at the histogram given below and answer the questions that follow:



(a) What information is being given by the histogram?

(b) How many teachers are of age less than 45 years?

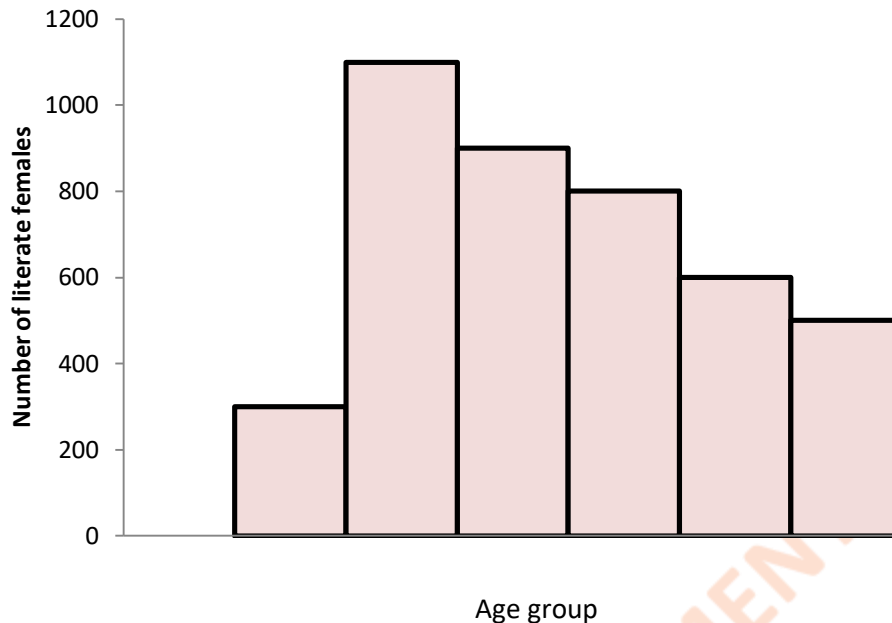
(c) How many teachers are of age 40 years or more but less than 55 years?

Solution: (i) It shows that the number of teachers in the school in the age groups from 25-30 to 55-60

(ii) Number of teachers having age less than 45 years = $3 + 5 + 7 + 6 = 21$

(iii) Number of teachers having age 40 years or more but less than 55 years = $6 + 4 + 3 = 13$

Question 11 – The histogram given below shows the number of literate females in the age group of 10 to 40 years. Study the histogram carefully and answer the questions that follow.



- (a) Write the classes, assuming that all the classes are of equal width.**
- (b) What is the class width?**
- (c) In which age group are the literate females the least?**
- (d) In which age group is the number of literate females the highest?**

Solution: i) On assuming that all the classes are of the equal size, the classes are as follows:

10-15, 15-20, 20-25, 25-30, 30-35 and 35-40

- (ii) Class width is 5
- (iii) Number of literate female is least in the age group 10-15
- (iv) Number of literate female is highest in the age group 15-20