Introduction

Square: The square of a number is the product of the number with the number itself.

Ex: Square of x is (x * x) = x^2

If x = 4 then square of 4 is (4*4) = 4^2 = 16

Perfect Squares: A natural number is called a perfect square if it is the square of some natural number.

Note: A perfect square is always expressible as the product of pairs of equal factors.

Examples:

Example 1 - Is 196 a perfect square? If so, find the number whose square is 196.

Solution - To see 196 is perfect square or not, we resolve 196 into prime factors

2	196
2	98
7	49
7	7
	1

Here we can see that 196 = $2 \times 2 \times 7 \times 7$

=> $196 = (2 \times 2) \times (7 \times 7) = 2^2 \times 7^2 = (2 \times 7)^2 = 14^2$

Since 196 can be expressed as product of pairs of equal factors

∴ 196 is a perfect square

And 14 is the number whose square is 196

Example 2 - Show that 1764 is a perfect square. Find the number whose square is 1764.

Solution - To see 1764 is perfect square or not, we resolve 1764 into prime factors

2	1764
2	882
3	441
3	147
7	49
7	7
	1

Here we can see that 1764 = $2 \times 2 \times 3 \times 3 \times 7 \times 7$

=> 1764 = $(2 \times 2) \times (7 \times 7) \times (3 \times 3) = 2^2 \times 7^2 \times 3^2 = (2 \times 7 \times 3)^2 = 42^2$

Since 1764 can be expressed as product of pairs of equal factors

∴ 1764 is a perfect square

And 42 is the number whose square is 1764.

Example 3 - Show that 6292 is not a perfect square.

Solution - To see 6292 is perfect square or not, we resolve 6292 into prime factors

2	6292
2	3146
11	1573
11	143
13	13
	1

Here we can see that 6292 = $2 \times 2 \times 11 \times 11 \times 13$

=> 6292 = $(2 \times 2) \times (11 \times 11) \times 13 = 2^2 \times 11^2 \times 13$

Since 6292 cannot be expressed as product of pairs of equal factors

∴ 6292 is not a perfect square.

Example 4 - By what least number should 3675 be multiplied to get a perfect square number? Also, find the number whose square is the new number.

Solution - First we resolve 3675 into prime factors

5	3675
5	735
3	147
7	49
7	7
	1

Here we can see that 3675 = $5 \times 5 \times 7 \times 7 \times 3$

=> 3675 = $(5 \times 5) \times (7 \times 7) \times 3 = 5^2 \times 7^2 \times 3$

Now to get a perfect square number, 3675 must be multiplied by 3.

Thus new number will be $(5^2 \times 7^2 \times 3^2) = (5 \times 7 \times 3)^2 = 105^2$ = 11025

And 105 is the number whose square is 11025

Example 5 - By what least number should 6300 be divided to get a perfect square number? Find the number whose square is the new number.

Solution - First we resolve 6300 into prime factors

2	6300
2	3150
3	1575
3	525
5	175
5	35
7	7
	1

Here we can see that 6300 = $2 \times 2 \times 3 \times 3 \times 5 \times 5 \times 7$

=> 6300 = $(5 \times 5) \times (2 \times 2) \times (3 \times 3) \times 7 = 2^2 \times 5^2 \times 3^2 \times 7$

Now to get a perfect square number, 6300 must be divided by 7.

Thus new number will be $\frac{2^2 \times 5^2 \times 3^2 \times 7}{7} = (2 \times 5 \times 3)^2 = 30^2 = 900$

And 30 is the number whose square is 900.

Exercise 3A

Question 1 - Using the prime factorization method, Find which of the following numbers is perfect squares:

(a) 441

Solution - To see 441 is perfect square or not, we resolve 441 into prime factors

3	441
3	147
7	49
7	7
	1

Here we can see that 441 = $3 \times 3 \times 7 \times 7$

=> 441 = $(3 \times 3) \times (7 \times 7) = 3^2 \times 7^2 = (3 \times 7)^2 = 21^2$

Since 441 can be expressed as product of pairs of equal factors

∴ 441 is a perfect square

(b) 576

Solution - To see 576 is perfect square or not, we resolve 576 into prime factors

2	576
2	288
2	144
2	72
2	36
2	18
3	9
3	3
	1

Here we can see that $576 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3$

=> $576 = (2 \times 2) \times (2 \times 2) \times (2 \times 2) \times (3 \times 3) = 2^2 \times 2^2 \times 2^2 \times 3^2 = (2 \times 2 \times 2 \times 3)^2 = 24^2$

Since 576 can be expressed as product of pairs of equal factors

∴ 576 is a perfect square

(c) 11025

Solution - To see 11025 is perfect square or not, we resolve 11025 into prime factors

5	11025
5	2205
3	441
3	147
7	49
7	7
	1

Here we can see that $11025 = 5 \times 5 \times 7 \times 7 \times 3 \times 3$

=> $11025 = (5 \times 5) \times (7 \times 7) \times (3 \times 3) = 5^2 \times 7^2 \times 3^2 = (5 \times 7 \times 3)^2 = 105^2$

Since 11025 can be expressed as product of pairs of equal factors

∴ 11025 is a perfect square

(d) 1176

Solution - To see 1176 is perfect square or not, we resolve 1176 into prime factors

2	1176
2	588
2	294
3	147
7	49
7	7
	1

Here we can see that 1176 = 2 × 2× 2 ×7× 7 × 3

=> 1176 = (2 × 2 × 2) × (7 × 7) × 3

Since 1176 cannot be expressed as product of pairs of equal factors

∴ 1176 is not a perfect square

(e) 5625

Solution - To see 5625 is perfect square or not, we resolve 5625 into prime factors

5	5625
5	1125
5	225
5	45
3	9
3	3
	1

Here we can see that 5625 = 5 × 5× 5 ×5× 3 × 3

=> 5625 = (5 × 5) × (5 × 5) × (3 × 3) = $5^2 \times 5^2 \times 3^2 = (5 \times 5 \times 3)^2 = 75^2$

Since 5625 can be expressed as product of pairs of equal factors

∴ 5625 is a perfect square

(f) 9075

Solution - To see 9075 is perfect square or not, we resolve 9075 into prime factors

5	9075
5	1815
3	363
11	121
11	11
	1

Here we can see that 9075 = 5 × 5× 11 ×11× 3

=> 9075 = (5 × 5) × (11 × 11) × 3

Since 9075 cannot be expressed as product of pairs of equal factors

∴ 9075 is not a perfect square

(g) 4225

Solution - To see 4225 is perfect square or not, we resolve 4225 into prime factors

5	4225
5	845
13	169
13	13
	1

Here we can see that $4225 = 5 \times 5 \times 13 \times 13$

=> $4225 = (5 \times 5) \times (13 \times 13) = 5^2 \times 13^2 = (5 \times 13)^2 = 65^2$

Since 4225 can be expressed as product of pairs of equal factors

∴ 4225 is a perfect square

(h) 1089

Solution - To see 1089 is perfect square or not, we resolve 1089 into prime factors

3	1089
3	363
11	121
11	11
	1

Here we can see that $1089 = 11 \times 11 \times 3 \times 3$

=> $1089 = (11 \times 11) \times (3 \times 3) = 11^2 \times 3^2 = (11 \times 3)^2 = 33^2$

Since 1089 can be expressed as product of pairs of equal factors

∴ 1089 is a perfect square

Question 2 - Show that each of the following numbers is a perfect square. In each case, find the number whose square is the given number:

(a) 1225

Solution - To see 1225 is perfect square or not, we resolve 1225 into prime factors

5	1225
5	245
7	49
7	7
	1

Here we can see that $1225 = 5 \times 5 \times 7 \times 7$

$\Rightarrow 1225 = (5 \times 5) \times (7 \times 7) = 5^2 \times 7^2 = (5 \times 7)^2 = 35^2$

Since 1225 can be expressed as product of pairs of equal factors

∴ 1225 is a perfect square

And 35 is the number whose square is 1225.

(b) 2601

Solution **-** To see 2601 is perfect square or not, we resolve 2601 into prime factors

3	2601
3	867
17	289
17	17
	1

Here we can see that $2601 = 3 \times 3 \times 17 \times 17$

$\Rightarrow 2601 = (17 \times 17) \times (3 \times 3) = 17^2 \times 3^2 = (17 \times 3)^2 = 51^2$

Since 2601 can be expressed as product of pairs of equal factors

∴ 2601 is a perfect square

And 51 is the number whose square is 2601.

(c) 5929

Solution **-** To see 5929 is perfect square or not, we resolve 5929 into prime factors

7	5929
7	847
11	121
11	11
	1

Here we can see that $5929 = 11 \times 11 \times 7 \times 7$

$\Rightarrow 5929 = (11 \times 11) \times (7 \times 7) = 11^2 \times 7^2 = (11 \times 7)^2 = 77^2$

Since 5929 can be expressed as product of pairs of equal factors

∴ 5929 is a perfect square

And 77 is the number whose square is 5929.

(d) 7056

Solution - To see 7056 is perfect square or not, we resolve 7056 into prime factors

2	7056
2	3528
2	1764
2	882
3	441
3	147
7	49
7	7
	1

Here we can see that $7056 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 7 \times 7$

$\Rightarrow 7056 = (2 \times 2) \times (2 \times 2) \times (7 \times 7) \times (3 \times 3) = 2^2 \times 2^2 \times 7^2 \times 3^2$

$= (2 \times 2 \times 7 \times 3)^2 = 84^2$

Since 7056 can be expressed as product of pairs of equal factors

∴ 7056 is a perfect square

And 84 is the number whose square is 7056.

(e) 8281

Solution - To see 8281 is perfect square or not, we resolve 8281 into prime factors

7	8281
7	1183
13	169
13	13
	1

Here we can see that $8281 = 13 \times 13 \times 7 \times 7$

$\Rightarrow 8281 = (7 \times 7) \times (13 \times 13) = 7^2 \times 13^2 = (7 \times 13)^2 = 91^2$

Since 8281 can be expressed as product of pairs of equal factors

∴ 8281 is a perfect square

And 91 is the number whose square is 8281.

Question 3 - By what least number should the given number be multiplied to get a perfect square number? In each case, find the number whose square is the new number.

(a) 3675

Solution - First we resolve 3675 into prime factors

5	3675
5	735
3	147
7	49
7	7
	1

Here we can see that $3675 = 5 \times 5 \times 7 \times 7 \times 3$

=> $3675 = (5 \times 5) \times (7 \times 7) \times 3 = 5^2 \times 7^2 \times 3$

Now to get a perfect square number, 3675 must be multiplied by 3.

Thus new number will be $(5^2 \times 7^2 \times 3^2) = (5 \times 7 \times 3)^2 = 105^2 = 11025$

And 105 is the number whose square is 11025

(b) 2156

Solution - First we resolve 2156 into prime factors

2	2156
2	1078
7	539
7	77
11	11
	1

Here we can see that $2156 = 2 \times 2 \times 7 \times 7 \times 11$

=> $2156 = (2 \times 2) \times (7 \times 7) \times 11 = 2^2 \times 7^2 \times 11$

Now to get a perfect square number, 2156 must be multiplied by 11.

Thus new number will be $(2^2 \times 7^2 \times 11^2) = (2 \times 7 \times 11)^2 = 154^2 = 23716$

And 154 is the number whose square is 23716.

(c) 3332

Solution - First we resolve 3332 into prime factors

2	3332
2	1666
7	833
7	119
17	17
	1

Here we can see that 3332 = $2 \times 2 \times 7 \times 7 \times 17$

=> 3332 = $(2 \times 2) \times (7 \times 7) \times 17 = 2^2 \times 7^2 \times 17$

Now to get a perfect square number, 3332 must be multiplied by 17.

Thus new number will be $(2^2 \times 7^2 \times 17^2) = (2 \times 7 \times 17)^2 = 238^2$

And 238 is the number whose square is the new number.

(d) 2925

Solution - First we resolve 2925 into prime factors

5	2925
5	585
3	117
3	39
13	13
	1

Here we can see that 2925 = $5 \times 5 \times 3 \times 3 \times 13$

=> 2925 = $(5 \times 5) \times (3 \times 3) \times 13 = 5^2 \times 3^2 \times 13$

Now to get a perfect square number, 2925 must be multiplied by 13.

Thus new number will be $(5^2 \times 3^2 \times 13^2) = (5 \times 3 \times 13)^2 = 195^2$

And 195 is the number whose square is the new number.

(e) 9075

Solution - First we resolve 9075 into prime factors

5	9075
5	1815
3	363
11	121
11	11
	1

Here we can see that $9075 = 5 \times 5 \times 11 \times 11 \times 3$

=> $9075 = (5 \times 5) \times (11 \times 11) \times 3 = 5^2 \times 11^2 \times 3$

Now to get a perfect square number, 9075 must be multiplied by 3.

Thus new number will be $(5^2 \times 11^2 \times 3^2) = (5 \times 11 \times 3)^2 = 165^2$

And 165 is the number whose square is the new number.

(f) 7623

Solution - First we resolve 7623 into prime factors

3	7623
3	2541
7	847
11	121
11	11
	1

Here we can see that $7623 = 3 \times 3 \times 11 \times 11 \times 7$

=> $7623 = (3 \times 3) \times (11 \times 11) \times 7 = 3^2 \times 11^2 \times 7$

Now to get a perfect square number, 7623 must be multiplied by 7.

Thus new number will be $(3^2 \times 11^2 \times 7^2) = (3 \times 11 \times 7)^2 = 231^2$

And 231 is the number whose square is the new number.

(g) 3380

Solution - First we resolve 3380 into prime factors

2	3380
2	1690
5	845
13	169
13	13
	1

Here we can see that $3380 = 2 \times 2 \times 13 \times 13 \times 5$

=> $3380 = (2 \times 2) \times (13 \times 13) \times 5 = 2^2 \times 13^2 \times 5$

Now to get a perfect square number, 3380 must be multiplied by 5.

Thus new number will be $(2^2 \times 13^2 \times 5^2) = (2 \times 13 \times 5)^2 = 130^2$

And 130 is the number whose square is the new number.

(h) 2475

Solution - First we resolve 2475 into prime factors

5	2475
5	495
3	99
3	33
11	11
	1

Here we can see that $2475 = 5 \times 5 \times 3 \times 3 \times 11$

=> $2475 = (5 \times 5) \times (3 \times 3) \times 11 = 5^2 \times 3^2 \times 11$

Now to get a perfect square number, 2475 must be multiplied by 11.

Thus new number will be $(5^2 \times 3^2 \times 11^2) = (5 \times 11 \times 3)^2 = 165^2$

And 165 is the number whose square is the new number.

Question 4 - By what least number should the given number be divided to get a perfect square number? In each case, find the number whose square is the new number.

(a) 1575

Solution - First we resolve 1575 into prime factors

3	1575
3	525
5	175
5	35
7	7
	1

Here we can see that 1575 = $3 \times 3 \times 5 \times 5 \times 7$

=> 1575 = $(5 \times 5) \times (3 \times 3) \times 7 = 5^2 \times 3^2 \times 7$

Now to get a perfect square number, 1575 must be divided by 7.

Thus, new number will be $\frac{5^2 \times 3^2 \times 7}{7} = (5 \times 3)^2 = 15^2$

And 15 is the number whose square is the new number.

(b) 9075

Solution - First we resolve 9075 into prime factors

3	9075
5	3025
5	605
11	121
11	11
	1

Here we can see that 9075 = $3 \times 5 \times 5 \times 11 \times 11$

=> 9075 = $(5 \times 5) \times (11 \times 11) \times 3 = 5^2 \times 11^2 \times 3$

Now to get a perfect square number, 9075 must be divided by 3.

Thus new number will be $\frac{5^2 \times 11^2 \times 3}{3} = (5 \times 11)^2 = 55^2$

And 55 is the number whose square is the new number.

(c) 4851

Solution - First we resolve 4851 into prime factors

3	4851
3	1617
7	539
7	77
11	11
	1

Here we can see that 4851 = $3 \times 3 \times 7 \times 7 \times 11$

=> 4851 = $(7 \times 7) \times (3 \times 3) \times 11 = 7^2 \times 3^2 \times 11$

Now to get a perfect square number, 4851 must be divided by 11.

Thus new number will be $\frac{7^2 \times 3^2 \times 11}{11}$ = $(7 \times 3)^2$ = 21^2

And 21 is the number whose square is the new number

(d) 3380

Solution - First we resolve 3380 into prime factors

2	3380
2	1690
5	845
13	169
13	13
	1

Here we can see that 3380 = $2 \times 2 \times 13 \times 13 \times 5$

=> 3380 = $(2 \times 2) \times (13 \times 13) \times 5 = 2^2 \times 13^2 \times 5$

Now to get a perfect square number, 3380 must be divided by 5.

Thus new number will be $\frac{2^2 \times 13^2 \times 5}{5}$ = $(2 \times 13)^2$ = 26^2

And 26 is the number whose square is the new number.

(e) 4500

Solution - First we resolve 4500 into prime factors

2	4500
2	2250
5	1125
5	225
5	45
3	9
3	3
	1

Here we can see that 4500 = $2 \times 2 \times 3 \times 3 \times 5 \times 5 \times 5$

=> 4500 = $(5 \times 5 \times 5) \times (2 \times 2) \times (3 \times 3) = 2^2 \times 3^2 \times 5^2 \times 5$

Now to get a perfect square number, 4500 must be divided by 5.

Thus new number will be $\frac{2^2 \times 5^2 \times 3^2 \times 5}{5} = (2 \times 5 \times 3)^2 = 30^2$

And 30 is the number whose square is the new number.

(f) 7776

Solution **-** First we resolve 7776 into prime factors

2	7776
2	3888
2	1944
2	972
2	486
3	243
3	81
3	27
3	9
3	3
	1

Here we can see that 7776 = $2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 3$

=> 7776 = $(2 \times 2) \times (2 \times 2) \times (3 \times 3) \times (3 \times 3) \times (2 \times 3) = 2^2 \times 2^2 \times 3^2 \times 3^2 \times 6$

Now to get a perfect square number, 7776 must be divided by 6.

Thus new number will be $\frac{2^2 \times 2^2 \times 3^2 \times 3^2 \times 6}{6} = (2 \times 2 \times 3 \times 3)^2 = 36^2$

And 36 is the number whose square is the new number.

(g) 8820

Solution - First we resolve 8820 into prime factors

2	8820
2	4410
5	2205
3	441
3	147
7	49
7	7
	1

Here we can see that $8820 = 2 \times 2 \times 3 \times 3 \times 7 \times 7 \times 5$

=> $8820 = (7 \times 7) \times (2 \times 2) \times (3 \times 3) \times 5 = 2^2 \times 7^2 \times 3^2 \times 5$

Now to get a perfect square number, 8820 must be divided by 5.

Thus new number will be $\frac{2^2 \times 7^2 \times 3^2 \times 5}{5} = (2 \times 7 \times 3)^2 = 42^2$

And 42 is the number whose square is the new number.

(h) 4056

Solution - First we resolve 4056 into prime factors

2	4056
2	2028
2	1014
3	507
13	169
13	13
	1

Here we can see that $4056 = 2 \times 2 \times 2 \times 3 \times 13 \times 13$

=> $4056 = (2 \times 2) \times (2 \times 3) \times (13 \times 13) = 2^2 \times 13^2 \times 6$

Now to get a perfect square number, 4056 must be divided by 6.

Thus new number will be $\frac{2^2 \times 13^2 \times 6}{6} = (2 \times 13)^2 = 26^2$

And 26 is the number whose square is the new number.

Question 5 - Find the largest number of 2 digits which is a perfect square.

Solution - We know that first three digit number is 100

And 100 = $10 \times 10 = 10^2$

Thus 100 is the perfect square

As a number before 10 is 9

Now (9^2) = 9×9 = 81

Hence largest 2 digit number which is a perfect square is 81.

Question 6 - Find the largest number of 3 digits which is a perfect square.

Solution - We know that largest three digit number = 999

Since square root of 999 is approximately 31.60

Thus square of 31 will be the largest three digit number.

=> $(31)^2 = 31 \times 31$ = 961

Properties of perfect squares:

Property1 A number ending in 2, 3, 7 or 8 is never a perfect square.

Property2 A number ending in an odd number of zeros is never a perfect square.

Property3 If a number when divided by 3 leaves a remainder 2, then it is not a perfect square.

Property4 If a number when divided by 4 leaves a remainder 2 0r 3, then it is not a perfect square.

Property5 the square of an even number is always even.

Property6 the square of an odd number is always odd.

Property7 the square of a proper fraction is smaller than the fraction.

Property8 For every natural number n, we have

$$(n+1)^2 - n^2 = \{(n+1) + n\}$$

Property9 For every natural number n, we have

Sum of first n odd natural numbers = n^2

Property10 (Pythagorean triplets): Three natural numbers m, n, p are said to form a Pythagorean triplet (m, n, p) if $(m^2 + n^2) = p^2$.

Note: For each natural number m>1, (2m, $m^2 - 1, m^2 + 1)$ is a Pythagorean triplet.

Property11: Between two consecutive square numbers n^2 and$(n + 1)^2$, there are 2n non perfect square numbers.

Examples:

Example 1 - Give reasons to show that none of the numbers given below is a perfect square.

(a) 2162

(b) 6843

(c) 9637

(d) 6598

Solution **-** According to property1, we know that number ending in 2, 3, 7 or 8 cannot be a perfect square. Thus 2162, 6843, 9637, 6598 are not perfect squares.

Example 2 - Give reason to show that none of the numbers 640, 81000 and 3600000 is a perfect square

Solution **-** According to property 2, we know that number ending in odd number of zeros cannot be a perfect square. Thus 640, 81000 and 3600000 are not perfect squares.

Example 3 - State whether the square of the given number is even or odd:

(a) 523

(b) 654

(c) 6776

(d) 7025

Solution **-** According to property 5 and 6, square of odd number is odd and square of even number is even.

(a) Since 523 is an odd number then $(523)^2$ is also odd.

(b) Since 654 is an even number then $(654)^2$ is also even.

(c) Since 6776 is an even number then $(6776)^2$ is also even

(d) Since 7025 is an odd number then $(7025)^2$ is also odd.

Example 4 - Without adding, find the sum:

(a) $(1 + 3 + 5 + 7 + 9 + 11)$

Solution - According to property 9, sum of first n odd natural numbers = n^2

Thus, $(1 + 3 + 5 + 7 + 9 + 11) = 6^2 = 36$

(b) $(1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17)$

Solution - According to property 9, sum of first n odd natural numbers = n^2

Thus, $(1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17) = 9^2 = 81$

Example 5 - Express

(a) 64 as the sum of 8 odd natural numbers

Solution - We know that sum of first n odd natural numbers = n^2

Thus, $64 = 8^2 = (1 + 3 + 5 + 7 + 9 + 11 + 13 + 15)$

(b) 121 as the sum of 11 odd natural numbers

Solution - We know that sum of first n odd natural numbers = n^2

Thus, $121 = 11^2 = (1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 + 21)$

Example 6 - Find the Pythagorean triplet whose smallest member is 12.

Solution - We know that for each natural number m>1, (2m, $m^2 - 1, m^2 + 1$) is a Pythagorean triplet.

One member = 12

Let us suppose 2m = 12

=> m = 6

$m^2 - 1 = 6^2 - 1 = 36 - 1 = 35$

$m^2 + 1 = 6^2 + 1 = 36+1 = 37$

Thus, (12, 35, 37) is a Pythagorean triplet.

Example 7 - Evaluate [$(\mathbf{337})^{\mathbf{2}} - (\mathbf{336})^{\mathbf{2}}$]

Solution - We know that $(n+1)^2 - n^2 = \{(n+1)+n\}$

Thus, [$(337)^2 - (336)^2$] = $\{(336+1)+336\}$ = {337+336} = 673

Example 8 - Using the identity $(\boldsymbol{a}+\boldsymbol{b})^{\mathbf{2}} = (\boldsymbol{a}^{\mathbf{2}} + \mathbf{2}\boldsymbol{ab} + \boldsymbol{b}^{\mathbf{2}})$, evaluate:

(a) $(\mathbf{609})^{\mathbf{2}}$

Solution - $(609)^2$ = $(600+9)^2$

= $(600)^2 + 2(600)(9) + 9^2$

= $360000 + 10800 + 81$ = 370881

(b) $(\mathbf{725})^{\mathbf{2}}$

Solution - $(725)^2$ = $(700+25)^2$

= $(700)^2 + 2(700)(25) + (25)^2$

= $490000 + 35000 + 625$ = 525625

Example 8 - Using the identity $(\boldsymbol{a}-\boldsymbol{b})^{\mathbf{2}} = (\boldsymbol{a}^{\mathbf{2}} - \mathbf{2}\boldsymbol{ab} + \boldsymbol{b}^{\mathbf{2}})$, evaluate

(a) $(\mathbf{491})^{\mathbf{2}}$

Solution - $(491)^2$ = $(500-9)^2$

= $(500)^2 - 2(500)(9) + (9)^2$

= $250000 - 9000 + 81$ = 241081

(b) $(\mathbf{491})^{\mathbf{2}}$

Solution - $(289)^2$ = $(300-11)^2$

= $(300)^2 - 2(300)(11) + (11)^2$

= $90000 - 6600 + 121$ = 83521

Example 10 - Evaluate

(a) $\mathbf{49} \times \mathbf{51}$

Solution - We can rewrite 49×51 as

= $(50-1) \times (50+1)$

Now using identity$(a+b)(a-b) = a^2 - b^2$

= $((50)^2 - 1^2)$ = $(2500 - 1)$ = 2499

(b) 30×32

Solution - we can rewrite 30×32 as

= $(31 - 1) \times (31 + 1)$

Now using identity$(a+b)(a-b) = a^2 - b^2$

= $((31)^2 - 1^2)$ = $(961 - 1)$ = 960

Exercise 3B

Question 1 - Give reason to show that none of the numbers given below is a perfect square:

(a) 5372 (b) 5963 (c) 8457 (d) 9468 (e) 360 (f) 64000 (g) 2500000

Solution - According to property1, we know that number ending in 2, 3, 7 or 8 cannot be a perfect square. Thus 5372, 5963, 8457 and 9468 are not perfect squares.

According to property 2, we know that number ending in odd number of zeros cannot be a perfect square. Thus 360, 64000 and 2500000 are not perfect squares.

Question 2 - Which of the following are squares of even numbers?

(a) 196 (b) 441 (c) 900 (d) 625 (e) 324

Solution - Since the square of an even number is always even

Thus, 196, 900 and 324 are squares of even numbers.

Question 3 - Which of the following are squares of odd numbers?

(a) 484 (b) 961 (c) 7396 (d) 8649 (e) 4225

Solution - Since the square of an odd number is always odd

Thus, 961, 8649 and 4225 are squares of odd numbers.

Question 4 - Without adding, find the sum:

(a) $(1 + 3 + 5 + 7 + 9 + 11 + 13)$

Solution - According to property 9, sum of first n odd natural numbers = n^2

Thus, $(1 + 3 + 5 + 7 + 9 + 11 + 13)$ = 7^2 = 49

(b) $(1+3+5+7+9+11+13+15+17+19)$

Solution - According to property 9, sum of first n odd natural numbers = n^2

Thus, $(1+3+5+7+9+11+13+15+17+19) = (10)^2 = 100$

(c) $(1+3+5+7+9+11+13+15+17+19+21+23)$

Solution - According to property 9, sum of first n odd natural numbers = n^2

Thus, $(1+3+5+7+9+11+13+15+17+19+21+23) = (12)^2 = 144$

Question 5 (a) - Express 81 as the sum of 9 odd numbers.

Solution - We know that sum of first n odd natural numbers = n^2

Thus, $81 = 9^2 = (1+3+5+7+9+11+13+15+17)$

(b) Express 100 as the sum of 10 odd numbers.

Solution - We know that sum of first n odd natural numbers = n^2

Thus, $100 = (10)^2 = (1+3+5+7+9+11+13+15+17+19)$

Question 6 - Write a Pythagorean triplet whose smallest member is

(a) 6

Solution - We know that for each natural number m>1, (2m, m^2-1, m^2+1) is a Pythagorean triplet.

One member = 6

Let us suppose 2m = 6

=> m = 3

$m^2-1 = 3^2-1 = 9-1 = 8$

$m^2+1 = 3^2+1 = 9+1 = 10$

Thus, (6, 8,10) is a Pythagorean triplet.

(b) 14

Solution - We know that for each natural number m>1, (2m, m^2-1, m^2+1) is a Pythagorean triplet.

One member = 14

Let us suppose 2m = 14

=> m = 7

$m^2 - 1$ = $7^2 - 1$ = 49 − 1= 48

$m^2 + 1$ = $7^2 + 1$= 49+1 = 50

Thus, (14, 48, 50) is a Pythagorean triplet.

(c) 16

Solution - We know that for each natural number m>1, (2m, $m^2 - 1, m^2 + 1$) is a Pythagorean triplet.

One member = 16

Let us suppose 2m = 16

=> m = 8

$m^2 - 1$ = $8^2 - 1$ = 64 − 1= 63

$m^2 + 1$ = $8^2 + 1$= 64+1 = 65

Thus, (16, 63, 65) is a Pythagorean triplet.

(d) 20

Solution - We know that for each natural number m>1, (2m, $m^2 - 1, m^2 + 1$) is a Pythagorean triplet.

One member = 20

Let us suppose 2m = 20

=> m = 10

$m^2 - 1$ = $10^2 - 1$ = 100 − 1= 99

$m^2 + 1$ = $10^2 + 1$= 100+1 = 101

Thus, (20, 99, 101) is a Pythagorean triplet.

Question 7 - Evaluate

(a) $(38)^2 - (37)^2$

Solution - We know that $(n+1)^2 - n^2 = \{(n+1) + n\}$

Thus, $[(38)^2 - (37)^2]$ = $[(37 + 1) + 37]$ = [38+37] = 75

(b) $(\mathbf{75})^2 - (\mathbf{74})^2$

Solution - We know that $(n + 1)^2 - n^2 = \{(n + 1) + n\}$

Thus, $[(75)^2 - (74)^2]$ = $[(74 + 1) + 74]$ = [75+74] = 149

(c) $(\mathbf{92})^2 - (\mathbf{91})^2$

Solution - We know that $(n + 1)^2 - n^2 = \{(n + 1) + n\}$

Thus, $[(92)^2 - (91)^2]$ = $[(91 + 1) + 91]$ = [92+91] = 183

(d) $(\mathbf{105})^2 - (\mathbf{104})^2$

Solution - We know that $(n + 1)^2 - n^2 = \{(n + 1) + n\}$

Thus, $[(105)^2 - (104)^2]$ = $[(104 + 1) + 104]$ = [105+104] = 209

(e) $(\mathbf{141})^2 - (\mathbf{140})^2$

Solution - We know that $(n + 1)^2 - n^2 = \{(n + 1) + n\}$

Thus, $[(141)^2 - (140)^2]$ = $[(140 + 1) + 140]$ = [141+140] = 281

(f) $(\mathbf{218})^2 - (\mathbf{217})^2$

Solution - We know that $(n + 1)^2 - n^2 = \{(n + 1) + n\}$

Thus, $[(218)^2 - (217)^2]$ = $[(217 + 1) + 217]$ = [218+217] = 435

Question 8 - Using the formula $(\boldsymbol{a} + \boldsymbol{b})^2 = (\boldsymbol{a}^2 + 2\boldsymbol{ab} + \boldsymbol{b}^2)$, evaluate:

(a) $(\mathbf{310})^2$

Solution**:** $(310)^2$ = $(300 + 10)^2$

= $(300)^2 + 2(300)(10) + (10)^2$

= $90000 + 6000 + 100$ = 96100

(b) $(\mathbf{508})^2$

Solution: $(508)^2$ = $(500 + 8)^2$

= $(500)^2 + 2(500)(8) + 8^2$

= $250000 + 8000 + 64$ = 258064

(c) $(\mathbf{630})^{\mathbf{2}}$

Solution: $(630)^2 = (600 + 30)^2$

$= (600)^2 + 2(600)(30) + (30)^2$

$= 360000 + 36000 + 900$ = 396900

Question 9 - Using the formula, $(\boldsymbol{a} - \boldsymbol{b})^{\mathbf{2}} = (\boldsymbol{a}^{\mathbf{2}} - \mathbf{2}\boldsymbol{ab} + \boldsymbol{b}^{\mathbf{2}})$**, evaluate:**

(a) $(\mathbf{196})^{\mathbf{2}}$

Solution: $(196)^2 = (200 - 4)^2$

$= (200)^2 - 2(200)(4) + (4)^2$

$= 40000 - 1600 + 16$ = 38416

(b) $(\mathbf{689})^{\mathbf{2}}$

Solution: $(689)^2 = (700 - 11)^2$

$= (700)^2 - 2(700)(11) + (11)^2$

$= 490000 - 15400 + 121$ = 474721

(c) $(\mathbf{891})^{\mathbf{2}}$

Solution: $(891)^2 = (900 - 9)^2$

$= (900)^2 - 2(900)(9) + (9)^2$

$= 810000 - 16200 + 81$ = 793881

Q10 Evaluate:

(a) $\mathbf{69} \times \mathbf{71}$

Solution: We can rewrite 69×71 as

$= (70 - 1) \times (70 + 1)$

Now using identity $(a + b)(a - b) = a^2 - b^2$

$= ((70)^2 - 1^2) = (4900 - 1)$ = 4899

(b) 94×106

Solution: We can rewrite 94×106 as

= $(100 - 6) \times (100 + 6)$

Now using identity$(a + b)(a - b) = a^2 - b^2$

= $((100)^2 - 6^2)$ = $(10000 - 36)$ = 9964

Question 11 - Evaluate

(a) 88×92

Solution: We can rewrite 88×92 as

= $(90 - 2) \times (90 + 2)$

Now using identity$(a + b)(a - b) = a^2 - b^2$

= $((90)^2 - 2^2)$ = $(8100 - 4)$ = 8096

(a) 78×82

Solution: We can rewrite 88×92 as

= $(80 - 2) \times (80 + 2)$

Now using identity$(a + b)(a - b) = a^2 - b^2$

= $((80)^2 - 2^2)$ = $(6400 - 4)$ = 6396

Question 12 - Fill in the blanks:

(a) The square of an even number is …..

Solution - even

(b) The square of an odd number is …..

Solution - odd

(c) The square of a proper fraction is ….. Than the given fraction

Solution - smaller

(d) n^2 = the sum of first n …. Natural numbers

Solution - odd

Question 13 - Write true and false for each of the statement given below:

(a) The number of digits in a perfect square is even

Solution - false as 100 has 3 digits (odd) and is a perfect square.

(b) The square of a prime number is prime

Solution - False as 2 is prime number and $2^2 = 4$ which is not prime.

(c) The sum of two perfect squares is a perfect square.

Solution - false as 4 and 9 are perfect squares but $4 + 9 = 13$ which is not a perfect square.

(d) The difference of two perfect squares is a perfect square.

Solution - false as 4 and 9 are perfect squares but $9 - 4 = 5$ which is not a perfect square.

(e) The product of two perfect squares is a perfect square.

Solution - true

<u>Short-cut methods for squaring a number</u>

There are two shortcut methods

Method (1) Column method for squaring a two digit number

Let the tens digit of given number = a and units digit = b

Step1: firstly make three columns I, II, III, headed bya^2, $(2 \times a \times b)$ and b^2 respectively. Write the values ofa^2, $(2 \times a \times b)$ and b^2 in columns I, II and III respectively.

Step2: Now In column III, underline the units digit of b^2 and carry over the tens digit of it to column II and add it to the value of $(2 \times a \times b)$

Step 3 in column II, underline the units digit of the number obtained in step2 and carry over the tens digit of it to column I and add it to the value of a^2.

Step4: Underline the number obtained in step3 in column I. The underlined digits give the required square number.

Example 1 - Find the square of:

(a) 47

Solution - Here a = 4 and b = 7

I	II	III
a^2	$(2 \times a \times b)$	b^2
16 +6 22	56 +4 60	49

Thus, $(47)^2 = 2209$

(b) 86

Solution - Here a = 8 and b = 6

I	II	III
a^2	$(2 \times a \times b)$	b^2
64 +9 73	96 +3 99	36

Thus, $(86)^2 = 7396$

Method (2) Diagonal method for squaring a number

Let us understand this by examples.

Example 2 - Find the square of 39 by using the diagonal method

Solution - Step1: The given number has two digits. So, draw a square and divide it into 4 sub squares as shown below. Write down the digits 3 and 9 horizontally and vertically as shown below.

Step2 Multiply each digit on the left of the square with each digit on the top, one by one. Write the product in the corresponding sub square.

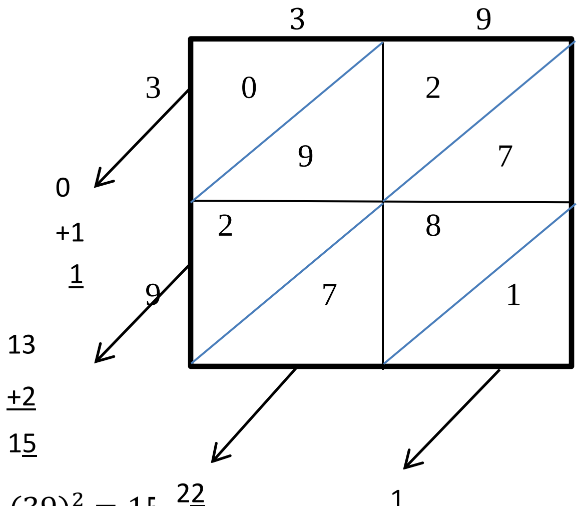
If the product is a one digit number, write it below the diagonal and put 0 above the diagonal.

In case the product is a two digit number, write the tens digit above the diagonal and the units digit below the diagonal.

Step3 Starting below the lowest diagonal, Sum the digits diagonally. If the sum is a two-digit number, underline the units digit and carry over the tens digit to the next diagonal.

Step4 Underline all the digits in the sum above the topmost diagonal.

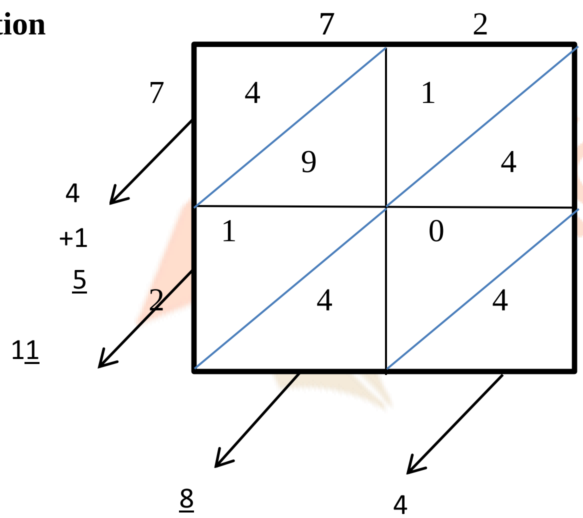
Step5 the underlined digits give the required square number.



Thus, $(39)^2 = 15$

Example 3 - Find the square of 72 by using the diagonal method.

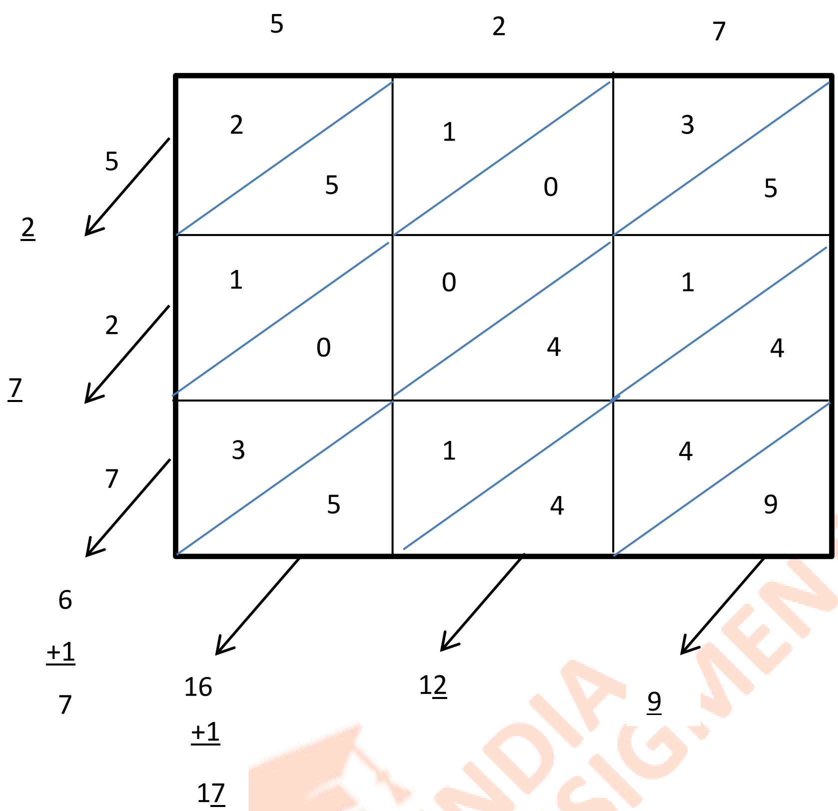
Solution



Thus, $(72)^2 = 5184$

Example 4 - Find the square of 527 by using the diagonal method.

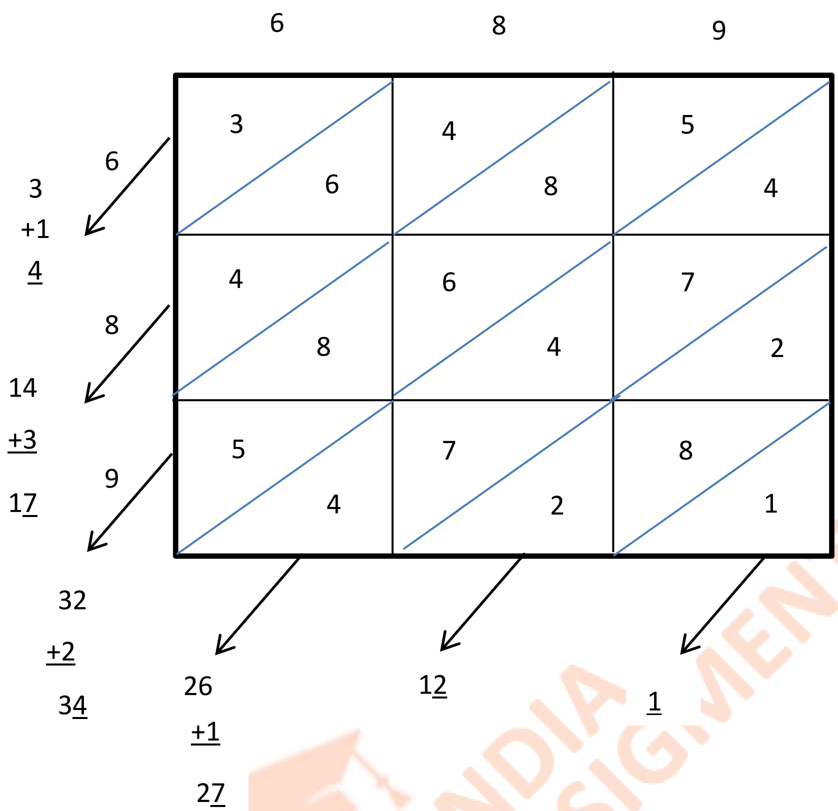
Solution



Thus $(527)^2 = 277729$

Example 5 - Find the square of 689 by using the diagonal method.

Solution



Thus, $(689)^2 = 474721$

Exercise 3C

Find the value of each of the following, using the column method:

(a) $(\mathbf{23})^2$

Solution - Here a = 2 and b = 3

I	II	III
a^2	$(2 \times a \times b)$	b^2
4 +1 5	12	9

Thus, $(23)^2 = 529$

(b) $(\mathbf{35})^2$

Solution - Here a = 3 and b = 5

I	II	III
a^2	$(2 \times a \times b)$	b^2
9 +3 12	30 +2 32	25

Thus, $(35)^2 = 1225$

(c) $(\mathbf{52})^2$

Solution - Here a = 5 and b = 2

I	II	III
a^2	$(2 \times a \times b)$	b^2
25 +2 27	20	4

Thus, $(52)^2 = 2704$

(d) $(\mathbf{96})^{\mathbf{2}}$

Solution - Here a = 9 and b = 6

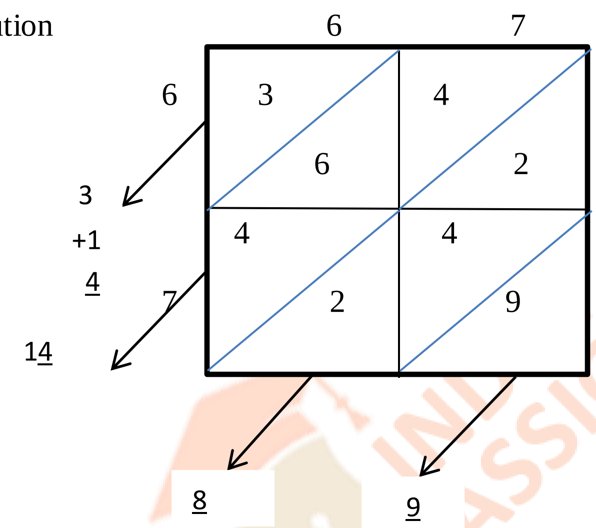
I	II	III
a^2	$(2 \times a \times b)$	b^2
81 +11 92	108 +3 111	36

Thus, $(96)^2 = 9216$

Find the value of each of the following, using the diagonal method:

(e) $(\mathbf{67})^{\mathbf{2}}$

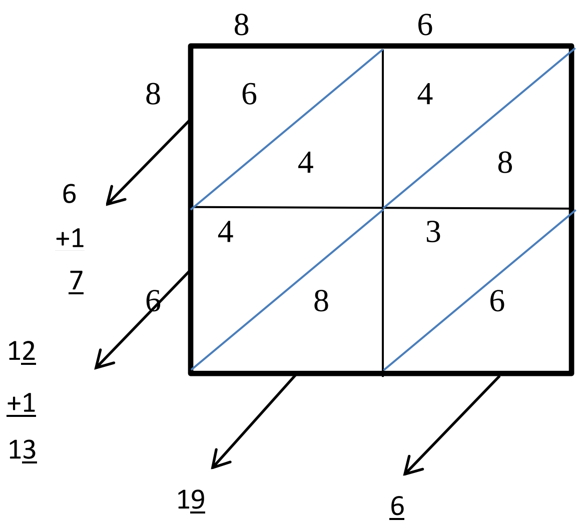
Solution



Thus, $(67)^2 = 4489$

(f) $(\mathbf{86})^2$

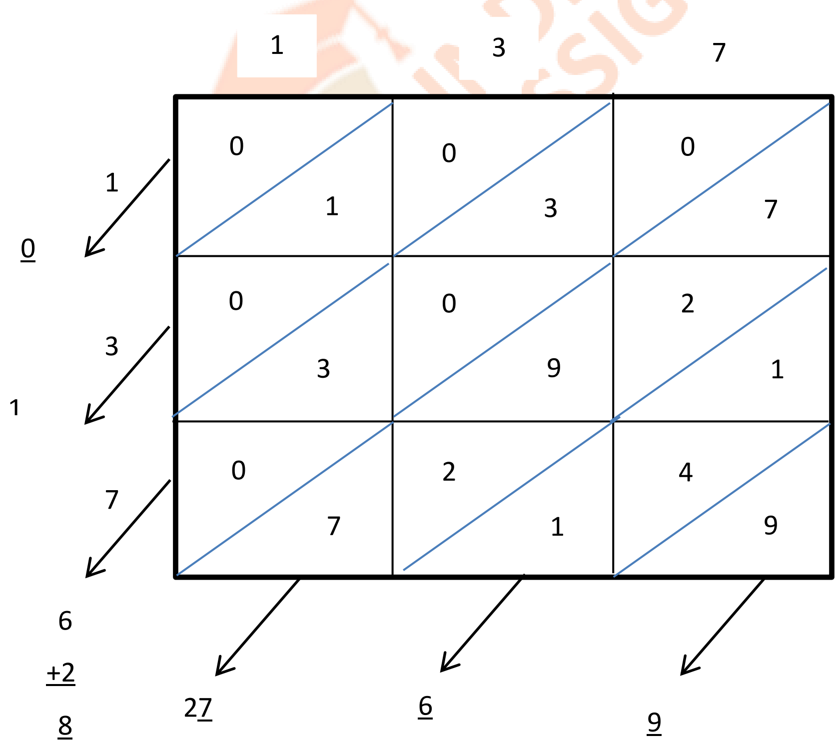
Solution



Thus, $(86)^2 = 7396$

(g) $(\mathbf{137})^2$

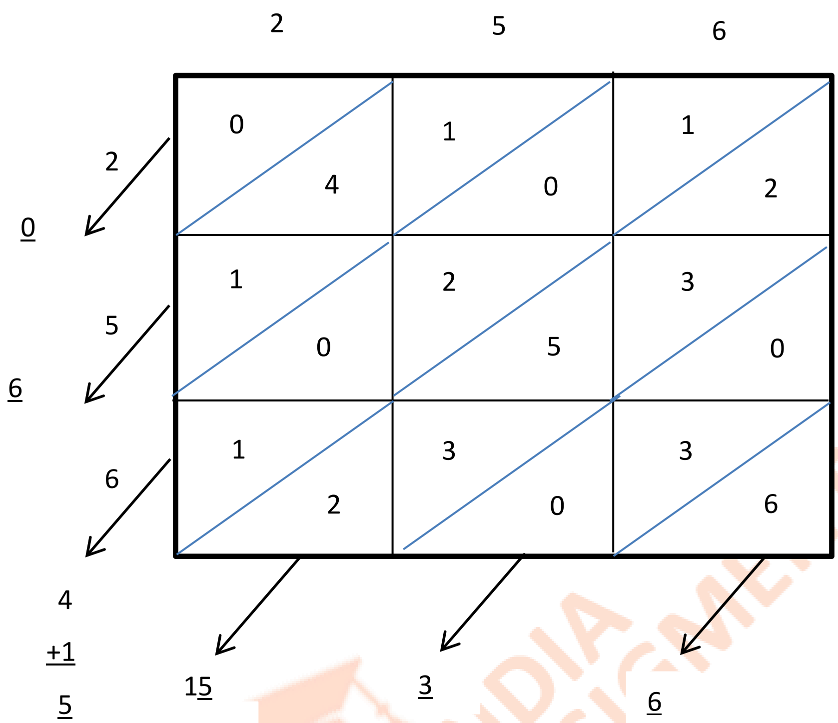
Solution



Thus, $(137)^2 = 18769$

(h) $(\mathbf{256})^{\mathbf{2}}$

Solution



Thus, $(256)^2 = 65536$

Square roots

The square root of a number x is that number which when multiplied by itself gives x as the product. We denote square root of x by $\sqrt{x}$

<u>Prime factorization method to find square root of a number</u>:

Step1 Resolve the given number into prime factors.

Step2 Make pairs of similar factors

Step3 Take the product of the prime factors, choosing one factor out of every pair.

Examples:

Example 1 - Find the square root of 324.

Solution - First we resolve 324 into prime factors

2	324
2	162
3	81
3	27
3	9
3	3
	1

Here we can see that 324 = $2 \times 2 \times 3 \times 3 \times 3 \times 3$

Thus, $\sqrt{324} = \sqrt{\underline{2 \times 2} \times \underline{3 \times 3} \times \underline{3 \times 3}}$

We have 3 pairs, so we take out one factor out of each pair

Therefore, $\sqrt{324} = 2 \times 3 \times 3 = 18$

Example 2 - Find the square root of 1764

Solution - First we resolve 1764 into prime factors

2	1764
2	882
3	441
3	147
7	49
7	7
	1

Here we can see that 1764 = $2 \times 2 \times 3 \times 3 \times 7 \times 7$

Thus, $\sqrt{1764} = \sqrt{\underline{2 \times 2} \times \underline{3 \times 3} \times \underline{7 \times 7}}$

We have 3 pairs, so we take out one factor out of each pair

Therefore, $\sqrt{1764} = 2 \times 3 \times 7 = 42$

Example 3 - Evaluate $\sqrt{4356}$

Solution - First we resolve 4356 into prime factors

2	4356
2	2178
3	1089
3	363
11	121
11	11
	1

Here we can see that 4356 = 2 × 2 × 3 × 3 × 11 × 11

Thus, $\sqrt{4356} = \sqrt{\underline{2 \times 2} \times \underline{3 \times 3} \times \underline{11 \times 11}}$

We have 3 pairs, so we take out one factor out of each pair

Therefore, $\sqrt{4356} = 2 \times 3 \times 11$ = 66

Example 4 - Evaluate $\sqrt{11025}$

Solution - First we resolve 11025 into prime factors

5	11025
5	2205
3	441
3	147
7	49
7	7
	1

Here we can see that 11025 = 5 × 5 × 3 × 3 × 7 × 7

Thus, $\sqrt{11025} = \sqrt{\underline{5 \times 5} \times \underline{3 \times 3} \times \underline{7 \times 7}}$

We have 3 pairs, so we take out one factor out of each pair

Therefore, $\sqrt{11025} = 5 \times 3 \times 7$ = 105

Example 5 - In an auditorium, the number of rows is equal to the number of chairs in each row. If the capacity of the auditorium is 2025, find the number of chairs in each row.

Solution - It is given that:

Capacity of auditorium = 2025

Number of rows = Number of chairs in each row

So let us assume the number of chairs in each row be x

Then number of rows = x

Thus total number of chairs in the auditorium = x^2

Since Capacity = 2025

=> x^2 = 2025

=> x = $\sqrt{2025}$

First we resolve 2025 into prime factors

5	2025
5	405
3	81
3	27
3	9
3	3
	1

Here we can see that 2025 = 5 × 5 × 3 × 3 × 3 × 3

Thus, $\sqrt{2025} = \sqrt{\underline{5 \times 5} \times \underline{3 \times 3} \times \underline{3 \times 3}}$

We have 3 pairs, so we take out one factor out of each pair

Therefore, $\sqrt{2025} = 5 \times 3 \times 3$ = 45

=> x = 45

Hence, number of chairs in each row = 45

Example 6 - Find the smallest number by which 396 must be multiplied so that the product becomes a perfect square.

Solution - First we resolve 396 into prime factors

2	396
2	198
3	99
3	33
11	11
	1

Here we can see that 396 = $2 \times 2 \times 3 \times 3 \times 11$

Thus, $\sqrt{396} = \sqrt{\underline{2 \times 2} \times \underline{3 \times 3} \times 11}$

It is clear from the prime factorization that one more 11 is required for getting a perfect square.

Thus, 11 must be multiplied to the given number to get a perfect square.

Example 7 - Find the least square number divisible by each one of 8, 9 and 10

Solution - Since least number divisible by each one of 8, 9 and 10 is their LCM.

LCM of 8, 9 and 10 = $2 \times 2 \times 2 \times 3 \times 3 \times 5 = 360$

2	8 , 9 , 10
2	4 , 9 , 5
2	2 , 9 , 5
3	1 , 9 , 5
3	1 , 3 , 5
5	1 , 1 , 5
	1 , 1 , 1

Now, by prime factorization

360 = $\underline{2 \times 2} \times 2 \times \underline{3 \times 3} \times 5$

To make it a perfect square, we must multiply it by 10

=> Required number = $360 \times 10 = 3600$

2	360
2	180
2	90
3	45
3	15
5	5
	1

Exercise 3D

Find the square root of each of the following numbers by using the method of prime factorization:

Question 1- 225

Solution - First we resolve 225 into prime factors

3	225
3	75
5	25
5	5
	1

Here, we can see that 225 = 3 × 3 × 5 × 5

Thus, $\sqrt{225} = \sqrt{\underline{3 \times 3} \times \underline{5 \times 5}}$

We have 2 pairs, so we take out one factor out of each pair

Therefore, $\sqrt{225} = 3 \times 5 = 15$

Question 2 - 441

Solution - First we resolve 441 into prime factors

3	441
3	147
7	49
7	7
	1

Here, we can see that 441 = 3 × 3 × 7 × 7

Thus, $\sqrt{441} = \sqrt{\underline{3 \times 3} \times \underline{7 \times 7}}$

We have 2 pairs, so we take out one factor out of each pair

Therefore, $\sqrt{441} = 3 \times 7 = 21$

Question 3 - 729

Solution - First we resolve 729 into prime factors

3	729
3	243
3	81
3	27
3	9
3	3
	1

Here, we can see that 729 = 3 × 3 × 3 × 3 × 3 × 3

Thus, $\sqrt{729} = \sqrt{\underline{3 \times 3} \times \underline{3 \times 3} \times \underline{3 \times 3}}$

We have 3 pairs, so we take out one factor out of each pair

Therefore, $\sqrt{729} = 3 \times 3 \times 3 = 27$

Question 4 - 1296

Solution - First we resolve 1296 into prime factors

2	1296
2	648
2	324
2	162
3	81
3	27
3	9
3	3
	1

Here, we can see that 1296 = 2 × 2 × 2 × 2 × 3 × 3 × 3 × 3

Thus, $\sqrt{1296} = \sqrt{\underline{2 \times 2} \times \underline{2 \times 2} \times \underline{3 \times 3} \times \underline{3 \times 3}}$

We have 4 pairs, so we take out one factor out of each pair

Therefore, $\sqrt{1296} = 2 \times 2 \times 3 \times 3 = 36$

Question 5 - 2025

Solution - First we resolve 2025 into prime factors

5	2025
5	405
3	81
3	27
3	9
3	3
	1

Here, we can see that 2025 = 5 × 5 × 3 × 3 × 3 × 3

Thus, $\sqrt{2025} = \sqrt{\underline{5 \times 5} \times \underline{3 \times 3} \times \underline{3 \times 3}}$

We have 3 pairs, so we take out one factor out of each pair

Therefore, $\sqrt{2025} = 5 \times 3 \times 3 = 45$

Question 6 - 4096

Solution - First we resolve 4096 into prime factors

2	4096
2	2048
2	1024
2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

Here, we can see that 4096 = 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2

Thus, $\sqrt{4096} = \sqrt{\underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2}}$

We have 6 pairs, so we take out one factor out of each pair

Therefore, $\sqrt{4096} = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$

Question 7 - 7056

Solution - First we resolve 7056 into prime factors

2	7056
2	3528
2	1764
2	882
3	441
3	147
7	49
7	7
	1

Here, we can see that $7056 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 7 \times 7$

Thus, $\sqrt{7056} = \sqrt{\underline{2 \times 2} \times \underline{2 \times 2} \times \underline{3 \times 3} \times \underline{7 \times 7}}$

We have 4 pairs, so we take out one factor out of each pair

Therefore, $\sqrt{7056} = 2 \times 2 \times 3 \times 7 = 84$

Question 8 - 8100

Solution - First we resolve 8100 into prime factors

2	8100
2	4050
5	2025
5	405
3	81
3	27
3	9
3	3
	1

Here, we can see that $8100 = 2 \times 2 \times 5 \times 5 \times 3 \times 3 \times 3 \times 3$

Thus, $\sqrt{8100} = \sqrt{\underline{2 \times 2} \times \underline{5 \times 5} \times \underline{3 \times 3} \times \underline{3 \times 3}}$

We have 4 pairs, so we take out one factor out of each pair

Therefore, $\sqrt{8100} = 2 \times 5 \times 3 \times 3 = 90$

Question 9 - 9216

Solution - First we resolve 9216 into prime factors

2	9216
2	4608
2	2304
2	1152
2	576
2	288
2	144
2	72
2	36
2	18
3	9
3	3
	1

Here, we can see that $9216 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3$

Thus, $\sqrt{9216} = \sqrt{\underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{3 \times 3}}$

We have 6 pairs, so we take out one factor out of each pair

Therefore, $\sqrt{9216} = 2 \times 2 \times 2 \times 2 \times 2 \times 3 = 96$

Question 10 - 11025

Solution - First we resolve 11025 into prime factors

5	11025
5	2205
3	441
3	147
7	49
7	7
	1

Here, we can see that $11025 = 5 \times 5 \times 3 \times 3 \times 7 \times 7$

Thus, $\sqrt{11025} = \sqrt{5 \times 5 \times 3 \times 3 \times 7 \times 7}$

We have 3 pairs, so we take out one factor out of each pair

Therefore, $\sqrt{11025} = 5 \times 3 \times 7 = 105$

Question 11 - 15876

Solution - First we resolve 15876 into prime factors

2	15876
2	7938
3	3969
3	1323
3	441
3	147
7	49
7	7
	1

Here, we can see that $15876 = 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 7 \times 7$

Thus, $\sqrt{15876} = \sqrt{2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 7 \times 7}$

We have 4 pairs, so we take out one factor out of each pair

Therefore, $\sqrt{15876} = 2 \times 3 \times 3 \times 7 = 126$

Question 12 - 17424

Solution - First we resolve 17424 into prime factors

2	17424
2	8712
2	4356
2	2178
3	1089
3	363
11	121
11	11
	1

Here, we can see that $17424 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 11 \times 11$

Thus, $\sqrt{17424} = \sqrt{\underline{2 \times 2} \times \underline{2 \times 2} \times \underline{3 \times 3} \times \underline{11 \times 11}}$

We have 4 pairs, so we take out one factor out of each pair

Therefore, $\sqrt{17424} = 2 \times 2 \times 3 \times 11 = 132$

Question 13 - Find the smallest number by which 252 must be multiplied to get a perfect square. Also, find the square root of the perfect square so obtained.

Solution - First we resolve 252 into prime factors

2	252
2	126
3	63
3	21
7	7
	1

Here, we can see that $252 = 2 \times 2 \times 3 \times 3 \times 7$

Thus, $\sqrt{252} = \sqrt{\underline{2 \times 2} \times \underline{3 \times 3} \times 7}$

It is clear from the prime factorization that one more 7 is required for getting a perfect square.

Thus, 7 must be multiplied to the given number to get a perfect square.

Thus, perfect square number $= 252 \times 7 = 1764$

And $\sqrt{1764} = \sqrt{\underline{2 \times 2} \times \underline{3 \times 3} \times \underline{7 \times 7}}$

We have 3 pairs, so we take out one factor out of each pair

Therefore, $\sqrt{1764} = 2 \times 3 \times 7 = 42$.

Question 14 - Find the smallest number by which 2925 must be divided to obtain a perfect square. Also, find the square root of the perfect square so obtained.

Solution - First we resolve 2925 into prime factors

5	2925
5	585
3	117
3	39
13	13
	1

Here, we can see that 2925 = $5 \times 5 \times 3 \times 3 \times 13$

Thus, $\sqrt{2925} = \sqrt{\underline{5 \times 5} \times \underline{3 \times 3} \times 13}$

It is clear from the prime factorization that one more 13 is required for getting a perfect square.

Thus, 7 must be divided by given number to get a perfect square.

Thus, perfect square number = $\frac{2925}{13} = 225$

And $\sqrt{225} = \sqrt{\underline{5 \times 5} \times \underline{3 \times 3}}$

We have 2 pairs, so we take out one factor out of each pair

Therefore, $\sqrt{225} = 5 \times 3$ = 15.

Question 15 - 1225 plants are to be planted in a garden in such a way that each row contains as many plants as the number of rows. Find the number of rows and the number of plants in each row.

Solution **-** It is given that:

Total plants to be planted in garden = 1225

Number of rows = Number of plants in each row

So let us assume the number of plants in each row be x

Then number of rows = x

Thus total number of plants in the garden = x^2

Since total plants = 1225

=> x^2 = 1225

=> x = $\sqrt{1225}$

First we resolve 1225 into prime factors

5	1225
5	245
7	49
7	7
	1

Here, we can see that 1225 = $5 \times 5 \times 7 \times 7$

Thus, $\sqrt{1225} = \sqrt{5 \times 5 \times 7 \times 7}$

We have 2 pairs, so we take out one factor out of each pair

Therefore, $\sqrt{1225} = 5 \times 7$ = 35

=> x = 35

Hence, number of plants in each row = 35

And number of rows = 35

Question 16 - The students of a class arranged a picnic. Each student contributed as many rupees as the number of students in the class. If the total contribution is Rs1156, find the strength of the class.

Solution **-** It is given that:

Total contribution = Rs1156

Number of students = contribution of each student

Let total students be x

Then contribution of each student = x

Since total contribution = 1156

=> x^2 = 1156

=> x = $\sqrt{1156}$

First we resolve 1156 into prime factors

2	1156
2	578
17	289
17	17
	1

Here, we can see that 1176 = 2 × 2 × 17 × 17

Thus, $\sqrt{1176} = \sqrt{\underline{2 \times 2} \times \underline{17 \times 17}}$

We have 2 pairs, so we take out one factor out of each pair

Therefore, $\sqrt{1176} = 2 \times 17$ = 34

=> x = 34

Hence, number of students = 34

Question 17 - Find the least square number which is exactly divisible by each of the numbers 6, 9, 15 and 20.

Solution **-** Since least number divisible by each one of 6, 9, 15, and 20 is their LCM.

LCM of 6, 9, 15 and 20 = 2 × 2 × 3 × 3 × 5 = 180

2	6,9,15,20
2	3,9,15,10
3	3,9,15,5
3	1,3,5,5
5	1,1,5,5
	1,1,1,1

Now by prime factorization

180 = $\underline{2 \times 2} \times \underline{3 \times 3} \times 5$

To make it a perfect square, we must multiply it by 5

=> Required number = 180 × 5 = 900

Question 18 - Find the least square number which is exactly divisible by each of the numbers 8, 12, 15, and 20

Solution - Since least number divisible by each one of 8, 12, 15, and 20 is their LCM.

LCM of 8, 12, 15 and 20 = $2 \times 2 \times 2 \times 3 \times 5 = 120$

2	8,12,15,20
2	4,6,15,10
2	2,3,15,5
3	1,3,15,5
5	1,1,5,5
	1,1,1,1

Now by prime factorization

120 = $2 \times 2 \times 2 \times 3 \times 5$

To make it a perfect square, we must multiply it by 30

=> Required number = $120 \times 30 = 3600$

Square root of a perfect square by the long division method

Step1 Group the digits in pairs, starting with the digit in the units place. Each pair and the remaining digit is called a period.

Step2 think of the largest number whose square is equal to or just less than the first period. Take this number as the divisor and also as a quotient.

Step3 subtract the product of the divisor and the quotient from the first period and bring down to the next period to the right of the remainder. This becomes the new dividend.

Step4 Now, the new divisor is obtained by taking two times the quotient and annexing with it a suitable digit which is also taken as the next digit of the quotient, chosen in such a way that the product of the new divisor and this digit is equal to or just less than the new dividend

Step5 Repeat steps (2), (3) and 4 till all the periods have been taken up.

Now, the quotient so obtained is the required square root of the given number.

Examples:

Example 1 - Find the square root of 784 by the long-division method.

Solution

	28
2	$\overline{7}\ \overline{84}$
	-4
48	384
	-384
	0

Thus, $\sqrt{784} = 28$

Example 2 - Evaluate $\sqrt{5329}$ using long-division method.

Solution

	73
7	$\overline{53}\ \overline{29}$
	-49
14	429
	-429
	0

Thus, $\sqrt{5329} = 73$

Example 3 - Evaluate $\sqrt{16384}$

Solution

	128
1	$\overline{1}\ \overline{63}\ \overline{84}$
	-1
22	63
	-44
248	1984
	1984
	0

Thus, $\sqrt{16384} = 128$

Example 4 - Evaluate $\sqrt{10609}$

Solution

	103
1	$\overline{1}\ \overline{06}\ \overline{09}$
	-1
203	609
	-609
	0

Thus, $\sqrt{10609} = 103$

Example 5 - Evaluate $\sqrt{66049}$

Solution

	257
2	$\overline{6}\ \overline{60}\ \overline{49}$
	-4
45	260
	-225
507	3549
	-3549
	0

Thus, $\sqrt{66049} = 257$

Example 6 - Find the cost of erecting a fence around a square field whose area is 9 hectares if fencing costs Rs35 per meter

Solution - Area of square field = 9 hectares = $9 \times 10000 = 90000m^2$

We know that area of square = $(side)^2$

=> Side of square = $\sqrt{90000} = 300\ m$

Perimeter of the field = $4 \times side = 4 \times 300 = 1200\ m$

Cost of 1m = Rs35

Cost of 1200m = 35×1200 = Rs42000

Example 7 - What least number must be subtracted from 7250 to get a perfect square? Also, find the square root of this perfect square.

Solution - We will first find the square root of 7250 by long division method,

	85
8	$\overline{72}\ \overline{50}$
	-64
165	850
	-825
	25

Since from above we can see that $(85)^2$ < 7250 by 25

So, 25 is the least number which is to be subtracted from 7250 to get perfect square .

Required perfect square number = 7250-25 = 7225

And $\sqrt{7225} = 85$

Example 8 - Find the greatest number of four digits which is a perfect square.

Solution -We know that greatest 4 digit number = 9999

We will first find the square root of 9999 by long division method,

	99
9	$\overline{99}\ \overline{99}$
	-81
189	1899
	-1701
	198

Since from above we can see that $(99)^2$ < 9999 by 198

So, 198 is the least number which is to be subtracted from 9999 to get perfect square.

Required perfect square number = 9999-198 = 9801

And $\sqrt{9801} = 99$

Example 9 - What least number must be added to 5607 to make the sum a perfect square? Find this perfect square and its square root.

Solution - We will first find the square root of 5607 by long division method,

	74
7	$\overline{56}\ \overline{07}$
	-49
144	707
	-576
	131

Since from above we can see that $(74)^2 < 5607 < (75)^2$

Thus, number to be added is $(75)^2 - 5607 = (5625 - 5607) = 18$

So, 18 is the least number which is to be added to 5607 to get perfect square.

Required perfect square number = 5625 and $\sqrt{5625} = 75$

Example 10 - Find the least number of six digits which is a perfect square. Find the square root of this number.

Solution **-** We know that least 6 digit number = 100000

We will first find square root of 100000 by long division method

```
              316
      ___________________
  3  |  10 00 00
     |  -9
     |___________________
 61  |   100
     |   -61
     |___________________
626  |   3900
     |   -3756
     |___________________
     |    144
```

Since from above we can see that $(316)^2 < 100000 < (317)^2$

Thus, number to be added is $(317)^2 - 100000 = (100489 - 100000) = 489$

So, 489 is the least number which is to be added to 100000 to get perfect square.

Required perfect square number = 100489 and $\sqrt{100489} = 317$

Exercise 3E

Evaluate using long division method:

Question 1 - $\sqrt{576}$

Solution

	24
2	$\overline{5}$ $\overline{76}$
	-4
44	176
	-176
	0

Thus, $\sqrt{576} = 24$

Question 2 - $\sqrt{1444}$

Solution

	38
3	$\overline{14}$ $\overline{44}$
	-9
68	544
	-544
	0

Thus, $\sqrt{1444} = 38$

Question 3 - $\sqrt{4489}$

Solution

	67
6	$\overline{44}$ $\overline{89}$
	-36
127	889
	-889
	0

Thus, $\sqrt{4489} = 67$

Question 4 - $\sqrt{\mathbf{6241}}$

Solution

	79
7	$\overline{62}\ \overline{41}$
	-49
149	1341
	-1341
	0

Thus, $\sqrt{6241} = 79$

Question 5 - $\sqrt{\mathbf{7056}}$

Solution

	84
8	$\overline{70}\ \overline{56}$
	-64
16	656
	-656
	0

Thus, $\sqrt{7056} = 84$

Question 6 - $\sqrt{\mathbf{9025}}$

Solution

	95
9	$\overline{90}\ \overline{25}$
	-81
185	925
	-925
	0

Thus, $\sqrt{9025} = 95$

Question 7 - $\sqrt{11449}$

Solution

	107
1	$\overline{1}\ \overline{14}\ \overline{49}$
	-1
207	1449
	-1449
	0

Thus, $\sqrt{11449} = 107$

Question 8 - $\sqrt{14161}$

Solution

	119
1	$\overline{1}\ \overline{41}\ \overline{61}$
	-1
21	41
	-21
229	2061
	-2061
	0

Thus, $\sqrt{14161} = 119$

Question 9 - $\sqrt{10404}$

Solution

	102
1	$\overline{1}\ \overline{04}\ \overline{04}$
	-1
202	404
	-404
	0

Thus, $\sqrt{10404} = 102$

Question 10 - $\sqrt{17956}$

Solution

134

1 | $\overline{1}\,\overline{79}\,\overline{56}$

-1

23 | 79

-69

264 | 1056

-1056

0

Thus, $\sqrt{17956} = 134$

Question 11 - $\sqrt{19600}$

Solution

140

1 | $\overline{1}\,\overline{96}\,\overline{00}$

-1

240 | 9600

-9600

0

Thus, $\sqrt{19600} = 140$

Question 12 - $\sqrt{92416}$

Solution

304

3 | $\overline{9}\,\overline{24}\,\overline{16}$

-9

604 | 2416

-2416

0

Thus, $\sqrt{92416} = 304$

Question 13 - Find the least number which must be subtracted from 2509 to make it a perfect square.

Solution - We will first find the square root of 2509 by long division method,

	50
5	$\overline{25}\ \overline{09}$
	-25
100	09

Since from above we can see that $(50)^2$ < 2509 by 9

So, 9 is the least number which is to be subtracted from 2509 to get perfect square .

Required perfect square number = 2509-9 = 2500

And $\sqrt{2500} = 50$

Question 14 - Find the least number which must be subtracted from 7581 to obtain a perfect square.

Find this perfect square and its square root.

Solution - We will first find the square root of 7581 by long division method,

	87
8	$\overline{75}\ \overline{81}$
	-64
167	1181
	-1169
	12

Since from above we can see that $(87)^2$ < 7581 by 12

So, 12 is the least number which is to be subtracted from 7581 to get perfect square.

Required perfect square number = 7581-12 = 7569

And $\sqrt{7569} = 87$

Question 15 - Find the least number which must be added to 6203 to obtain a perfect square.

Find this perfect square and its square root.

Solution - We will first find the square root of 6203 by long division method,

```
          78
       ---------
  7   | 62  03
      | -49
      |---------
148   | 1303
      | -1184
      |---------
      |  119
```

Since from above we can see that $(78)^2 < 6203 < (79)^2$

Thus, number to be added is $(79)^2 - 6203 = (6241 - 6203) = 38$

So, 38 is the least number which is to be added to 6203 to get perfect square.

Required perfect square number = 6241 and $\sqrt{6241} = 79$

Question 16 - Find the least number which must be added to 8400 to obtain a perfect square. Find this perfect square and its square root

Solution - We will first find the square root of 8400 by long division method,

```
          91
       ---------
  9   | 84  00
      | -81
      |---------
181   | 300
      | -181
      |---------
      |  119
```

Since from above we can see that $(91)^2 < 8400 < (92)^2$

Thus, number to be added is $(92)^2 - 8400 = (8464 - 8400) = 64$

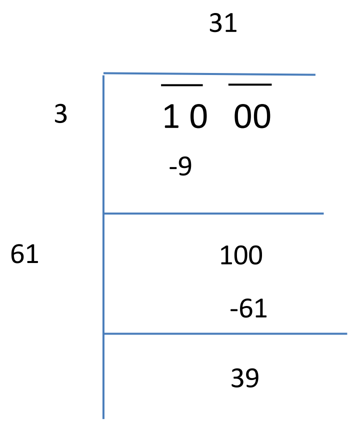
So, 64 is the least number which is to be added to 8400 to get perfect square.

Required perfect square number = 8464 and $\sqrt{8464} = 92$

Question 17 - Find the least number of four digits which is a perfect square. Also find the square root of the number so obtained.

Solution **-** We know that least 4 digit number = 1000

We will first find square root of 1000 by long division method



Since from above we can see that $(31)^2 < 1000 < (32)^2$

Thus, number to be added is $(32)^2 - 1000 = (1024 - 1000) = 24$

So, 24 is the least number which is to be added to 1000 to get perfect square.

Required perfect square number = 1024 and $\sqrt{1024} = 32$

Question 18 - Find the greatest number of five digits which is a perfect square. Also find the square root of the number so obtained.

Solution **-** We know that greatest 5 digit number = 999

We will first find the square root of 99999 by long division method, 316

3 | $\overline{9}\ \overline{99}\ \overline{99}$

-9

61 | 99

-61

626 | 3899

-3756

143

Since from above we can see that $(316)^2$ < 99999 by 143

So, 143 is the least number which is to be subtracted from 99999 to get perfect square.

Required perfect square number = 99999-143 = 99856

And $\sqrt{99856} = 316$

Question 19 - The area of a square field is 60025m^2. A man cycles along its boundary at 18 km/h. In how much time will he return to the starting point?

Solution **-** It is given that:

Area of square field = 60025m^2

=> $(side)^2$ = 60025 => Side = $\sqrt{60025}$

245

2 | $\overline{6}\ \overline{00}\ \overline{25}$

-4

200

44 | -176

485 | 2425

-2425

0

Thus, side = $\sqrt{60025} = 245$ m

Perimeter of square field = $4 \times side$ = $4 \times 245 = 980m$

Speed of man along its boundary = 18km/h

$$= \frac{18 \times 1000}{60 \times 60} = 5 \text{ m/sec}$$

Time taken to return back to the starting point = $\frac{distance}{speed} = \frac{980}{5} = 196$ sec

Square roots of numbers in decimal form

Let us understand by examples

Examples:

Example 1 - Evaluate $\sqrt{42.25}$

Solution

	6.5
6	$\overline{42}\ .\ \overline{25}$
	-36
125	625
	-625
	0

Thus, $\sqrt{42.25} = 6.5$

Example 2 - Evaluate $\sqrt{1.96}$

Solution

	1.4
1	$\overline{1}\ .\ \overline{96}$
	-1
24	96
	-96
	0

Thus, $\sqrt{1.96} = 1.4$

Example 3 - Evaluate $\sqrt{6.4009}$

Solution

	2.53
2	$\overline{6}.\ \overline{40}\ \overline{09}$
	-4
45	240
	-225
503	1509
	-1509
	0

Thus, $\sqrt{6.4009} = 2.53$

Example 4 - Evaluate $\sqrt{0.4225}$

Solution

	0.65
6	$\overline{0}.\ \overline{42}\ \overline{25}$
	-36
125	625
	-625
	0

Thus, $\sqrt{0.4225} = 0.65$

Example 5 - Evaluate $\sqrt{2}$ correct up to two places of decimal.

Solution - Using long division method, we will find $\sqrt{2}$

For finding value correct up to two places we will find up to three decimal place and then approximate it to 2 places

	1.414
1	$\overline{2}.\overline{00}\ \overline{00}\ \overline{00}$
	-1
24	100
	-96
281	400
	-281
2824	11900
	-11296
	604

Thus, $\sqrt{2} = 1.414 = 1.41$(upto 2 decimal place)

Example 6 - Evaluate $\sqrt{0.8}$ correct up to two places of decimal.

Solution - Using long division method, we will find $\sqrt{0.8}$

For finding value correct up to two places we will find up to three decimal place and then approximate it to 2 places

	0.894
8	$\overline{0}.\overline{80}\ \overline{00}\ \overline{00}$
	-64
169	1600
	-1521
1784	7900
	-7136
	764

Thus, $\sqrt{0.8} = 0.894 = 0.89$(upto 2 decimal place)

Exercise 3F

Evaluate

Question 1 - $\sqrt{\mathbf{1.69}}$

Solution

	1.3
1	$\overline{1}\ .\ \overline{69}$
	-1
23	69
	-69
	0

Thus, $\sqrt{1.69} = 1.3$

Question 2 - $\sqrt{33.64}$

Solution

```
            5.8
      ___________
 5   | 33 . 64
     | -25
     |__________
108  |  864
     | -864
     |__________
     |    0
```

Thus, $\sqrt{33.64} = 5.8$

Question 3 - $\sqrt{156.25}$

Solution

```
            12.5
      ___________
 1   | 1 56 . 25
     | -1
     |__________
22   |  56
     | -44
     |__________
245  |  1225
     | -1225
     |__________
     |    0
```

Thus, $\sqrt{156.25} = 12.5$

Question 4 - $\sqrt{75.69}$

Solution

	8.7
8	$\overline{75}\ .\ \overline{69}$
	-64
167	1169
	-1169
	0

Thus, $\sqrt{75.69} = 8.7$

Question 5 - $\sqrt{9.8596}$

Solution

	3.14
3	$\overline{9}\ .\ \overline{85}\ \overline{96}$
	-9
61	85
	-61
624	2496
	-2496
	0

Thus, $\sqrt{9.8596} = 3.14$

Question 6 - $\sqrt{\mathbf{10.0489}}$

Solution

	3.17
3	$\overline{10}\,.\,\overline{04}\ \overline{89}$
	-9
61	104
	-61
627	4389
	-4389
	0

Thus, $\sqrt{10.0489} = 3.17$

Question 7 - $\sqrt{\mathbf{1.0816}}$

Solution

	1.04
1	$\overline{1}.\,\overline{08}\ \overline{16}$
	-1
204	816
	-816
	0

Thus, $\sqrt{1.0816} = 1.04$

Question 8 - $\sqrt{\mathbf{0.2916}}$

Solution

	0.54
5	$\overline{0}.\ \overline{29}\ \overline{16}$
	-25
104	416
	-416
	0

Thus, $\sqrt{0.2916} = 0.54$

Question 9 - Evaluate $\sqrt{3}$ **up to two places of decimal.**

Solution **-** Using long division method, we will find $\sqrt{3}$

For finding value correct up to two places we will find up to three decimal place and then approximate it to 2 places

	1.732
1	$\overline{3}.\ \overline{00}\ \overline{00}\ \overline{00}$
	-1
27	200
	-189
343	1100
	-1029
3462	7100
	-6924
	176

Thus, $\sqrt{3} = 1.732 = 1.73$(upto 2 decimal place)

Question 10 - Evaluate $\sqrt{2.8}$ correct up to two places of decimal.

Solution - Using long division method, we will find $\sqrt{2.8}$

For finding value correct up to two places we will find up to three decimal place and then approximate it to 2 places

	1.673
1	$\overline{2}.\ \overline{80}\ \overline{00}\ \overline{00}$
	-1
26	180
	-156
327	2400
	-2289
3343	11100
	-10029
	1071

Thus, $\sqrt{2.8} = 1.673 = 1.67$(upto 2 decimal place)

Question 11 - Evaluate $\sqrt{0.9}$ correct up to two places of decimal.

Solution - Using long division method, we will find $\sqrt{0.9}$

For finding value correct up to two places we will find up to three decimal place and then approximate it to 2 places

	0.948
9	$\overline{0}.\,\overline{90}\ \overline{00}\ \overline{00}$
	-81
184	900
	-736
1888	16400
	-15104
	1296

Thus, $\sqrt{0.9} = 0.948 = 0.95$(upto 2 decimal place)

Square roots of fractions

For any positive numbers a and b,

(I) $\sqrt{ab} = (\sqrt{a} \times \sqrt{b})$

(II) $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

Examples:

Example 1 - Evaluate $\sqrt{\frac{\mathbf{441}}{\mathbf{961}}}$

Solution**:** $\sqrt{\frac{\mathbf{441}}{\mathbf{961}}} = \frac{\sqrt{441}}{\sqrt{961}}$

We solve each $\sqrt{441}$ and $\sqrt{961}$ by prime factorization method

First we resolve 441 into prime factors

3	441
3	147
7	49
7	7
	1

Here we can see that 441 = 3 × 3 × 7 × 7

Thus, $\sqrt{441} = \sqrt{\underline{3 \times 3} \times \underline{7 \times 7}}$

We have 2 pairs, so we take out one factor out of each pair

Therefore, $\sqrt{441} = 3 \times 7 = 21$

Secondly we resolve 961 into prime factors

31	961
31	31
	1

Here, we can see that 961 = 31 × 31

Thus, $\sqrt{961} = \sqrt{\underline{31 \times 31}}$

We have 1 pairs, so we take out one factor out of each pair

Therefore, $\sqrt{961} = 31$

Thus, $\sqrt{\frac{\mathbf{441}}{\mathbf{961}}} = \frac{\sqrt{441}}{\sqrt{961}} = \frac{21}{31}$

Example 2 - Find the square root of $1\frac{\mathbf{56}}{\mathbf{169}}$

Solution - Since $1\frac{56}{169} = \frac{225}{169}$

We have to find $\sqrt{\frac{225}{169}} = \frac{\sqrt{225}}{\sqrt{169}}$

We solve each $\sqrt{225}$ and $\sqrt{169}$ by prime factorization method

First we resolve 225 into prime factors

3	225
3	75
5	25
5	5
	1

Here we can see that 225 = 3 × 3 × 5 × 5

Thus, $\sqrt{225} = \sqrt{\underline{3 \times 3} \times \underline{5 \times 5}}$

We have 2 pairs, so we take out one factor out of each pair

Therefore $\sqrt{225} = 3 \times 5 = 15$

Secondly we resolve 169 into prime factors

13	169
13	13
	1

Here, we can see that 169 = 13 × 13

Thus, $\sqrt{169} = \sqrt{\underline{13 \times 13}}$

We have 1 pairs, so we take out one factor out of each pair

Therefore, $\sqrt{169} = 13$

Thus, $\sqrt{\frac{\mathbf{225}}{\mathbf{169}}} = \frac{\sqrt{225}}{\sqrt{169}} = \frac{15}{13} = 1\frac{2}{13}$

Example 3 - Find the value of $\frac{\sqrt{\mathbf{243}}}{\sqrt{\mathbf{363}}}$

Solution - We solve each $\sqrt{243}$ and $\sqrt{363}$ by prime factorization method

First we resolve 243 into prime factors

3	243
3	81
3	27
3	9
3	3
	1

Here, we can see that 243 = $3 \times 3 \times 3 \times 3 \times 3$

Thus, $\sqrt{243} = \sqrt{\underline{3 \times 3} \times \underline{3 \times 3} \times 3}$

We have 2 pairs, so we take out one factor out of each pair

Therefore, $\sqrt{243} = 3 \times 3 \times \sqrt{3} = 9\sqrt{3}$

Secondly we resolve 363 into prime factors

3	363
11	121
11	11
	1

Here, we can see that 363 = $3 \times 11 \times 11$

Thus, $\sqrt{363} = \sqrt{3 \times \underline{11 \times 11}}$

We have 1 pairs, so we take out one factor out of each pair

Therefore, $\sqrt{363} = 11\sqrt{3}$

Thus, $\frac{\sqrt{243}}{\sqrt{363}} = \frac{9\sqrt{3}}{11\sqrt{3}} = \frac{9}{11}$

Example 4 - Find the value of $\sqrt{45} \times \sqrt{20}$

Solution - Since $\sqrt{45} \times \sqrt{20} = \sqrt{45 \times 20} = \sqrt{900}$

First we resolve 900 into prime factors

3	900
3	300
2	100
2	50
5	25
5	5
	1

Here we can see that 900 = 3 × 3 × 2 × 2 × 5 × 5

Thus, $\sqrt{900} = \sqrt{\underline{3 \times 3} \times \underline{2 \times 2} \times \underline{5 \times 5}}$

We have 3 pairs, so we take out one factor out of each pair

Therefore, $\sqrt{900} = 3 \times 2 \times 5$ = 30

Exercise 3G

Evaluate:

Question 1 $\sqrt{\frac{16}{81}}$

Solution - Since $\sqrt{\frac{16}{81}} = \frac{\sqrt{16}}{\sqrt{81}}$

We solve each 16 and $\sqrt{81}$ by prime factorization method

First we resolve 16 into prime factors

2	16
2	8
2	4
2	2
	1

Here we can see that 16 = 2 × 2 × 2 × 2

Thus, $\sqrt{16} = \sqrt{\underline{2 \times 2} \times \underline{2 \times 2}}$

We have 2 pairs, so we take out one factor out of each pair

Therefore, $\sqrt{16} = 2 \times 2 = 4$

Secondly we resolve 81 into prime factors

3	81
3	27
3	9
3	3
	1

Here we can see that $81 = 3 \times 3 \times 3 \times 3$

Thus, $\sqrt{81} = \sqrt{\underline{3 \times 3} \times \underline{3 \times 3}}$

We have 2 pairs, so we take out one factor out of each pair

Therefore, $\sqrt{81} = 3 \times 3 = 9$

Thus, $\sqrt{\frac{\mathbf{16}}{\mathbf{81}}} = \frac{\sqrt{16}}{\sqrt{81}} = \frac{4}{9}$

Question 2 - $\sqrt{\frac{\mathbf{64}}{\mathbf{225}}}$

Solution - Since $\sqrt{\frac{64}{225}} = \frac{\sqrt{64}}{\sqrt{225}}$

We solve each $\sqrt{64}$ and $\sqrt{225}$ by prime factorization method

First we resolve 64 into prime factors

2	64
2	32
2	16
2	8
2	4
2	2
	1

Here we can see that $64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$

Thus, $\sqrt{64} = \sqrt{\underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2}}$

We have 3 pairs, so we take out one factor out of each pair

Therefore, $\sqrt{64} = 2 \times 2 \times 2 = 8$

Secondly we resolve 225 into prime factors

3	225
3	75
5	25
5	5
	1

Here, we can see that 225 = 3 × 3 × 5 × 5

Thus, $\sqrt{225} = \sqrt{3 \times 3 \times 5 \times 5}$

We have 2 pairs, so we take out one factor out of each pair

Therefore, $\sqrt{225} = 3 \times 5 = 15$

Thus, $\sqrt{\frac{64}{225}} = \frac{\sqrt{64}}{\sqrt{225}} = \frac{8}{15}$

Question 3 - $\sqrt{\frac{121}{256}}$

Solution - Since $\sqrt{\frac{121}{256}} = \frac{\sqrt{121}}{\sqrt{256}}$

We solve each $\sqrt{121}$ and $\sqrt{256}$ by prime factorization method

First we resolve 121 into prime factors

11	121
11	11
	1

Here, we can see that 121 = 11 × 11

Thus, $\sqrt{121} = \sqrt{11 \times 11}$

We have 1 pairs, so we take out one factor out of each pair

Therefore, $\sqrt{121} = 11$

Secondly we resolve 256 into prime factors

2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

Here, we can see that 256 = 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2

Thus, $\sqrt{256} = \sqrt{\underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2}}$

We have 4 pairs, so we take out one factor out of each pair

Therefore, $\sqrt{256} = 2 \times 2 \times 2 \times 2$ = 16

Thus, $\sqrt{\frac{\mathbf{121}}{\mathbf{256}}} = \frac{\sqrt{121}}{\sqrt{256}} = \frac{11}{16}$

Question 4 - $\sqrt{\frac{\mathbf{625}}{\mathbf{729}}}$

Solution - Since $\sqrt{\frac{625}{729}} = \frac{\sqrt{625}}{\sqrt{729}}$

We solve each $\sqrt{625}$ and $\sqrt{729}$ by prime factorization method

5	625
5	125
5	25
5	5
	1

First we resolve 625 into prime factors

Here we can see that 625 = 5 × 5 × 5 × 5

Thus, $\sqrt{625} = \sqrt{\underline{5 \times 5} \times \underline{5 \times 5}}$

We have 2 pairs, so we take out one factor out of each pair

Therefore, $\sqrt{625}$ = $5 \times 5 = 25$

Secondly we resolve 729 into prime factors

3	729
3	243
3	81
3	27
3	9
3	3
	1

Here, we can see that 729 = 3 × 3 × 3 × 3 × 3 × 3

Thus, $\sqrt{729} = \sqrt{\underline{3 \times 3} \times \underline{3 \times 3} \times \underline{3 \times 3}}$

We have 3 pairs, so we take out one factor out of each pair

Therefore, $\sqrt{729} = 3 \times 3 \times 3 = 27$

Thus, $\sqrt{\frac{625}{729}} = \frac{\sqrt{625}}{\sqrt{729}} = \frac{25}{27}$

Question 5 - $\sqrt{3\frac{13}{36}}$

Solution - Since $3\frac{13}{36} = \frac{121}{36}$

We have to find $\sqrt{\frac{121}{36}} = \frac{\sqrt{121}}{\sqrt{36}}$

We solve each $\sqrt{121}$ and $\sqrt{36}$ by prime factorization method

First we resolve 121 into prime factors

11	121
11	11
	1

Here, we can see that 121 = 11 × 11

Thus, $\sqrt{121} = \sqrt{\underline{11 \times 11}}$

We have 1 pairs, so we take out one factor out of each pair

Therefore, $\sqrt{121} = 11$

Secondly we resolve 36 into prime factors

2	36
2	18
3	9
3	3
	1

Here, we can see that 36 = 2 × 2 × 3 × 3

Thus, $\sqrt{36} = \sqrt{\underline{2 \times 2} \times \underline{3 \times 3}}$

We have 2 pairs, so we take out one factor out of each pair

Therefore, $\sqrt{36} = 2 \times 3 = 6$

Thus, $\sqrt{\frac{\mathbf{121}}{\mathbf{36}}} = \frac{\sqrt{121}}{\sqrt{36}} = \frac{11}{6} = 1\frac{5}{6}$

Question 6 - $\sqrt{\mathbf{4\frac{73}{324}}}$

Solution - Since $4\frac{73}{324} = \frac{1369}{324}$

We have to find $\sqrt{\frac{1369}{324}} = \frac{\sqrt{1369}}{\sqrt{324}}$

We solve each $\sqrt{1369}$ and $\sqrt{324}$ by prime factorization method

First we resolve 1369 into prime factors

37	1369
37	37
	1

Here, we can see that 1369 = 37 × 37

Thus, $\sqrt{1369} = \sqrt{\underline{37 \times 37}}$

We have 1 pairs, so we take out one factor out of each pair

Therefore, $\sqrt{1369} = 37$

Secondly we resolve 324 into prime factors

2	324
2	162
3	81
3	27
3	9
3	3
	1

Here, we can see that 324 = 2 × 2 × 3 × 3 × 3 × 3

Thus, $\sqrt{324} = \sqrt{\underline{2 \times 2} \times \underline{3 \times 3} \times \underline{3 \times 3}}$

We have 3 pairs, so we take out one factor out of each pair

Therefore, $\sqrt{324} = 2 \times 3 \times 3 = 18$

Thus, $\sqrt{\frac{\mathbf{1369}}{\mathbf{324}}} = \frac{\sqrt{1369}}{\sqrt{324}} = \frac{37}{18} = 2\frac{1}{18}$

Question 7 - $\sqrt{\mathbf{3\frac{33}{289}}}$

Solution - Since $3\frac{33}{289} = \frac{900}{289}$

We have to find $\sqrt{\frac{900}{289}} = \frac{\sqrt{900}}{\sqrt{289}}$

We solve each $\sqrt{900}$ and $\sqrt{289}$ by prime factorization method

First we resolve 900 into prime factors

3	900
3	300
2	100
2	50
5	25
5	5
	1

Here, we can see that 900 = 3 × 3 × 2 × 2 × 5 × 5

Thus, $\sqrt{900} = \sqrt{\underline{3 \times 3} \times \underline{2 \times 2} \times \underline{5 \times 5}}$

We have 3 pairs, so we take out one factor out of each pair

Therefore, $\sqrt{900} = 2 \times 5 \times 3 = 30$

Secondly we resolve 289 into prime factors

17	289
17	17
	1

Here, we can see that $289 = 17 \times 17$

Thus, $\sqrt{289} = \sqrt{17 \times 17}$

We have 1 pairs, so we take out one factor out of each pair

Therefore, $\sqrt{289} = 17$

Thus, $\sqrt{\frac{900}{289}} = \frac{\sqrt{900}}{\sqrt{289}} = \frac{30}{17} = 1\frac{13}{17}$

Question 8 - $\frac{\sqrt{80}}{\sqrt{405}}$

Solution - We solve each $\sqrt{80}$ and $\sqrt{405}$ by prime factorization method

First we resolve 80 into prime factors

2	80
2	40
2	20
2	10
5	5
	1

Here, we can see that $80 = 2 \times 2 \times 2 \times 2 \times 5$

Thus, $\sqrt{80} = \sqrt{2 \times 2 \times 2 \times 2 \times 5}$

We have 2 pairs, so we take out one factor out of each pair

Therefore, $\sqrt{80} = 2 \times 2 \times \sqrt{5} = 4\sqrt{5}$

Secondly we resolve 405 into prime factors

3	405
3	135
3	45
3	15
5	5
	1

Here we can see that 405 = 3 × 3 × 3 × 3 × 5

Thus, $\sqrt{405} = \sqrt{3 \times 3 \times 3 \times 3 \times 5}$

We have 2 pairs, so we take out one factor out of each pair

Therefore, $\sqrt{405} = 3 \times 3 \times \sqrt{5} = 9\sqrt{5}$

Thus, $\frac{\sqrt{80}}{\sqrt{405}} = \frac{4\sqrt{5}}{9\sqrt{5}} = \frac{4}{9}$

Question 9 - $\frac{\sqrt{1183}}{\sqrt{2023}}$

Solution - We solve each $\sqrt{1183}$ and $\sqrt{2023}$ by prime factorization method

First we resolve 1183 into prime factors

7	1183
13	169
13	13
	1

Here, we can see that 1183 = 7 × 13 × 13

Thus, $\sqrt{1183} = \sqrt{7 \times 13 \times 13}$

We have 1 pairs, so we take out one factor out of each pair

Therefore, $\sqrt{1183} = 13 \times \sqrt{7} = 13\sqrt{7}$

Secondly we resolve 2023 into prime factors

7	2023
17	289
17	17
	1

Here, we can see that 2023 = 7 × 17 × 17

Thus, $\sqrt{2023} = \sqrt{17 \times 17 \times 7}$

We have 1 pairs, so we take out one factor out of each pair

Therefore, $\sqrt{2023} = 17 \times \sqrt{7} = 17\sqrt{7}$

Thus, $\frac{\sqrt{1183}}{\sqrt{2023}} = \frac{13\sqrt{7}}{17\sqrt{7}} = \frac{13}{7} = 1\frac{6}{7}$

Question 10 - $\sqrt{98} \times \sqrt{162}$

Solution - Since $\sqrt{98} \times \sqrt{162} = \sqrt{98 \times 162}$

First we resolve 98 into prime factors

2	98
7	49
7	7
	1

Here, we can see that $98 = 2 \times 7 \times 7$

Now, we resolve 162 into prime factors

2	162
3	81
3	27
3	9
3	3
	1

Here, we can see that $162 = 2 \times 3 \times 3 \times 3 \times 3$

Thus, $\sqrt{98 \times 162} = \sqrt{2 \times 7 \times 7 \times 2 \times 3 \times 3 \times 3 \times 3}$

We have 4 pairs, so we take out one factor out of each pair

Therefore, $\sqrt{98} \times \sqrt{162} = 2 \times 7 \times 3 \times 3 = 126$

Exercise 3H

Question 1 - Which of the following numbers is not a perfect square?

(a) 7056
(b) 3969
(c) 5478

(d) 4624

Solution **-** As we know that a number ending with 2, 3, 7 or 8 is never a perfect square

Thus, 5478 is not a perfect square.

Question 2 - Which of the following number is not a perfect square?

(a) 1444
(b) 3136
(c) 961
(d) 2222

Solution **-** As we know that a number ending with 2, 3, 7 or 8 is never a perfect square

Thus, 2222 is not a perfect square

Question 3 - Which of the following number is not a perfect square?

(a) 1843
(b) 3721
(c) 1024
(d) 1296

Solution **-** As we know that a number ending with 2, 3, 7 or 8 is never a perfect square

Thus, 1843 is not a perfect square

Question 4 - Which of the following number is not a perfect square?

(a) 1156
(b) 4787
(c) 2704
(d) 3969

Solution **-** As we know that a number ending with 2, 3, 7 or 8 is never a perfect square

Thus, 4787 is not a perfect square

Question 5 - Which of the following number is not a perfect square?

(a) 3600
(b) 6400
(c) 81000

(d) 2500

Solution - As we know that a number ending in an odd number of zeros is never a perfect square.

Thus, 81000 is not a perfect square

Question 6 - Which of the following cannot be the unit digit of a perfect square number?

(a) 6
(b) 1
(c) 9
(d) 8

Solution - Since we know that a number ending with 2, 3, 7 or 8 is never a perfect square

Thus, 8 cannot be the unit digit of a perfect square number

Question 7 - The Square of a proper fraction is ….

Solution - The Square of a proper fraction is smaller than the fraction

Question 8 - If n is odd, then $(1 + 3 + 5 \dots \dots$ to n terms) is equal to ….

Solution - n^2

Question 9 - Which of the following is a Pythagorean triplet?

(a) (2,3,5)
(b) (5,7,9)
(c) (6,9,11)
(d) (8,15,17)

Solution - Since we know that three natural numbers m, n and p are said to form a Pythagorean triplet (m, n, p) if $m^2 + n^2 = p^2$

First we take (2, 3, 5)

Here, m = 2, n= 3, p = 5

$m^2 + n^2 = 3^2 + 2^2 = 9+4=13$

$p^2 = 5^2 = 25$

Thus, $m^2 + n^2 \neq p^2$

Now take (5, 7, 9)

Here, m = 5, n= 7, p = 9

$m^2 + n^2$ = $5^2 + 7^2$ = 25+49=74

$p^2 = 9^2 = 81$

Thus, $m^2 + n^2 \neq p^2$

Now take (6, 9, 11)

Here, m = 6, n= 9, p = 11

$m^2 + n^2$ = $6^2 + 9^2$ = 36+81=117

$p^2 = 11^2 = 121$

Thus, $m^2 + n^2 \neq p^2$

Now take (8, 15, 17)

Here, m = 8, n= 15, p = 17

$m^2 + n^2$ = $8^2 + 15^2$ = 64+225=289

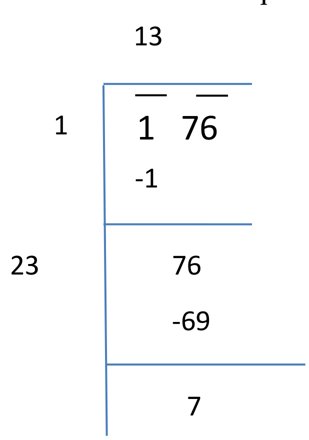
$p^2 = 17^2 = 289$

Thus, $m^2 + n^2 = p^2$

Therefore, (8, 15,17) is a Pythagorean triplet

Question 10 - What least number must be subtracted from 176 to make it a perfect square?

Solution - We will first find the square root of 176 by long division method,



Since from above we can see that $(13)^2$ < 176 by 7

So, 7 is the least number which is to be subtracted from 176 to get perfect square

Required perfect square number = 176-7 = 169

And $\sqrt{169} = 13$

Question 11 - What least number must be added to 526 to make it a perfect square?

Solution **-** We will first find the square root of 526 by long division method,

```
         22
      ______
 2   | 5 26
     | -4
     |______
42   |  126
     |  -84
     |_______
     |    42
```

Since from above we can see that $(22)^2$ < 526 < $(23)^2$

So, the number to added to get perfect square is $(23)^2 - 526 = 529 - 526 = 3$

Thus, 3 is added to 526 to get a perfect square

Question 12 - What least number must be added to 15370 to make it a perfect square?

Solution - We will first find the square root of 15370 by long division method,

```
          123
       ________
  1   | 1 53 70
      | -1
      |________
 22   |  53
      | -44
      |_________
243   |   970
      |  -729
      |___________
      |    241
```

Since from above we can see that $(123)^2 < 15370 < (124)^2$

So, the number to added to get perfect square is $(124)^2 - 15370 = 15376 - 15370 = 6$

Thus, 6 is added to 15370 to get a perfect square

Question 13 - $\sqrt{\mathbf{0.9}}$ =?

Solution - Using long division method, we will find $\sqrt{0.9}$

	0.948
9	$0.\overline{90}\ \overline{00}\ \overline{00}$
	-81
184	900
	-736
1888	16400
	-15104
	1296

Thus, $\sqrt{0.9} = 0.948 = 0.94$

Question 14 - $\sqrt{\mathbf{0.1}}$ =?

Solution - Using long division method, we will find $\sqrt{0.1}$

	0.316
3	$0.\overline{10}\ \overline{00}\ \overline{00}$
	-9
61	100
	-61
626	3900
	-3756
	144

Thus, $\sqrt{0.1} = 0.316$

Question 15 - $\sqrt{\mathbf{0.9}} \times \sqrt{\mathbf{1.6}} = ?$

Solution - Since $\sqrt{0.9} \times \sqrt{1.6} = \sqrt{0.9 \times 1.6} = \sqrt{1.44}$

Using long division method, we will find $\sqrt{1.44}$

```
            1.2
       ___________
  1   | 1. 44
      | -1
      |________
 22   |   44
      |  -44
      |________
      |    0
```

Thus, $\sqrt{1.44} = 1.2$

Question 16 - $\frac{\sqrt{\mathbf{288}}}{\sqrt{\mathbf{128}}}$

Solution - We solve each $\sqrt{288}$ and $\sqrt{128}$ by prime factorization method

First we resolve 288 into prime factors

Here, we can see that 288 = 2 × 2 × 2 × 2 × 2 × 3 × 3

2	288
2	144
2	72
2	36
2	18
3	9
3	3
	1

Thus, $\sqrt{288} = \sqrt{2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3}$

We have 3 pairs, so we take out one factor out of each pair

Therefore, $\sqrt{288} = 2 \times 2 \times 3 \times \sqrt{2} = 12\sqrt{2}$

Secondly we resolve 128 into prime factors

2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

Here, we can see that $128 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

Thus, $\sqrt{128} = \sqrt{\underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times 2}$

We have 3 pairs, so we take out one factor out of each pair

Therefore, $\sqrt{128} = 2 \times 2 \times 2\sqrt{2} = 8\sqrt{2}$

Thus, $\frac{\sqrt{288}}{\sqrt{128}} = \frac{12\sqrt{2}}{8\sqrt{2}} = \frac{12}{8} = \frac{3}{2}$

Question 17: $\sqrt{2\frac{1}{4}} = ?$

Solution - Since $2\frac{1}{4} = \frac{9}{4}$

We have to find $\sqrt{\frac{9}{4}} = \frac{\sqrt{9}}{\sqrt{4}}$

We solve each $\sqrt{9}$ and $\sqrt{4}$ by prime factorization method

First we resolve 9 into prime factors

3	9
3	3
	1

Here, we can see that 9 = 3 × 3

Thus, $\sqrt{9} = \sqrt{3 \times 3}$

We have 1 pairs, so we take out one factor out of each pair

Therefore, $\sqrt{9} = 3$

Secondly we resolve 4 into prime factors

2	4
2	2
	1

Here we can see that 4 = 2 × 2

Thus, $\sqrt{4} = \sqrt{2 \times 2}$

We have 1 pairs, so we take out one factor out of each pair

Therefore, $\sqrt{4} = 2$

Thus, $\sqrt{\frac{9}{4}} = \frac{\sqrt{9}}{\sqrt{4}} = \frac{3}{2} = 1\frac{1}{2}$

Question 18 - Which of the following is the square of an even number?

(a) 196
(b) 441
(c) 625
(d) 529

Solution - Since we know that the square of an even number is even

Thus, 196 is the square of an even number.

Question 19 - Which of the following is the square of an odd number?

- **(a) 2116**
- **(b) 3844**
- **(c) 1369**
- **(d) 2500**

Solution **-** Since the square of an odd number is odd

Thus, 1369 is the square of an odd number.