Chapter 4

Rational Numbers

Introduction

Rational numbers are those numbers which can be written in the form of $\frac{p}{q}$ where p and q are integers and $q \neq 0$. In a rational number $\frac{p}{q}$, p is the numerator of fraction and q is the denominator of the fraction.

For example: $\frac{1}{3}, \frac{5}{7}, -3$ are rational numbers.

Positive rational numbers are those numbers in which its numerator and denominator are either positive integers or both negative integers.

Negative rational numbers are those in which numerator and denominator are in such a way that one of them is positive and other is negative.

Useful results:

(1) All natural numbers are rational numbers but a rational number need not be a natural number. Natural numbers are 1, 2, 3….. , all of which can be written in the fraction form as $\frac{1}{1}, \frac{2}{1}, \frac{3}{1}$ $etc.$

If we take a rational number $\frac{4}{5}$ then we see that it is not a natural number.

(2) Zero is a rational number: It is rational number because we can write it as $0 = \frac{0}{1}$ where, 0 and 1 are integers and $1 \neq 0$

(3) All integers are rational numbers but a rational number need not be an integer.

For example: …,-2, -1, 0, 1, 2…., are integers which can be written in the fraction form. Thus, all are rational numbers also. But if we take a rational number as $-\frac{1}{2}$ which can never be an integer.

(4) Each fraction is a rational number but a rational number need not be a fraction.

Exercise 4.1

Question 1 – Write down the numerator of each of the following rational numbers:

(i) $\frac{-7}{5}$

Solution: In the fraction $\frac{-7}{5}$, numerator is -7

(ii) $\frac{15}{-4}$

Solution: In the fraction $\frac{15}{-4}$, numerator is 15

(iii) $\frac{-17}{-21}$

Solution: In the fraction $\frac{-17}{-21}$, numerator is -17

(iv) $\frac{8}{9}$

Solution: In the fraction $\frac{8}{9}$, numerator is 8

(v) 5

Solution: 5 can be written as $\frac{5}{1}$. Thus, numerator is 5

Question 2 – Write down the denominator of each of the following rational numbers:

(i) $\frac{-4}{5}$

Solution: In the fraction $\frac{-4}{5}$, denominator is 5

(ii) $\frac{11}{-34}$

Solution: In the fraction $\frac{11}{-34}$, denominator is -34

(iii) $\frac{-15}{-82}$

Solution: In the fraction $\frac{-15}{-82}$, denominator is -82

(iv) 15

Solution: Since 15 can written as $\frac{15}{1}$. In the fraction $\frac{15}{1}$, denominator is 1

(v) 0

Solution: Since, we can write 0 as $0 = \frac{0}{1} = \frac{0}{2} = \frac{0}{3} = \frac{0}{-1} = \frac{0}{-2}$ and so on

Therefore, in this case, denominator can be any integer other than 0.

Question 3 – Write down the rational number whose numerator is (-3) ×4, and whose denominator is $(\mathbf{34-23}) \times (\mathbf{7-4})$.

Solution: Numerator = $(-3) \times 4 = -12$

Denominator = $(34-23) \times (7-4) = 11 \times 3 = 33$

Thus, rational number = $\frac{numerator}{denominator} = \frac{-12}{33}$

Question 4 – Write the following rational numbers as integers:

$$\mathbf{\frac{7}{1}, \frac{-12}{1}, \frac{34}{1}, \frac{-73}{1}, \frac{95}{1}}$$

Solution: Since integers are whole numbers which are not represented in fractional form.

Thus, these rational numbers in integer form are $7, -12, 34, -73, 95$

Question 5 – Write the following integers as rational numbers with denominator 1:

$$\mathbf{-15, 17, 85, -100}$$

Solution: Since a rational number always have a numerator and a denominator.

Thus, these integers can be written as rational numbers as follows:

$$\frac{-15}{1}, \frac{17}{1}, \frac{85}{1}, \frac{-100}{1}$$

Question 6 – Write down the rational number whose numerator is the smallest three digit number and denominator is the largest four digit number.

Solution: Since smallest three digit number = 100 and largest four digit number = 9999

Thus, rational number = $\frac{numerator}{denominator} = \frac{100}{9999}$

Question 7 – Separate positive and negative rational numbers from the following rational numbers: $\frac{-5}{-7}, \frac{12}{-5}, \frac{7}{4}, \frac{13}{-9}, 0, \frac{-18}{-7}, \frac{-95}{116}, \frac{-1}{-9}$

Solution: $\frac{-5}{-7}, \frac{7}{4}, \frac{-18}{-7}$ *and* $\frac{-1}{-9}$ are positive rational numbers because numerator and denominator are either positive or both negative integers whereas $\frac{12}{-5}, \frac{13}{-9}$ *and* $\frac{-95}{116}$ are negative rational numbers because numerator and denominator are in such a way that one of them is negative and other one is positive.

Question 8 – Which of the following rational numbers are positive?

(i) $\frac{-8}{7}$

Solution: $\frac{-8}{7}$ is not a positive rational number as numerator and denominator are of opposite sign.

(ii) $\frac{9}{8}$

Solution: $\frac{9}{8}$ is a positive rational number as numerator and denominator are of same sign (positive sign).

(iii) $\frac{-19}{-13}$

Solution: $\frac{-19}{-13}$ is a positive rational number as numerator and denominator are of same sign (negative sign).

(iv) $\frac{-21}{13}$

Solution: $\frac{-21}{13}$ is not a positive rational number as numerator and denominator are of opposite sign.

Question 9 – Which of the following rational numbers are negative?

(i) $\frac{-3}{7}$

Solution: $\frac{-3}{7}$ is a negative rational number as numerator and denominator are of opposite sign.

(ii) $\frac{-5}{-8}$

Solution: $\frac{-5}{-8}$ is a positive rational number as numerator and denominator are of same sign.

(iii) $\frac{\mathbf{9}}{\mathbf{-83}}$

Solution: $\frac{9}{-83}$ is a negative rational number as numerator and denominator are of opposite sign.

(iv) $\frac{\mathbf{-115}}{\mathbf{-197}}$

Solution: $\frac{-115}{-197}$ is a positive rational number as numerator and denominator are of same sign.

Properties of Rational Numbers:

Property 1: If $\frac{p}{q}$ is any rational number and m is a non-zero integer, then we have

$$\frac{p}{q} = \frac{p \times m}{q \times m}$$

Here, $\frac{p \times m}{q \times m}$ is a rational number equivalent to $\frac{p}{q}$

According to this property, if we multiply the numerator and denominator by the same non-zero integer then, rational number remains unchanged.

Property 2: If $\frac{p}{q}$ is any rational number and m is a common divisor of p and q, then we have

$$\frac{p}{q} = \frac{p \div m}{q \div m}$$

According to this property, if we divide the numerator and denominator by the common divisor of numerator and denominator then, rational number remains unchanged.

Examples:

Example 1 – Write each of the following rational numbers with positive denominator:

$$\frac{\mathbf{5}}{\mathbf{-7}}, \frac{\mathbf{15}}{\mathbf{-28}}, \frac{\mathbf{-17}}{\mathbf{-13}}$$

Solution: We can write the rational numbers with positive denominator by multiplying the numerator and denominator by (-1).

$\frac{5}{-7} = \frac{5 \times (-1)}{-7 \times (-1)} = \frac{-5}{7}$;

$\frac{15}{-28} = \frac{15 \times (-1)}{-28 \times (-1)} = \frac{-15}{28}$;

$\frac{-17}{-13} = \frac{-17\times(-1)}{-13\times(-1)} = \frac{17}{13}$

Example 2 – Express $\frac{-5}{6}$ as a rational number with numerator:

(i) -15

Solution: In order to make numerator as -15, we have to multiply the numerator by $\frac{-15}{-5} = 3$. Thus, to make the given rational number equivalent, we will multiply both numerator and denominator by 3.

Therefore, $\frac{-5}{6} = \frac{-5\times3}{6\times3} = \frac{-15}{18}$

(ii) 10

Solution: In order to make numerator as 10, we have to multiply the numerator by $\frac{10}{-5} = -2$. Thus, to make the given rational number equivalent, we will multiply both numerator and denominator by -2.

Therefore, $\frac{-5}{6} = \frac{-5\times(-2)}{6\times(-2)} = \frac{10}{-12}$

Example 3 – Express $\frac{-4}{5}$ as a rational number with denominator:

(i) 20

Solution: In order to make denominator as 20, we have to multiply the denominator by $\frac{20}{5} = 4$. Thus, to make the given rational number equivalent, we will multiply both numerator and denominator by 4.

Therefore, $\frac{-4}{5} = \frac{-4\times4}{5\times4} = \frac{-16}{20}$

(ii) -30

Solution: In order to make denominator as -30, we have to multiply the denominator by $\frac{-30}{5} = -6$. Thus, to make the given rational number equivalent, we will multiply both numerator and denominator by -6.

Therefore, $\frac{-4}{5} = \frac{-4\times(-6)}{5\times(-6)} = \frac{24}{-30}$

Example 4 – Express $\frac{-48}{60}$ as a rational number with denominator 5.

Solution: In order to make denominator as 5, we have to divide the denominator by $\frac{60}{5} = 12$. Thus, to make the given rational number equivalent, we will divide both numerator and denominator by 12.

Therefore, $\frac{-48}{60} = \frac{-48 \div 12}{60 \div 12} = \frac{-4}{5}$

Example 5 – Express $\frac{42}{-63}$ as a rational number with denominator 3.

Solution: In order to make denominator as 3, we have to divide the denominator by $\frac{-63}{3} = -21$. Thus, to make the given rational number equivalent, we will divide both numerator and denominator by -21.

Therefore, $\frac{42}{-63} = \frac{42 \div (-21)}{-63 \div (-21)} = \frac{-2}{3}$

Example 6 – Fill in the blanks:

(i) $\frac{5}{-7} = \frac{-}{35} = \frac{-}{-77}$

Solution: In order to fill the blank spaces, we need to make these fractions as equivalent rational numbers.

Firstly, in order to make denominator as 35, we have to multiply the denominator by $\frac{35}{-7} = -5$. Thus, to make the given rational number equivalent, we will multiply both numerator and denominator by -5.

Therefore, $\frac{5}{-7} = \frac{5 \times (-5)}{-7 \times (-5)} = \frac{-25}{35}$

Secondly, in order to make denominator as -77, we have to multiply the denominator by $\frac{-77}{-7} =$ 11. Thus, to make the given rational number equivalent, we will multiply both numerator and denominator by 11.

Therefore, $\frac{5}{-7} = \frac{5 \times (11)}{-7 \times (11)} = \frac{55}{-77}$

Therefore, $\frac{5}{-7} = \frac{-25}{35} = \frac{55}{-77}$

(ii) $\frac{7}{13} = \frac{35}{-} = \frac{-63}{-}$

Solution: In order to fill the blank spaces, we need to make these fractions as equivalent rational numbers.

Firstly, in order to make numerator as 35, we have to multiply the numerator by $\frac{35}{7} = 5$. Thus, to make the given rational number equivalent, we will multiply both numerator and denominator by 5.

Therefore, $\frac{7}{13} = \frac{7\times(5)}{13\times(5)} = \frac{35}{65}$

Secondly, in order to make numerator as -63, we have to multiply the numerator by $\frac{-63}{7} = -9$. Thus, to make the given rational number equivalent, we will multiply both numerator and denominator by -9.

Therefore, $\frac{7}{13} = \frac{7\times(-9)}{13\times(-9)} = \frac{-63}{-117}$

Therefore, $\frac{7}{13} = \frac{35}{65} = \frac{-63}{-117}$

Example 7 – In each of the following, find an equivalent form of the rational numbers having a common denominator

(i) $\frac{5}{6}\, and\, \frac{7}{9}$

Solution: We will take LCM of (6 and 9) in order to make the denominators of each rational number equal.

Now, LCM of (6 and 9) is 18

Firstly, in order to make denominator as 18 in $\frac{5}{6}$, we have to multiply the denominator by $\frac{18}{6} = 3$. Thus, to make the given rational number equivalent, we will multiply both numerator and denominator by 3.

Therefore, $\frac{5}{6} = \frac{5\times(3)}{6\times(3)} = \frac{15}{18}$

Secondly, in order to make denominator as 18 in $\frac{7}{9}$, we have to multiply the denominator by $\frac{18}{9} = 2$. Thus, to make the given rational number equivalent, we will multiply both numerator and denominator by 2.

Therefore, $\frac{7}{9} = \frac{7\times(2)}{9\times(2)} = \frac{14}{18}$

Therefore, rational numbers with common denominator are $\frac{15}{18}$ and $\frac{14}{18}$

(ii) $\frac{2}{3}, \frac{5}{6}$ $\boldsymbol{and}$ $\frac{7}{12}$

Solution: We will take LCM of (3, 6 and 12) in order to make the denominators of each rational number equal.

Now, LCM of (3, 6 and 12) is 12

Firstly, in order to make denominator as 12 in $\frac{2}{3}$, we have to multiply the denominator by $\frac{12}{3} = 4$. Thus, to make the given rational number equivalent, we will multiply both numerator and denominator by 4.

Therefore, $\frac{2}{3} = \frac{2\times(4)}{3\times(4)} = \frac{8}{12}$

Secondly, in order to make denominator as 12 in $\frac{5}{6}$, we have to multiply the denominator by $\frac{12}{6} = 2$. Thus, to make the given rational number equivalent, we will multiply both numerator and denominator by 2.

Therefore, $\frac{5}{6} = \frac{5\times(2)}{6\times(2)} = \frac{10}{12}$

Thirdly, in order to make denominator as 12 in $\frac{7}{12}$, we have to multiply the denominator by $\frac{12}{12} = 1$. Thus, to make the given rational number equivalent, we will multiply both numerator and denominator by 1.

Therefore, $\frac{7}{12} = \frac{7\times(1)}{12\times(1)} = \frac{7}{12}$

Therefore, rational numbers with common denominator are $\frac{8}{12}, \frac{10}{12}$ and $\frac{7}{12}$

Exercise 4.2

Question 1 – Express each of the following as a rational number with positive denominator:

(i) $\frac{-15}{-28}$

Solution: We can write the rational numbers with positive denominator by multiplying the numerator and denominator by (-1).

$\frac{-15}{-28} = \frac{-15\times(-1)}{-28\times(-1)} = \frac{15}{28}$

(ii) $\frac{6}{-9}$

Solution: We can write the rational numbers with positive denominator by multiplying the numerator and denominator by (-1).

$\frac{6}{-9} = \frac{6\times(-1)}{-9\times(-1)} = \frac{-6}{9}$

(iii) $\frac{-28}{-11}$

Solution: We can write the rational numbers with positive denominator by multiplying the numerator and denominator by (-1).

$\frac{-28}{-11} = \frac{-28\times(-1)}{-11\times(-1)} = \frac{28}{11}$

(iv) $\frac{19}{-7}$

Solution: We can write the rational numbers with positive denominator by multiplying the numerator and denominator by (-1).

$\frac{19}{-7} = \frac{19\times(-1)}{-7\times(-1)} = \frac{-19}{7}$

Question 2 – Express $\frac{3}{5}$ as a rational number with numerator:

(i) 6

Solution: In order to make numerator as 6, we have to multiply the numerator by $\frac{6}{3} = 2$. Thus, to make the given rational number equivalent, we will multiply both numerator and denominator by 2.

Therefore, $\frac{3}{5} = \frac{3\times2}{5\times2} = \frac{6}{10}$

(ii) -15

Solution: In order to make numerator as -15, we have to multiply the numerator by $\frac{-15}{3} = -5$. Thus, to make the given rational number equivalent, we will multiply both numerator and denominator by -5.

Therefore, $\frac{3}{5} = \frac{3\times(-5)}{5\times(-5)} = \frac{-15}{-25}$

(iii) 21

Solution: In order to make numerator as 6, we have to multiply the numerator by$\frac{21}{3} = 7$. Thus, to make the given rational number equivalent, we will multiply both numerator and denominator by 7.

Therefore, $\frac{3}{5} = \frac{3\times7}{5\times7} = \frac{21}{35}$

(iv) -27

Solution: In order to make numerator as -27, we have to multiply the numerator by$\frac{-27}{3} = -9$. Thus, to make the given rational number equivalent, we will multiply both numerator and denominator by -9.

Therefore, $\frac{3}{5} = \frac{3\times(-9)}{5\times(-9)} = \frac{-27}{-45}$

Question 3 – Express $\frac{5}{7}$ as a rational number with denominator:

(i) -14

Solution: In order to make denominator as -14, we have to multiply the denominator by$\frac{-14}{7} = -2$. Thus, to make the given rational number equivalent, we will multiply both numerator and denominator by -2.

Therefore, $\frac{5}{7} = \frac{5\times(-2)}{7\times(-2)} = \frac{-10}{-14}$

(ii) 70

Solution: In order to make denominator as 70, we have to multiply the denominator by$\frac{70}{7} = 10$. Thus, to make the given rational number equivalent, we will multiply both numerator and denominator by 10.

Therefore, $\frac{5}{7} = \frac{5\times(10)}{7\times(10)} = \frac{50}{70}$

(iii) -28

Solution: In order to make denominator as -28, we have to multiply the denominator by$\frac{-28}{7} = -4$. Thus, to make the given rational number equivalent, we will multiply both numerator and denominator by -4.

Therefore, $\frac{5}{7} = \frac{5\times(-4)}{7\times(-4)} = \frac{-20}{-28}$

(iv) -84

Solution: In order to make denominator as -84, we have to multiply the denominator by $\frac{-84}{7} = -12$. Thus, to make the given rational number equivalent, we will multiply both numerator and denominator by -12.

Therefore, $\frac{5}{7} = \frac{5\times(-12)}{7\times(-12)} = \frac{-60}{-84}$

Question 4 – Express $\frac{3}{4}$ as a rational number with denominator:

(i) 20

Solution: In order to make denominator as 20, we have to multiply the denominator by $\frac{20}{4} = 5$. Thus, to make the given rational number equivalent, we will multiply both numerator and denominator by 5.

Therefore, $\frac{3}{4} = \frac{3\times(5)}{4\times(5)} = \frac{15}{20}$

(ii) 36

Solution: In order to make denominator as 36, we have to multiply the denominator by $\frac{36}{4} = 9$. Thus, to make the given rational number equivalent, we will multiply both numerator and denominator by 9.

Therefore, $\frac{3}{4} = \frac{3\times(9)}{4\times(9)} = \frac{27}{36}$

(iii) 44

Solution: In order to make denominator as 44, we have to multiply the denominator by $\frac{44}{4} = 11$. Thus, to make the given rational number equivalent, we will multiply both numerator and denominator by 11.

Therefore, $\frac{3}{4} = \frac{3\times(11)}{4\times(11)} = \frac{33}{44}$

(iv) -80

Solution: In order to make denominator as -80, we have to multiply the denominator by $\frac{-80}{4} = -20$. Thus, to make the given rational number equivalent, we will multiply both numerator and denominator by -20.

Therefore, $\frac{3}{4} = \frac{3\times(-20)}{4\times(-20)} = \frac{-60}{-80}$

Question 5 – Express $\frac{2}{5}$ as a rational number with numerator

(i) -56

Solution: In order to make numerator as -56, we have to multiply the numerator by $\frac{-56}{2} = -28$. Thus, to make the given rational number equivalent, we will multiply both numerator and denominator by -28.

Therefore, $\frac{2}{5} = \frac{2\times(-28)}{5\times(-28)} = \frac{-56}{-140}$

(ii) 154

Solution: In order to make numerator as 154, we have to multiply the numerator by $\frac{154}{2} = 77$. Thus, to make the given rational number equivalent, we will multiply both numerator and denominator by 77.

Therefore, $\frac{2}{5} = \frac{2\times(77)}{5\times(77)} = \frac{154}{385}$

(iii) -750

Solution: In order to make numerator as -750, we have to multiply the numerator by $\frac{-750}{2} = -375$. Thus, to make the given rational number equivalent, we will multiply both numerator and denominator by -375.

Therefore, $\frac{2}{5} = \frac{2\times(-375)}{5\times(-375)} = \frac{-750}{-1875}$

(iv) 500

Solution: In order to make numerator as 500, we have to multiply the numerator by $\frac{500}{2} = 250$. Thus, to make the given rational number equivalent, we will multiply both numerator and denominator by 250.

Therefore, $\frac{2}{5} = \frac{2\times(250)}{5\times(250)} = \frac{500}{1250}$

Question 6 – Express $\frac{-192}{108}$ as a rational number with numerator:

(i) 64

Solution: In order to make numerator as 64, we have to divide the numerator by $\frac{-192}{64} = -3$. Thus, to make the given rational number equivalent, we will divide both numerator and denominator by -3.

Therefore, $\frac{-192}{108} = \frac{-192\div(-3)}{108\div(-3)} = \frac{64}{-36}$

(ii) -16

Solution: In order to make numerator as -16, we have to divide the numerator by $\frac{-192}{-16} = 12$. Thus, to make the given rational number equivalent, we will divide both numerator and denominator by 12.

Therefore, $\frac{-192}{108} = \frac{-192\div(12)}{108\div(12)} = \frac{-16}{9}$

(iii) 32

Solution: In order to make numerator as 32, we have to divide the numerator by $\frac{-192}{32} = -6$. Thus, to make the given rational number equivalent, we will divide both numerator and denominator by -6.

Therefore, $\frac{-192}{108} = \frac{-192\div(-6)}{108\div(-6)} = \frac{32}{-18}$

(iv) -48

Solution: In order to make numerator as -48, we have to divide the numerator by $\frac{-192}{-48} = 4$. Thus, to make the given rational number equivalent, we will divide both numerator and denominator by 4.

Therefore, $\frac{-192}{108} = \frac{-192\div(4)}{108\div(4)} = \frac{-48}{27}$

Question 7 – Express $\frac{168}{-294}$ as a rational number with denominator:

(i) 14

Solution: In order to make denominator as 14, we have to divide the denominator by $\frac{-294}{14} = -21$. Thus, to make the given rational number equivalent, we will divide both numerator and denominator by -21.

Therefore, $\frac{168}{-294} = \frac{168\div(-21)}{-294\div(-21)} = \frac{-8}{14}$

(ii) -7

Solution: In order to make denominator as -7, we have to divide the denominator by $\frac{-294}{-7} = 42$. Thus, to make the given rational number equivalent, we will divide both numerator and denominator by 42.

Therefore, $\frac{168}{-294} = \frac{168\div(42)}{-294\div(42)} = \frac{4}{-7}$

(iii) -49

Solution: In order to make denominator as -49, we have to divide the denominator by $\frac{-294}{-49} = 6$. Thus, to make the given rational number equivalent, we will divide both numerator and denominator by 6.

Therefore, $\frac{168}{-294} = \frac{168\div(6)}{-294\div(6)} = \frac{28}{-49}$

(iv) 1470

Solution: In order to make denominator as 1470, we have to multiply the denominator by $\frac{1470}{-294} = -5$. Thus, to make the given rational number equivalent, we will multiply both numerator and denominator by -5.

Therefore, $\frac{168}{-294} = \frac{168\times(-5)}{-294\times(-5)} = \frac{-840}{1470}$

Question 8 – Write $\frac{-14}{42}$ in a form so that the numerator is equal to:

(i) -2

Solution: In order to make numerator as -2, we have to divide the numerator by $\frac{-14}{2} = -7$. Thus, to make the given rational number equivalent, we will divide both numerator and denominator by -7.

Therefore, $\frac{-14}{42} = \frac{-14\div(-7)}{42\div(-7)} = \frac{2}{-6}$

(ii) 7

Solution: In order to make numerator as 7, we have to divide the numerator by $\frac{-14}{7} = -2$. Thus, to make the given rational number equivalent, we will divide both numerator and denominator by -2.

Therefore, $\frac{-14}{42} = \frac{-14 \div (-2)}{42 \div (-2)} = \frac{7}{-21}$

(iii) 42

Solution: In order to make numerator as 42, we have to multiply the numerator by $\frac{42}{-14} = -3$. Thus, to make the given rational number equivalent, we will multiply both numerator and denominator by -3.

Therefore, $\frac{-14}{42} = \frac{-14 \times (-3)}{42 \times (-3)} = \frac{42}{-126}$

(iv) -70

Solution: In order to make numerator as -70, we have to multiply the numerator by $\frac{-70}{-14} = 5$. Thus, to make the given rational number equivalent, we will multiply both numerator and denominator by 5.

Therefore, $\frac{-14}{42} = \frac{-14 \times (5)}{42 \times (5)} = \frac{-70}{210}$

Question 9 – Select those rational numbers which can be written as a rational number with numerator 6:

$$\frac{1}{22}, \frac{2}{3}, \frac{3}{4}, \frac{4}{-5}, \frac{5}{6}, \frac{-6}{7}, \frac{-7}{8}$$

Solution: Below are the rational numbers which can be written as a rational number with numerator 6:

$\frac{1}{22} = \frac{1 \times 6}{22 \times 6} = \frac{6}{132}$ (Multiplying each term by 6)

$\frac{2}{3} = \frac{2 \times 3}{3 \times 3} = \frac{6}{9}$ (Multiplying each term by 3)

$\frac{3}{4} = \frac{3 \times 2}{4 \times 2} = \frac{6}{8}$ (Multiplying each term by 2)

$\frac{-6}{7} = \frac{-6 \times (-1)}{7 \times (-1)} = \frac{6}{-7}$ (Multiplying each term by -1)

Question 10 – Select those rational numbers which can be written as a rational number with denominator 4:

$$\frac{7}{8}, \frac{64}{16}, \frac{36}{-12}, -\frac{16}{17}, \frac{5}{-4}, \frac{140}{28}$$

Solution: Below are the rational numbers which can be written as a rational number with denominator 4:

$\frac{7}{8} = \frac{7 \div 2}{8 \div 2} = \frac{3.5}{4}$ (Dividing each term by 2)

$\frac{64}{16} = \frac{64 \div 4}{16 \div 4} = \frac{16}{4}$ (Dividing each term by 4)

$\frac{36}{-12} = \frac{36 \div (-3)}{-12 \div (-3)} = \frac{-12}{4}$ (Dividing each term by -3)

$\frac{5}{-4} = \frac{5 \times (-1)}{-4 \times (-1)} = \frac{-5}{4}$ (Multiplying each term by -1)

$\frac{140}{28} = \frac{140 \div 7}{28 \div 7} = \frac{20}{4}$ (Dividing each term by 7)

Question 11 – In each of the following, find an equivalent form of the rational number having a common denominator:

(i) $\boldsymbol{\frac{3}{4} \; and \; \frac{5}{12}}$

Solution: We will take LCM of (4 and 12) in order to make the denominators of each rational number equal.

Now, LCM of (4 and 12) is 12

Firstly, in order to make denominator as 12 in $\frac{3}{4}$, we have to multiply the denominator by $\frac{12}{4} = 3$. Thus, to make the given rational number equivalent, we will multiply both numerator and denominator by 3.

Therefore, $\frac{3}{4} = \frac{3 \times (3)}{4 \times (3)} = \frac{9}{12}$

Secondly, in order to make denominator as 12 in $\frac{5}{12}$, we have to multiply the denominator by $\frac{12}{12} = 1$. Thus, to make the given rational number equivalent, we will multiply both numerator and denominator by 1.

Therefore, $\frac{5}{12} = \frac{5 \times (1)}{12 \times (1)} = \frac{5}{12}$

Therefore, rational numbers with common denominator are $\frac{9}{12}$ and $\frac{5}{12}$

(ii) $\frac{2}{3}, \frac{7}{6}$ $\boldsymbol{and}$ $\frac{11}{12}$

Solution: We will take LCM of (3, 6 and 12) in order to make the denominators of each rational number equal.

Now, LCM of (3, 6 and 12) is 12

Firstly, in order to make denominator as 12 in $\frac{2}{3}$, we have to multiply the denominator by $\frac{12}{3} = 4$. Thus, to make the given rational number equivalent, we will multiply both numerator and denominator by 4.

Therefore, $\frac{2}{3} = \frac{2\times(4)}{3\times(4)} = \frac{8}{12}$

Secondly, in order to make denominator as 12 in $\frac{7}{6}$, we have to multiply the denominator by $\frac{12}{6} = 2$. Thus, to make the given rational number equivalent, we will multiply both numerator and denominator by 2.

Therefore, $\frac{7}{6} = \frac{7\times(2)}{6\times(2)} = \frac{14}{12}$

Thirdly, in order to make denominator as 12 in $\frac{11}{12}$, we have to multiply the denominator by $\frac{12}{12} = 1$. Thus, to make the given rational number equivalent, we will multiply both numerator and denominator by 1.

Therefore, $\frac{11}{12} = \frac{11\times(1)}{12\times(1)} = \frac{11}{12}$

Therefore, rational numbers with common denominator are $\frac{8}{12}, \frac{14}{12}$ and $\frac{11}{12}$

(iii) $\frac{5}{7}, \frac{3}{8}, \frac{9}{14}$ $\boldsymbol{and}$ $\frac{20}{21}$

Solution: We will take LCM of (7, 8, 14 and 21) in order to make the denominators of each rational number equal.

Now, LCM of (7, 8, 14 and 21) is 168

Firstly, in order to make denominator as 168 in $\frac{5}{7}$, we have to multiply the denominator by $\frac{168}{7} = 24$. Thus, to make the given rational number equivalent, we will multiply both numerator and denominator by 24.

Therefore, $\frac{5}{7} = \frac{5\times(24)}{7\times(24)} = \frac{120}{168}$

Secondly, in order to make denominator as 168 in $\frac{3}{8}$, we have to multiply the denominator by $\frac{168}{8} = 21$. Thus, to make the given rational number equivalent, we will multiply both numerator and denominator by 21.

Therefore, $\frac{3}{8} = \frac{3\times(21)}{8\times(21)} = \frac{63}{168}$

Thirdly, in order to make denominator as 168 in $\frac{9}{14}$, we have to multiply the denominator by $\frac{168}{14} = 12$. Thus, to make the given rational number equivalent, we will multiply both numerator and denominator by 12.

Therefore, $\frac{9}{14} = \frac{9\times(12)}{14\times(12)} = \frac{108}{168}$

Therefore, rational numbers with common denominator are $\frac{120}{168}, \frac{63}{168}, \frac{108}{168}\ and\ \frac{160}{168}$

Lowest form of a rational number

Suppose $\frac{p}{q}$ is any rational number where p and q are integers, then it is said to be in its lowest or simplest form when p and q have no common factor other than 1.

Examples

Example 1 – Find whether the following rational numbers are in the lowest form or not.

(i) $\mathbf{\frac{17}{79}}$

Solution: Firstly, we will find: m = HCF of (17 and 79) as follows:

17 = 17×1

79 = 79 × 1

We can see that only 1 is common factor between 17 and 79

Thus, m = HCF (17, 79) = 1

Therefore, $\frac{17}{79}$ is in its lowest form.

(ii) $\frac{24}{320}$

Solution: Firstly, we will find: m = HCF of (24 and 320) as follows:

24 = 2 × 2 × 2 × 3

320 = 2 × 2 × 2 × 2 × 2 × 2× 5

We can see that $2 \times 2 \times 2 = 8$ is common factor between 24 and 320

Thus, m = HCF (24, 320) = 8

Since m ≠ 1, thus $\frac{24}{320}$ is not in its lowest form.

Therefore, we must divide each term by m = 8 in order to get its lowest term

=> $\frac{24}{320} = \frac{24 \div 8}{320 \div 8} = \frac{3}{40}$

Example 2 – Express each of the following rational numbers to the lowest form:

(i) $\frac{12}{16}$

Solution: Firstly, we will find: m = HCF of (12 and 16) as follows:

12 = 2 × 2 × 3

16 = 2 × 2 × 2 × 2

We can see that $2 \times 2 = 4$ is common factor between 12 and 16

Thus, m = HCF (12, 16) = 4

Since m ≠ 1, thus $\frac{12}{16}$ is not in its lowest form.

Therefore, we must divide each term by m = 4 in order to get its lowest term

=> $\frac{12}{16} = \frac{12 \div 4}{16 \div 4} = \frac{3}{4}$

(ii) $\frac{-60}{72}$

Solution: Firstly, we will find: m = HCF of (60 and 72) as follows:

60 = 2 × 2 × 3 × 5

72 = 2 × 2 × 2 × 3 × 3

We can see that $2 \times 2 \times 3 = 12$ is common factor between 60 and 72

Thus, m = HCF (60, 72) = 12

Since m ≠ 1, thus $\frac{-60}{72}$ is not in its lowest form.

Therefore, we must divide each term by m = 12 in order to get its lowest term

=> $\frac{-60}{72} = \frac{-60 \div 12}{72 \div 12} = \frac{-5}{6}$

(iii) $\frac{-24}{-36}$

Solution: Firstly, we will find: m = HCF of (24 and 36) as follows:

24 = 2 × 2 × 2 × 3

36 = 2 × 2 × 3 × 3

We can see that $2 \times 2 \times 3 = 12$ is common factor between 24 and 36

Thus, m = HCF (24, 36) = 12

Since m ≠ 1, thus $\frac{-24}{-36}$ is not in its lowest form.

Therefore, we must divide each term by m = 12 in order to get its lowest term

=> $\frac{-24}{-36} = \frac{-24 \div 12}{-36 \div 12} = \frac{-2}{-3}$

(iv) $\frac{91}{-364}$

Solution: Firstly, we will find: m = HCF of (91 and 364) as follows:

91 = 7 × 13

364 = 2 × 2 × 7 × 13

We can see that $7 \times 13 = 91$ is common factor between 91 and 364

Thus, m = HCF (91, 364) = 91

Since m ≠ 1, thus $\frac{91}{-364}$ is not in its lowest form.

Therefore, we must divide each term by m = 91 in order to get its lowest term

=> $\frac{91}{-364} = \frac{91 \div 91}{-364 \div 91} = \frac{1}{-4}$

Example 3 – Fill in the blanks:

$$\frac{\mathbf{90}}{\mathbf{165}} = \frac{\mathbf{-6}}{\mathbf{-}} = \frac{\mathbf{-}}{\mathbf{-55}}$$

Solution: We will make them equivalent rational numbers as follows:

Consider, $\frac{90}{165}$

We will find: m = HCF of (90 and 165) as follows:

90 = 2 × 3 × 3 × 5

165 = 3 × 5 × 11

We can see that $3 \times 5 = 15$ is common factor between 90 and 165

Thus, m = HCF (90, 165) = 15

Since m ≠ 1, thus $\frac{90}{165}$ is not in its lowest form.

Therefore, we must divide each term by m = 15 in order to get its lowest term

=> $\frac{90}{165} = \frac{90 \div 15}{165 \div 15} = \frac{6}{11}$

Now, we multiply each term by (-1), $\frac{6}{11} = \frac{6 \times (-1)}{11 \times (-1)} = \frac{-6}{-11}$

Now, in order to make denominator as -55, we will multiply each term by $\frac{-55}{-11} = 5$. Thus we get $\frac{-6}{-11} = \frac{-6 \times 5}{-11 \times 5} = \frac{-30}{-55}$

Therefore, we have, $\frac{90}{165} = \frac{-6}{-11} = \frac{-30}{-55}$

Exercise 4.3

Question 1 – Determine whether the following rational numbers are in the lowest form or not:

(i) $\frac{\mathbf{65}}{\mathbf{84}}$

Solution: Firstly, we will find: m = HCF of (65 and 84) as follows:

65 = 5×13

84 = 2×2× 3×7

We can see that there is no common factor between 65 and 84

Thus, m = HCF (17, 79) = 1

Therefore, $\frac{65}{84}$ is in its lowest form.

(ii) $\frac{-15}{32}$

Solution: Firstly, we will find: m = HCF of (15 and 32) as follows:

15 = 5×3

32 = 2×2× 2×2 ×2

We can see that there is no common factor between 15 and 32

Thus, m = HCF (15, 32) = 1

Therefore, $\frac{-15}{32}$ is in its lowest form.

(iii) $\frac{24}{128}$

Solution: Firstly, we will find: m = HCF of (24 and 128) as follows:

24 = 2 × 2 × 2 × 3

128 = 2 × 2 × 2 × 2 × 2 × 2× 2

We can see that $2 \times 2 \times 2 = 8$ is common factor between 24 and 128

Thus, m = HCF (24, 128) = 8

Since m ≠ 1, thus $\frac{24}{128}$ is not in its lowest form.

Therefore, we must divide each term by m = 8 in order to get its lowest term

=> $\frac{24}{128} = \frac{24 \div 8}{128 \div 8} = \frac{3}{16}$

(iv) $\frac{-56}{-32}$

Solution: Firstly, we will find: m = HCF of (56 and 32) as follows:

56 = 2 × 2 × 2 × 7

32 = 2 × 2 × 2 × 2 × 2

We can see that $2 \times 2 \times 2 = 8$ is common factor between 56 and 32

Thus, m = HCF (56, 32) = 8

Since m ≠ 1, thus $\frac{-56}{-32}$ is not in its lowest form.

Therefore, we must divide each term by m = 8 in order to get its lowest term

=> $\frac{-56}{-32} = \frac{-56 \div 8}{-32 \div 8} = \frac{-7}{-4}$

Question 2 – Express each of the following rational numbers to the lowest form:

(i) $\mathbf{\frac{4}{22}}$

Solution: Firstly, we will find: m = HCF of (4 and 22) as follows:

4 = 2 × 2

22 = 2 × 11

We can see that 2 is common factor between 4 and 22

Thus, m = HCF (4, 22) = 2

Since m ≠ 1, thus $\frac{4}{22}$ is not in its lowest form.

Therefore, we must divide each term by m = 2 in order to get its lowest term

=> $\frac{4}{22} = \frac{4 \div 2}{22 \div 2} = \frac{2}{11}$

(ii) $\mathbf{\frac{-36}{180}}$

Solution: Firstly, we will find: m = HCF of (36 and 180) as follows:

36 = 2 × 2 × 3 × 3

180 = 2 × 2 × 3 × 3 × 5

We can see that $2 \times 2 \times 3 \times 3 = 36$ is common factor between 36 and 180

Thus, m = HCF (36, 180) = 36

Since m ≠ 1, thus $\frac{-36}{180}$ is not in its lowest form.

Therefore, we must divide each term by m = 36 in order to get its lowest term

=> $\frac{-36}{180} = \frac{-36 \div 36}{180 \div 36} = \frac{-1}{5}$

(iii) $\frac{\mathbf{132}}{\mathbf{-428}}$

Solution: Firstly, we will find: m = HCF of (132 and 428) as follows:

132 = 2 × 2 × 3 × 11

428 = 2 × 2 × 107

We can see that $2 \times 2 = 4$ is common factor between 132 and 428

Thus, m = HCF (132, 428) = 4

Since m ≠ 1, thus $\frac{132}{428}$ is not in its lowest form.

Therefore, we must divide each term by m = 4 in order to get its lowest term

=> $\frac{132}{-428} = \frac{132 \div 4}{-428 \div 4} = \frac{33}{-107}$

(iv) $\frac{\mathbf{-32}}{\mathbf{-56}}$

Solution: Firstly, we will find: m = HCF of (32 and 56) as follows:

32 = 2 × 2 × 2 × 2 × 2

56 = 2 × 2 × 2 × 7

We can see that $2 \times 2 \times 2 = 8$ is common factor between 56 and 32

Thus, m = HCF (32, 56) = 8

Since m ≠ 1, thus $\frac{-32}{-56}$ is not in its lowest form.

Therefore, we must divide each term by m = 8 in order to get its lowest term

=> $\frac{-32}{-56} = \frac{-32 \div 8}{-56 \div 8} = \frac{-4}{-7} = \frac{4}{7}$

Question 3 – Fill in the blanks:

(i) $\frac{\mathbf{-5}}{\mathbf{7}} = \frac{\mathbf{-}}{\mathbf{35}} = \frac{\mathbf{-}}{\mathbf{49}}$

Solution: We will make them equivalent rational numbers as follows:

Consider, $\frac{-5}{7}$

Now, in order to make denominator as 35, we must multiply each term by $\frac{35}{7} = 5$. Thus, we get

=> $\frac{-5}{7} = \frac{-5\times5}{7\times5} = \frac{-25}{35}$

Also, in order to make denominator as 49, we must multiply each term by $\frac{49}{7} = 7$. Thus, we get

=> $\frac{-5}{7} = \frac{-5\times7}{7\times7} = \frac{-35}{49}$

Therefore, we have, $\frac{-5}{7} = \frac{-25}{35} = \frac{-35}{49}$

(ii) $\boldsymbol{\frac{-4}{-9} = \frac{-}{18} = \frac{12}{-}}$

Solution: We will make them equivalent rational numbers as follows:

Consider, $\frac{-4}{-9}$

Now, in order to make denominator as 18, we must multiply each term by $\frac{18}{-9} = -2$. Thus, we get

=> $\frac{-4}{-9} = \frac{-4\times(-2)}{-9\times(-2)} = \frac{8}{18}$

Also, in order to make numerator as 12, we must multiply each term by $\frac{12}{-4} = -3$. Thus, we get

=> $\frac{-4}{-9} = \frac{-4\times(-3)}{-9\times(-3)} = \frac{12}{27}$

Therefore, we have, $\frac{-4}{-9} = \frac{8}{18} = \frac{12}{27}$

(iii) $\boldsymbol{\frac{6}{-13} = \frac{-12}{-} = \frac{24}{-}}$

Solution: We will make them equivalent rational numbers as follows:

Consider, $\frac{6}{-13}$

Now, in order to make numerator as -12, we must multiply each term by $\frac{-12}{6} = -2$. Thus, we get

=> $\frac{6}{-13} = \frac{6\times(-2)}{-13\times(-2)} = \frac{-12}{26}$

Also, in order to make numerator as 24, we must multiply each term by $\frac{24}{6} = 4$. Thus, we get

=> $\frac{6}{-13} = \frac{6\times(4)}{-13\times(4)} = \frac{24}{-52}$

Therefore, we have, $\frac{6}{-13} = \frac{-12}{26} = \frac{24}{-52}$

(iv) $\mathbf{\frac{-6}{-} = \frac{3}{11} = \frac{-}{-55}}$

Solution: We will make them equivalent rational numbers as follows:

Consider, $\frac{3}{11}$

Now, in order to make numerator as -6, we must multiply each term by$\frac{-6}{3} = -2$. Thus, we get

=> $\frac{3}{11} = \frac{3\times(-2)}{11\times(-2)} = \frac{-6}{-22}$

Also, in order to make denominator as -55, we must multiply each term by$\frac{-55}{11} = -5$. Thus, we get

=> $\frac{3}{11} = \frac{3\times(-5)}{11\times(-5)} = \frac{-15}{-55}$

Therefore, we have, $\frac{-6}{-22} = \frac{3}{11} = \frac{-15}{-55}$

Standard form of a Rational Number

Suppose $\frac{p}{q}$ be any rational number, then it is said to be in standard form if q is non-negative and p and q have no common factor other than 1

Examples

Example 1 – Express each of the following rational numbers in the standard form:

(i) $\mathbf{\frac{-8}{28}}$

Solution: We observe that denominator = 28 which is positive. Now, we convert $\frac{-8}{28}$ into its standard form as follows:

Firstly, we will find: m = HCF of (8 and 28)

8 = 2 × 2 × 2

28 = 2 × 2 × 7

We can see that $2 \times 2 = 4$ is common factor between 8 and 28

Thus, m = HCF (8, 28) = 4

Since m ≠ 1, thus $\frac{-8}{28}$ is not in its standard form

Therefore, we must divide each term by m = 4 in order to get its standard form

=> $\frac{-8}{28} = \frac{-8 \div 4}{28 \div 4} = \frac{-2}{7}$

Thus $\frac{-2}{7}$ is the standard form of $\frac{-8}{28}$

(ii) $\mathbf{\frac{-12}{-30}}$

Solution: We observe that denominator = -30 which is negative. Firstly, we will make denominator positive by multiplying each term by (-1).

$$\frac{-12}{-30} = \frac{-12 \times (-1)}{-30 \times (-1)} = \frac{12}{30}$$

Now, we convert $\frac{12}{30}$ into its standard form as follows:

Firstly, we will find: m = HCF of (12 and 30)

12 = 2 × 2 × 3

30 = 2 × 3 × 5

We can see that $2 \times 3 = 6$ is common factor between 12 and 30

Thus, m = HCF (12, 30) = 6

Since m ≠ 1, thus $\frac{12}{30}$ is not in its standard form

Therefore, we must divide each term by m = 6 in order to get its standard form

=> $\frac{12}{30} = \frac{12 \div 6}{30 \div 6} = \frac{2}{5}$

Thus $\frac{2}{5}$ is the standard form of $\frac{-12}{-30}$

(iii) $\frac{14}{-49}$

Solution: We observe that denominator = -49 which is negative. Firstly, we will make denominator positive by multiplying each term by (-1).

$$\frac{14}{-49} = \frac{14 \times (-1)}{-49 \times (-1)} = \frac{-14}{49}$$

Now, we convert $\frac{-14}{49}$ into its standard form as follows:

Firstly, we will find: m = HCF of (14 and 49)

14 = 2 × 7

49 = 7 × 7

We can see that 7 is common factor between 14 and 49

Thus, m = HCF (14, 49) = 7

Since m ≠ 1, thus $\frac{-14}{49}$ is not in its standard form

Therefore, we must divide each term by m = 7 in order to get its standard form

=> $\frac{-14}{49} = \frac{-14 \div 7}{49 \div 7} = \frac{-2}{7}$

Thus $\frac{-2}{7}$ is the standard form of $\frac{14}{-49}$

(iv) $\frac{-16}{-56}$

Solution: We observe that denominator = -56 which is negative. Firstly, we will make denominator positive by multiplying each term by (-1).

$$\frac{-16}{-56} = \frac{-16 \times (-1)}{-56 \times (-1)} = \frac{16}{56}$$

Now, we convert $\frac{16}{56}$ into its standard form as follows:

Firstly, we will find: m = HCF of (16 and 56)

16 = 2 × 2 × 2 × 2

56 = 2 × 2 × 2 × 7

We can see that $2 \times 2 \times 2 = 8$ is common factor between 16 and 56

Thus, m = HCF (16, 56) = 8

Since m ≠ 1, thus $\frac{16}{56}$ is not in its standard form

Therefore, we must divide each term by m = 8 in order to get its standard form

=> $\frac{16}{56} = \frac{16 \div 8}{56 \div 8} = \frac{2}{7}$

Thus $\frac{2}{7}$ is the standard form of $\frac{-16}{-56}$

Example 2 – Express each one of the following rational numbers in the standard form:

(i) $\frac{-247}{-228}$

Solution: We observe that denominator = -228 which is negative. Firstly, we will make denominator positive by multiplying each term by (-1).

$$\frac{-247}{-228} = \frac{-247 \times (-1)}{-228 \times (-1)} = \frac{247}{228}$$

Now, we convert $\frac{247}{228}$ into its standard form as follows:

Firstly, we will find: m = HCF of (247 and 228)

247 = 13 × 19

228 = 2 × 2 × 3 × 19

We can see that 19 is common factor between 247 and 228

Thus, m = HCF (247, 228) = 19

Since m ≠ 1, thus $\frac{247}{228}$ is not in its standard form

Therefore, we must divide each term by m = 19 in order to get its standard form

=> $\frac{247}{228} = \frac{247 \div 19}{228 \div 19} = \frac{13}{12}$

Thus $\frac{13}{12}$ is the standard form of $\frac{-247}{-228}$

(ii) $\frac{299}{-161}$

Solution: We observe that denominator = -161 which is negative. Firstly, we will make denominator positive by multiplying each term by (-1).

$$\frac{299}{-161} = \frac{299 \times (-1)}{-161 \times (-1)} = \frac{-299}{161}$$

Now, we convert $\frac{-299}{161}$ into its standard form as follows:

Firstly, we will find: m = HCF of (299 and 161)

299 = 13 × 23

161 = 23 × 7

We can see that 23 is common factor between 299 and 161

Thus, m = HCF (299, 161) = 23

Since m ≠ 1, thus $\frac{-299}{161}$ is not in its standard form

Therefore, we must divide each term by m = 23 in order to get its standard form

=> $\frac{-299}{161} = \frac{-299 \div 23}{161 \div 23} = \frac{-13}{7}$

Thus $\frac{-13}{7}$ is the standard form of $\frac{299}{-161}$

Exercise 4.4

Question 1 – Write each of the following rational numbers in the standard form:

(i) $\frac{2}{10}$

Solution: We observe that denominator = 10 which is positive. Now, we convert $\frac{2}{10}$ into its standard form as follows:

Firstly, we will find: m = HCF of (2 and 10)

2 = 2 × 1

10 = 2 × 5 × 1

We can see that $2 \times 1 = 2$ is common factor between 2 and 10

Thus, m = HCF (2, 10) = 2

Since m ≠ 1, thus $\frac{2}{10}$ is not in its standard form

Therefore, we must divide each term by m = 2 in order to get its standard form

=> $\frac{2}{10} = \frac{2 \div 2}{10 \div 2} = \frac{1}{5}$

Thus $\frac{1}{5}$ is the standard form of $\frac{2}{10}$

(ii) $\frac{-8}{36}$

Solution: We observe that denominator = 36 which is positive. Now, we convert $\frac{-8}{36}$ into its standard form as follows:

Firstly, we will find: m = HCF of (8 and 36)

8 = 2 × 2 × 2

36 = 2 × 2 × 2 × 3

We can see that $2 \times 2 = 4$ is common factor between 8 and 36

Thus, m = HCF (8, 36) = 4

Since m ≠ 1, thus $\frac{-8}{36}$ is not in its standard form

Therefore, we must divide each term by m = 4 in order to get its standard form

=> $\frac{-8}{36} = \frac{-8 \div 4}{36 \div 4} = \frac{-2}{9}$

Thus $\frac{-2}{9}$ is the standard form of $\frac{-8}{36}$

(iii) $\frac{4}{-16}$

Solution: We observe that denominator = -16 which is negative. Firstly, we will make denominator positive by multiplying each term by (-1).

$$\frac{4}{-16} = \frac{4 \times (-1)}{-16 \times (-1)} = \frac{-4}{16}$$

Now, we convert $\frac{-4}{16}$ into its standard form as follows:

Firstly, we will find: m = HCF of (4 and 16)

4 = 2 × 2

16 = 2 × 2 ×2 × 2

We can see that $2 \times 2 = 4$ is common factor between 4 and 16

Thus, m = HCF (4, 16) = 4

Since m ≠ 1, thus $\frac{-4}{16}$ is not in its standard form

Therefore, we must divide each term by m = 4 in order to get its standard form

=> $\frac{-4}{16} = \frac{-4 \div 4}{16 \div 4} = \frac{-1}{4}$

Thus $\frac{-1}{4}$ is the standard form of $\frac{4}{-16}$

(iv) $\frac{-15}{-35}$

Solution: We observe that denominator = -35 which is negative. Firstly, we will make denominator positive by multiplying each term by (-1).

$$\frac{-15}{-35} = \frac{-15 \times (-1)}{-35 \times (-1)} = \frac{15}{35}$$

Now, we convert $\frac{15}{35}$ into its standard form as follows:

Firstly, we will find: m = HCF of (15 and 35)

15 = 3 × 5

35 = 5 × 7

We can see that 5 is common factor between 15 and 35

Thus, m = HCF (15, 35) = 5

Since m ≠ 1, thus $\frac{15}{35}$ is not in its standard form

Therefore, we must divide each term by m = 5 in order to get its standard form

=> $\frac{15}{35} = \frac{15 \div 5}{35 \div 5} = \frac{3}{7}$

Thus $\frac{3}{7}$ is the standard form of $\frac{-15}{-35}$

(v) $\frac{299}{-161}$

Solution: We observe that denominator = -161 which is negative. Firstly, we will make denominator positive by multiplying each term by (-1).

$$\frac{299}{-161} = \frac{299 \times (-1)}{-161 \times (-1)} = \frac{-299}{161}$$

Now, we convert $\frac{-299}{161}$ into its standard form as follows:

Firstly, we will find: m = HCF of (299 and 161)

299 = 13 × 23

161 = 7 × 23

We can see that 23 is common factor between 299 and 161

Thus, m = HCF (299, 161) = 23

Since m ≠ 1, thus $\frac{-299}{161}$ is not in its standard form

Therefore, we must divide each term by m = 23 in order to get its standard form

=> $\frac{-299}{161} = \frac{-299 \div 23}{161 \div 23} = \frac{-13}{7}$

Thus $\frac{-13}{7}$ is the standard form of $\frac{299}{-161}$

(vi) $\frac{-63}{-210}$

Solution: We observe that denominator = -210 which is negative. Firstly, we will make denominator positive by multiplying each term by (-1).

$$\frac{-63}{-210} = \frac{-63 \times (-1)}{-210 \times (-1)} = \frac{63}{210}$$

Now, we convert $\frac{63}{210}$ into its standard form as follows:

Firstly, we will find: m = HCF of (63 and 210)

63 = 3 × 3 × 7

210 = 3 × 7 ×2 × 5

We can see that $3 \times 7 = 21$ is common factor between 63 and 210

Thus, m = HCF (63, 210) = 21

Since m ≠ 1, thus $\frac{63}{210}$ is not in its standard form

Therefore, we must divide each term by m = 21 in order to get its standard form

=> $\frac{63}{210} = \frac{63 \div 21}{210 \div 21} = \frac{3}{10}$

Thus $\frac{3}{10}$ is the standard form of $\frac{-63}{-210}$

(vii) $\frac{\mathbf{68}}{\mathbf{-119}}$

Solution: We observe that denominator = -119 which is negative. Firstly, we will make denominator positive by multiplying each term by (-1).

$$\frac{68}{-119} = \frac{68 \times (-1)}{-119 \times (-1)} = \frac{-68}{119}$$

Now, we convert $\frac{-68}{119}$ into its standard form as follows:

Firstly, we will find: m = HCF of (68 and 119)

68 = 2 × 2 ×17

119 = 7 × 17

We can see that 17 is common factor between 68 and 119

Thus, m = HCF (68, 119) = 17

Since m ≠ 1, thus $\frac{-68}{119}$ is not in its standard form

Therefore, we must divide each term by m = 17 in order to get its standard form

=> $\frac{-68}{119} = \frac{-68 \div 17}{119 \div 17} = \frac{-4}{7}$

Thus $\frac{-4}{7}$ is the standard form of $\frac{68}{-119}$

(viii) $\frac{\mathbf{-195}}{\mathbf{275}}$

Solution: We observe that denominator = 275 which is positive. Now, we convert $\frac{-195}{275}$ into its standard form as follows:

Firstly, we will find: m = HCF of (195 and 275)

195 = 5 × 3 × 13

275 = 5 × 5 × 11

We can see that 5 is common factor between 195 and 275

Thus, m = HCF (195, 275) = 5

Since m ≠ 1, thus $\frac{-195}{275}$ is not in its standard form

Therefore, we must divide each term by m = 5 in order to get its standard form

=> $\frac{-195}{275} = \frac{-195 \div 5}{275 \div 5} = \frac{-39}{55}$

Thus $\frac{-39}{55}$ is the standard form of $\frac{-195}{275}$

Equality of Rational Numbers

Method 1: In the first method, we will test the equality of rational numbers by converting them into standard form. If they have the same standard form, then they are equal.

Method 2: In the second method, we will multiply the numerator and denominator of first number by denominator of second number and multiply the numerator and denominator of second number by denominator of first number. If their numerators are equal then the given rational numbers are equal.

Method 3: For testing the equality of rational numbers, we will use the following result:

$$\frac{a}{b} = \frac{c}{d} <=> a \times d = c \times b$$

Examples

Example 1 – Are the rational numbers $\frac{8}{-12}$ *and* $\frac{-50}{75}$ equal?

Solution: We will first convert them into standard form:

Consider, $\frac{8}{-12} = \frac{8 \times (-1)}{-12 \times (-1)} = \frac{-8}{12}$

8 = 2 × 2 × 2 and

12 = 2 × 2 × 3

We can see that 2 × 2 = 4 is common factor between 8 and 12. So, in order to make it in standard form, we must divide each term by 4

$\frac{-8}{12} = \frac{-8 \div 4}{12 \div 4} = \frac{-2}{3}$

Now, consider $\frac{-50}{75}$

50 = 2 × 5 × 5 and

75 = 3 × 5 × 5

We can see that 5 × 5 = 25 is common factor between 50 and 75. So, in order to make it in standard form, we must divide each term by 25

$$\frac{-50}{75} = \frac{-50 \div 25}{75 \div 25} = \frac{-2}{3}$$

Now, we can see that given rational numbers has the same standard form$\frac{-2}{3}$.

Therefore, $\frac{8}{-12} = \frac{-50}{75}$

Example 2 – Are the rational numbers $\frac{-8}{28}$ *and* $\frac{28}{-49}$ equal?

Solution: We will first convert them into standard form:

Consider, $\frac{-8}{28}$

8 = 2 × 2 × 2 and

28 = 2 × 2 × 7

We can see that 2 × 2 = 4 is common factor between 8 and 28. So, in order to make it in standard form, we must divide each term by 4

$$\frac{-8}{28} = \frac{-8 \div 4}{28 \div 4} = \frac{-2}{7}$$

Now, consider $\frac{28}{-49} = \frac{28 \times (-1)}{-49 \times (-1)} = \frac{-28}{49}$

28 = 2 × 2 × 7 and

49 = 7 × 7

We can see that 7 is common factor between 28 and 49. So, in order to make it in standard form, we must divide each term by 7

$$\frac{-28}{49} = \frac{-28 \div 7}{49 \div 7} = \frac{-4}{7}$$

Now, we can see that given rational numbers does not have same standard form

Therefore, $\frac{-8}{28} \neq \frac{28}{-49}$

Example 3 – Are the rational numbers $\frac{-4}{6}$ *and* $\frac{16}{-24}$ equal?

Solution: Using method 2, we will test the equality of rational numbers:

Consider, $\frac{-4}{6}$

We will multiply each term by denominator of second number which is -24, we get

$$\frac{-4}{6} = \frac{-4 \times (-24)}{6 \times (-24)} = \frac{96}{-144}$$

Now consider, $\frac{16}{-24}$

We will multiply each term by denominator of first number which is 6, we get

$$\frac{16}{-24} = \frac{16 \times (6)}{-24 \times (6)} = \frac{96}{-144}$$

We can see that both the numerators are equal that is 96

Therefore, $\frac{-4}{6} = \frac{16}{-24}$

Example 4 – Show that the rational numbers $\frac{-15}{35}$ *and* $\frac{4}{-6}$ are not equal.

Solution: Using method 2, we will test the equality of rational numbers:

Consider, $\frac{-15}{35}$

We will multiply each term by denominator of second number which is -6, we get

$$\frac{-15}{35} = \frac{-15 \times (-6)}{35 \times (-6)} = \frac{90}{-210}$$

Now consider, $\frac{4}{-6}$

We will multiply each term by denominator of first number which is 35, we get

$$\frac{4}{-6} = \frac{4 \times (35)}{-6 \times (35)} = \frac{140}{-210}$$

We can see that both the numerators are not equal.

Therefore, $\frac{-15}{35} \neq \frac{4}{-6}$

Example 5 – Which of the following pairs of rational numbers are equal?

(i) $\boldsymbol{\frac{-7}{21}\ and\ \frac{3}{-9}}$

Solution: We will use the below result in order to test the equality of rational numbers:

$$\frac{a}{b} = \frac{c}{d} <=> a \times d = c \times b$$

Here, $a \times d = (-7) \times (-9) = 63$ and

$c \times b = (3) \times (21) = 63$

We can see that $a \times d = c \times b = 63$

Therefore, $\frac{-7}{21} = \frac{3}{-9}$

(ii) $\boldsymbol{\frac{-8}{-14}\ and\ \frac{13}{21}}$

Solution: We will use the below result in order to test the equality of rational numbers:

$$\frac{a}{b} = \frac{c}{d} <=> a \times d = c \times b$$

Here, $a \times d = (-8) \times (21) = -168$ and

$c \times b = (13) \times (-14) = -182$

We can see that $a \times d \neq c \times b$

Therefore, $\frac{-8}{-14} \neq \frac{13}{21}$

Example 6 – If $\boldsymbol{\frac{-5}{7} = \frac{x}{28}}$**, find the value of x.**

Solution: We know that: $\frac{a}{b} = \frac{c}{d} <=> a \times d = c \times b$

=> $\frac{-5}{7} = \frac{x}{28}$

=> $(-5) \times (28) = 7x$

=> $-140 = 7x$

=> $x = \frac{-140}{7} = -20$

Example 7 – Fill in the blank: $\frac{-3}{8} = \frac{-}{48}$

Solution: Let the blank space be 'x'

Then we have, $\frac{-3}{8} = \frac{x}{48}$

Now, we know that: $\frac{a}{b} = \frac{c}{d} <=> a \times d = c \times b$

=> $\frac{-3}{8} = \frac{x}{48}$

=> $(-3) \times (48) = 8x$

=> $-144 = 8x$

=> $x = \frac{-144}{8} = -18$

Exercise 4.5

Question 1 – Which of the following rational numbers are equal?

(i) $\frac{-9}{12}$ *and* $\frac{8}{-12}$

Solution: We will use the below result in order to test the equality of rational numbers:

$$\frac{a}{b} = \frac{c}{d} <=> a \times d = c \times b$$

Here, $a \times d = (-9) \times (-12) = 108$ and

$c \times b = (8) \times (12) = 96$

We can see that $a \times d \neq c \times b$

Therefore, $\frac{-9}{12} \neq \frac{8}{-12}$

(ii) $\frac{-16}{20}$ *and* $\frac{20}{-25}$

Solution: We will use the below result in order to test the equality of rational numbers:

$$\frac{a}{b} = \frac{c}{d} <=> a \times d = c \times b$$

Here, $a \times d = (-16) \times (-25) = 400$ and

$c \times b = (20) \times (20) = 400$

We can see that $a \times d = c \times b = 400$

Therefore, $\frac{-16}{20} = \frac{20}{-25}$

(iii) $\boldsymbol{\frac{-7}{21}\ and\ \frac{3}{-9}}$

Solution: We will use the below result in order to test the equality of rational numbers:

$$\frac{a}{b} = \frac{c}{d} <=> a \times d = c \times b$$

Here, $a \times d = (-7) \times (-9) = 63$ and

$c \times b = (21) \times (3) = 63$

We can see that $a \times d = c \times b = 63$

Therefore, $\frac{-7}{21} = \frac{3}{-9}$

(iv) $\boldsymbol{\frac{-8}{-14}\ and\ \frac{13}{21}}$

Solution: We will use the below result in order to test the equality of rational numbers:

$$\frac{a}{b} = \frac{c}{d} <=> a \times d = c \times b$$

Here, $a \times d = (-8) \times (-21) = 168$ and

$c \times b = (-14) \times (13) = -182$

We can see that $a \times d \neq c \times b$

Therefore, $\frac{-8}{-14} \neq \frac{13}{21}$

Question 2 – If each of the following pairs represents a pair of equivalent rational numbers, find the values of x:

(i) $\boldsymbol{\frac{2}{3}\ and\ \frac{5}{x}}$

Solution: It is given that above pair represents equivalent rational number.

Thus, by using $\frac{a}{b} = \frac{c}{d} <=> a \times d = c \times b$

We have: $\frac{2}{3} = \frac{5}{x}$

=> $(2) \times (x) = (5) \times (3)$

=> $2x = 15$

=> $x = \frac{15}{2}$

(ii) $\boldsymbol{\frac{-3}{7}\ and\ \frac{x}{4}}$

Solution: It is given that above pair represents equivalent rational number.

Thus, by using $\frac{a}{b} = \frac{c}{d} <=> a \times d = c \times b$

We have: $\frac{-3}{7} = \frac{x}{4}$

=> $(-3) \times (4) = (7) \times (x)$

=> $-12 = 7x$

=> $x = \frac{-12}{7}$

(iii) $\boldsymbol{\frac{3}{5}\ and\ \frac{x}{-25}}$

Solution: It is given that above pair represents equivalent rational number.

Thus, by using $\frac{a}{b} = \frac{c}{d} <=> a \times d = c \times b$

We have: $\frac{3}{5} = \frac{x}{-25}$

=> $(3) \times (-25) = (5) \times (x)$

=> $-75 = 5x$

=> $x = \frac{-75}{5} = -15$

(iv) $\boldsymbol{\frac{13}{6}\ and\ \frac{-65}{x}}$

Solution: It is given that above pair represents equivalent rational number.

Thus, by using $\frac{a}{b} = \frac{c}{d} <=> a \times d = c \times b$

We have: $\frac{13}{6} = \frac{-65}{x}$

=> $(13) \times (x) = (-65) \times (6)$

=> $13x = -390$

=> $x = \frac{-390}{13} = -30$

Question 3 – In each of the following, fill in the blanks so as to make the statement true:

(i) A number which can be expressed in the form $\frac{p}{q}$, where p and q are integers and q is not equal to zero, is called a …..

Solution: Rational number

(ii) If the integers p and q have no common divisor other than 1 and q is positive, then the rational number $\frac{p}{q}$ is said to be in the ….

Solution: Standard form

(iii) Two rational numbers are said to be equal, if they have the same ….form

Solution: Standard

(iv) If m is a common divisor of a and b, then $\frac{a}{b} = \frac{a \div m}{\ldots\ldots}$

Solution: $\frac{a}{b} = \frac{a \div m}{b \div m}$

(v) If p and q are positive integers, then $\frac{p}{q}$ is a ….rational number and $\frac{p}{-q}$ is a ….rational number.

Solution: $\frac{p}{q}$ is a positive rational number and $\frac{p}{-q}$ is a negative rational number.

(vi) The standard form of -1 is ….

Solution: The standard form of -1 is $\frac{-1}{1}$

(vii) If $\frac{p}{q}$ is a rational number, then q cannot be ….

Solution: q cannot be zero

(viii) Two rational numbers with different numerators are equal, if their numerators are in the same ….as their denominators.

Solution: Ratio

Question 4 – In each of the following state if the statement is true (T) or false (F):

(i) The quotient of two integers is always an integer.

Solution: False

Reasoning: It is not always true that the quotient of two integers is always an integer. For example:$\frac{-2}{4} = \frac{-1}{2}$. Here, $-\frac{2}{4}$ is a rational number but $-\frac{1}{2}$ is not an integer.

(ii) Every integer is a rational number.

Solution: True

Reasoning: It is true that every integer is a rational number because all integers $\ldots. -3, -2, -1, 0, 1, 2, 3$ ....can be written in the form as $\ldots. -\frac{3}{1}, -\frac{2}{1}, \frac{0}{1}, \frac{1}{1}, \frac{2}{1}$ ....which all are rational numbers.

(iii) Every rational number is an integer.

Solution: False

Reasoning: It is not always true that every rational number is an integer. For example: $\frac{2}{5}$ is a rational number but it is not an integer.

(iv) Every fraction is a rational number.

Solution: True

Reasoning: It is true that every fraction is a rational number.

(v) Every rational number is a fraction.

Solution: False

Reasoning: It is not always true that every rational number is a fraction. For example: $-\frac{1}{2}$ is a rational number but it is not a fraction.

(vi) If $\frac{a}{b}$ is a rational number and m any integer, then $\frac{a}{b} = \frac{a \times m}{b \times m}$

Solution: False

Reasoning: It is false because there is a condition that m must be a non-zero integer.

(vii) Two rational numbers with different numerators cannot be equal.

Solution: False

Reasoning: Two rational numbers with different numerators can be equal when their numerators are in the same ratio as their denominators.

(viii) 8 can be written as a rational number with any integer as denominator.

Solution: False

Reasoning: 8 cannot be written as a rational number with any integer as denominator. It can only be written as $8 = \frac{8}{1}$

(ix) 8 can be written as a rational number with any integer as numerator.

Solution: False

Reasoning: 8 cannot be written as a rational number with any integer as numerator.

(x) $\frac{2}{3}$ is equal to $\frac{4}{6}$

Solution: True

Reasoning: We can write $\frac{2}{3} = \frac{2\times2}{3\times2} = \frac{4}{6}$, where m = 2 is a non-zero integer.

Representation of Rational numbers on the number line:

Let us understand this by examples:

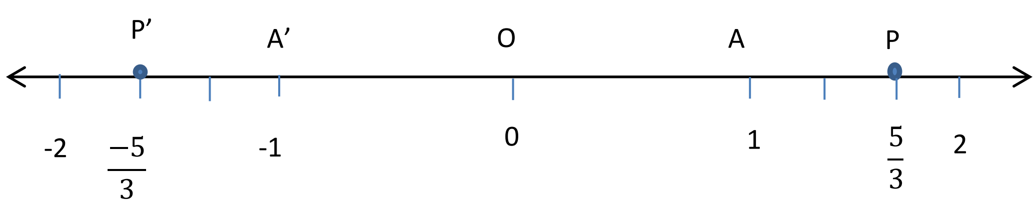
Example 1 – Represent $\frac{5}{3}$ *and* $\frac{-5}{3}$ on the number line.

Solution: We can write $\frac{5}{3} = 1\frac{2}{3}$

Firstly we draw a number line and mark point O on it representing zero.

Now, $\frac{5}{3}$ lies between 1 and 2 and $\frac{-5}{3}$ lies between -1 and -2.

Here, since denominator = 3. So we divide the distance between 1 and 2 into 3equal parts. Points P and P' represents $\frac{5}{3}$ and $-\frac{5}{3}$ respectively.



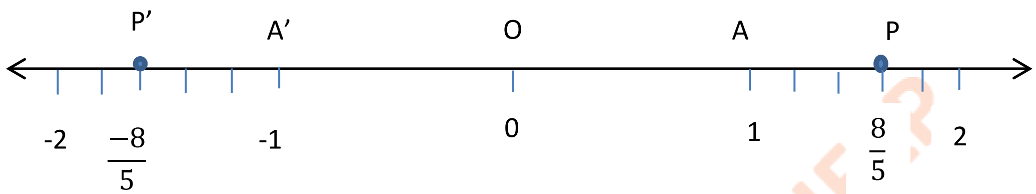
Example 2 – Represent $\frac{8}{5}$ *and* $-\frac{8}{5}$ on the number line.

Solution: We can write $\frac{8}{5} = 1\frac{3}{5}$

Firstly we draw a number line and mark point O on it representing zero.

Now, $\frac{8}{5}$ lies between 1 and 2 and $\frac{-8}{5}$ lies between -1 and -2.

Here, since denominator = 5. So we divide the distance between 1 and 2 into 5equal parts. Points P and P' represents $\frac{8}{5}$ and $-\frac{8}{5}$ respectively**.**



Comparison of rational Numbers: We can compare two or more rational numbers using a simple procedure as follows:

Step 1- Firstly; write each rational number with positive denominator.

Step2- Taking the LCM of these positive denominators.

Step 3- Express each rational number with this LCM as the common denominator.

Step 4- Start comparing the numerators of the rational numbers so obtained in step 3.

The number having the greater numerator is greater.

Examples

Example 1 – Which of the two rational numbers $\frac{3}{5}$ *and* $-\frac{2}{3}$ is greater?

Solution: It is clear that $\frac{3}{5}$ is a positive rational number and $-\frac{2}{3}$ is a negative rational number. Since, positive number is always greater than a negative number therefore, $\frac{3}{5} > -\frac{2}{3}$

Example 2 – Which of the two rational numbers $\frac{5}{7}$ *and* $\frac{3}{5}$ is greater?

Solution: Firstly, we will make the denominators of both rational numbers same by taking their LCM as follows:

Now, LCM of 7 and 5 is 35

Now, $\frac{5}{7} = \frac{5\times5}{7\times5} = \frac{25}{35}$ (Multiplying each term by 5)

$\frac{3}{5} = \frac{3\times7}{5\times7} = \frac{21}{35}$ (Multiplying each term by 7)

Clearly 25 > 21

Thus, $\frac{25}{35} > \frac{21}{35}$

Therefore, $\frac{5}{7} > \frac{3}{5}$

Example 3 – Which of the two rational numbers $\frac{-4}{9}$ *and* $\frac{5}{-12}$ is greater?

Solution: Firstly we will make the denominator positive in $\frac{5}{-12}$ as follows:

$\frac{5}{-12} = \frac{5\times(-1)}{-12\times(-1)} = \frac{-5}{12}$ (Multiplying each term by -1)

Now, we will make the denominators of both rational numbers same by taking their LCM as follows:

LCM of 9 and 12 is 36

Now, $\frac{-4}{9} = \frac{-4\times4}{9\times4} = \frac{-16}{36}$ (Multiplying each term by 4)

$\frac{-5}{12} = \frac{-5\times3}{12\times3} = \frac{-15}{36}$ (Multiplying each term by 3)

Clearly -15 > -16

Thus, $\frac{-15}{36} > \frac{-16}{36}$

Therefore, $\frac{-5}{12} > \frac{-4}{9}$ => $\frac{5}{-12} > \frac{-4}{9}$

Example 4 – Arrange the rational numbers $\frac{-7}{10}, \frac{5}{-8}, \frac{2}{-3}$ in ascending order:

Solution: Firstly we will make the denominator positive in all the rational numbers as follows:

One number = $\frac{-7}{10}$

Second number = $\frac{5}{-8} = \frac{5\times(-1)}{-8\times(-1)} = \frac{-5}{8}$

Third number = $\frac{2}{-3} = \frac{2\times(-1)}{-3\times(-1)} = \frac{-2}{3}$

Now, we will make the denominators of both rational numbers same by taking their LCM as follows:

LCM of 10, 8 and 3 is 120

Now, $\frac{-7}{10} = \frac{-7\times12}{10\times12} = \frac{-84}{120}$ (Multiplying each term by 12)

$\frac{-5}{8} = \frac{-5\times15}{8\times15} = \frac{-75}{120}$ (Multiplying each term by 15)

$\frac{-2}{3} = \frac{-2\times40}{3\times40} = \frac{-80}{120}$ (Multiplying each term by 40)

Clearly, $-84 < -80 < -75$

Thus, $\frac{-84}{120} < \frac{-80}{120} < \frac{-75}{120}$

Therefore, $\frac{-7}{10} < \frac{2}{-3} < \frac{5}{-8}$

Example 5 - Arrange the following rational numbers in descending order:

$\mathbf{\frac{4}{9}, \frac{-5}{6}, \frac{-7}{-12}, \frac{11}{-24}}$

Solution: Firstly we will make the denominator positive in all the rational numbers as follows:

One number = $\frac{4}{9}$

Second number = $\frac{-5}{6}$

Third number = $\frac{-7}{-12} = \frac{-7\times(-1)}{-12\times(-1)} = \frac{7}{12}$

Forth number = $\frac{11}{-24} = \frac{11\times(-1)}{-24\times(-1)} = \frac{-11}{24}$

Now, we will make the denominators of both rational numbers same by taking their LCM as follows:

LCM of 9, 6, 12 and 24 is 72

Now, $\frac{4}{9} = \frac{4\times8}{9\times8} = \frac{32}{72}$ (Multiplying each term by 8)

$\frac{-5}{6} = \frac{-5\times12}{6\times12} = \frac{-60}{72}$ (Multiplying each term by 12)

$\frac{7}{12} = \frac{7\times6}{12\times6} = \frac{42}{72}$ (Multiplying each term by 6)

$\frac{-11}{24} = \frac{-11\times3}{24\times3} = \frac{-33}{72}$ (Multiplying each term by 3)

Clearly, $42 > 32 > -33 > -60$

Thus, $\frac{42}{72} < \frac{32}{72} < \frac{-33}{72} < \frac{-60}{72}$

Therefore, $\frac{-7}{-12} < \frac{4}{9} < \frac{11}{-24} > \frac{-5}{6}$

Exercise 4.6

Question 1 – Draw the number line and represent the following rational numbers on it:

(i) $\frac{2}{3}$

Solution**:** Firstly we draw a number line and mark point O on it representing zero.

Now, $\frac{2}{3}$ lies between 0 and 1

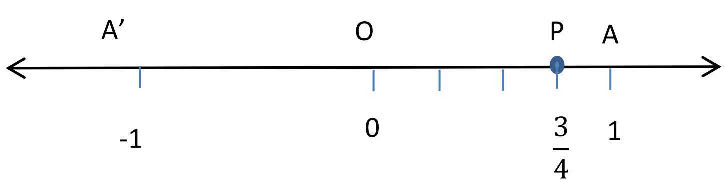
Here, since denominator = 3. So we divide the distance between 0 and 1 into 3equal parts. Points P represents $\frac{2}{3}$

(ii) $\frac{3}{4}$

Solution**:** Firstly we draw a number line and mark point O on it representing zero.

Now, $\frac{3}{4}$ lies between 0 and 1

Here, since denominator = 4. So we divide the distance between 0 and 1 into 4equal parts. Points P represents $\frac{3}{4}$

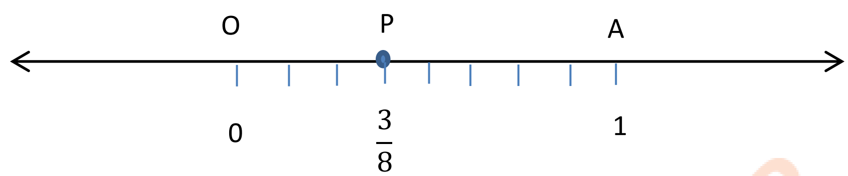


(iii) $\frac{3}{8}$

Solution**:** Firstly we draw a number line and mark point O on it representing zero.

Now, $\frac{3}{8}$ lies between 0 and 1

Here, since denominator = 8. So we divide the distance between 0 and 1 into 8 equal parts. Points P represents $\frac{3}{8}$

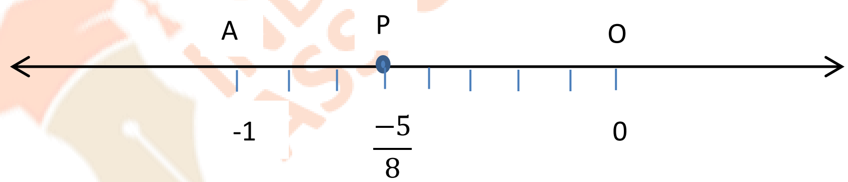


(iv) $\frac{-5}{8}$

Solution**:** Firstly we draw a number line and mark point O on it representing zero.

Now, $\frac{-5}{8}$ lies between 0 and -1

Here, since denominator = 8. So we divide the distance between 0 and -1 into 8 equal parts. Points P represents $\frac{-5}{8}$

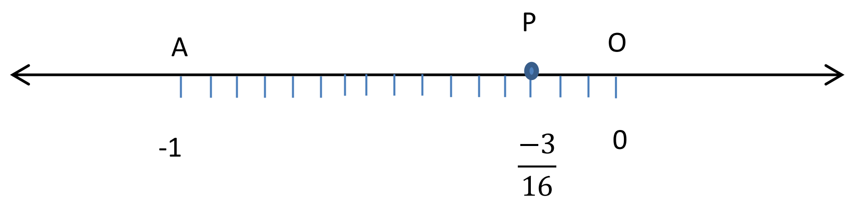


(v) $\frac{-3}{16}$

Solution**:** Firstly we draw a number line and mark point O on it representing zero.

Now, $\frac{-3}{16}$ lies between 0 and -1

Here, since denominator = 16. So we divide the distance between 0 and -1 into 16 equal parts. Points P represents $\frac{-5}{8}$



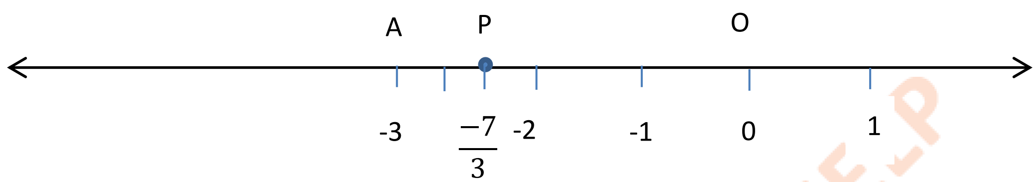
(vi) $\frac{-7}{3}$

Solution: We can write $\frac{-7}{3} = -2\frac{1}{3}$

Firstly we draw a number line and mark point O on it representing zero.

Now, $\frac{-7}{3}$ lies between -2 and -3

Here, since denominator = 3. So we divide the distance between -2 and -3 into 3equal parts. Points P represents $\frac{-7}{3}$



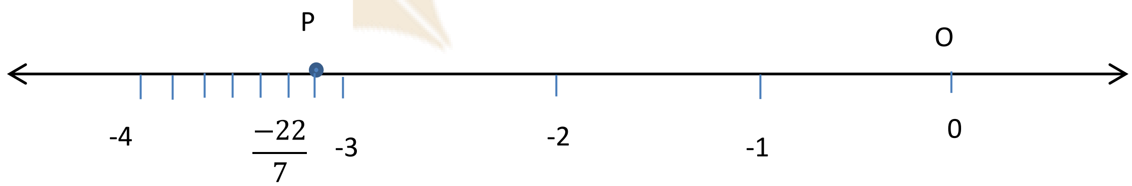
(vii) $\frac{22}{-7}$

Solution: We can write $\frac{22}{-7} = \frac{-22}{7} = -3\frac{1}{7}$

Firstly we draw a number line and mark point O on it representing zero.

Now, $\frac{-22}{7}$ lies between -3 and -4

Here, since denominator = 7. So we divide the distance between -3 and -4 into 7 equal parts. Points P represents $\frac{-22}{7}$



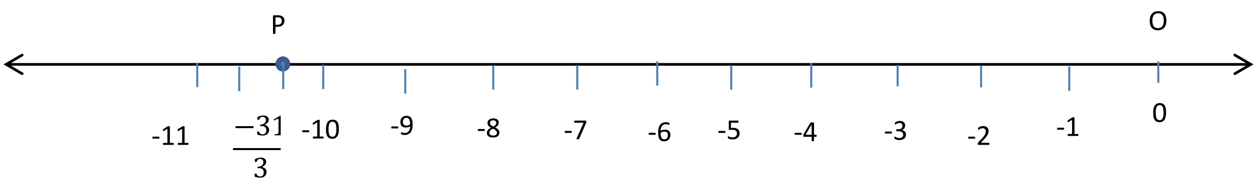
(viii) $\frac{-31}{3}$

Solution: We can write $\frac{-31}{3} = -10\frac{1}{3}$

Firstly we draw a number line and mark point O on it representing zero.

Now, $\frac{-31}{3}$ lies between -10 and -11

Here, since denominator = 3. So we divide the distance between -10 and -11 into 3 equal parts. Points P represents $\frac{-31}{3}$



Question 2 – Which of the two rational numbers in each of the following pairs of rational numbers is greater?

(i) $-\frac{3}{8}, 0$

Solution: It is clear that $\frac{-3}{8}$ is a negative rational number which is smaller than 0 since it comes on left side of zero on a real number line. Thus, $0 > \frac{-3}{8}$

(ii) $\frac{5}{2}, 0$

Solution: It is clear that $\frac{5}{8}$ is a positive rational number which is greater than 0 since it comes on right side of zero on a real number line. Thus, $\frac{5}{2} > 0$

(iii) $\frac{-4}{11}, \frac{3}{11}$

Solution: It is clear that $\frac{3}{11}$ is a positive rational number and $\frac{-4}{11}$ is a negative rational number. Since, positive number is always greater than a negative number therefore, $\frac{3}{11} > \frac{-4}{11}$

(iv) $\frac{-7}{12}, \frac{5}{-8}$

Solution: Firstly we will make the denominator positive in $\frac{5}{-8}$ as follows:

$\frac{5}{-8} = \frac{5\times(-1)}{-8\times(-1)} = \frac{-5}{8}$ (Multiplying each term by -1)

Now, we will make the denominators of both rational numbers same by taking their LCM as follows:

LCM of 12 and 8 is 24

Now, $\frac{-7}{12} = \frac{-7\times2}{12\times2} = \frac{-14}{24}$ (Multiplying each term by 2)

$\frac{-5}{8} = \frac{-5\times3}{8\times3} = \frac{-15}{24}$ (Multiplying each term by 3)

Clearly -14 > -15

Thus, $\frac{-14}{24} > \frac{-15}{24}$

Therefore, $\frac{-7}{12} > \frac{-5}{8}$ => $\frac{-7}{12} > \frac{5}{-8}$

(v) $\mathbf{\frac{4}{-9}, \frac{-3}{-7}}$

Solution: Firstly we will make the denominator positive in $\frac{4}{-9}$ and $\frac{-3}{-7}$as follows:

$\frac{4}{-9} = \frac{4\times(-1)}{-9\times(-1)} = \frac{-4}{9}$ (Multiplying each term by -1)

$\frac{-3}{-7} = \frac{-3\times(-1)}{-7\times(-1)} = \frac{3}{7}$ (Multiplying each term by -1)

Now, we will make the denominators of both rational numbers same by taking their LCM as follows:

LCM of 9 and 7 is 63

Now, $\frac{-4}{9} = \frac{-4\times7}{9\times7} = \frac{-28}{63}$ (Multiplying each term by 7)

$\frac{3}{7} = \frac{3\times9}{7\times9} = \frac{27}{63}$ (Multiplying each term by 9)

Clearly 27 > -28

Thus, $\frac{27}{63} > \frac{-28}{63}$

Therefore, $\frac{3}{7} > \frac{-4}{9}$ => $\frac{-3}{-7} > \frac{4}{-9}$

(vi) $\mathbf{\frac{-5}{8}, \frac{3}{-4}}$

Solution: Firstly we will make the denominator positive in $\frac{3}{-4}$ as follows:

$\frac{3}{-4} = \frac{3\times(-1)}{-4\times(-1)} = \frac{-3}{4}$ (Multiplying each term by -1)

Now, we will make the denominators of both rational numbers same by taking their LCM as follows:

LCM of 8 and 4 is 8

Now, $\frac{-5}{8} = \frac{-5\times1}{8\times1} = \frac{-5}{8}$ (Multiplying each term by 1)

$\frac{-3}{4} = \frac{-3\times2}{4\times2} = \frac{-6}{8}$ (Multiplying each term by 2)

Clearly -5 > -6

Thus, $\frac{-5}{8} > \frac{-6}{8}$

Therefore, $\frac{-5}{8} > \frac{-3}{4} \Rightarrow \frac{-5}{8} > \frac{3}{-4}$

(vii) $\mathbf{\frac{5}{9}, \frac{-3}{-8}}$

Solution: Firstly we will make the denominator positive in $\frac{-3}{-8}$ as follows:

$\frac{-3}{-8} = \frac{-3\times(-1)}{-8\times(-1)} = \frac{3}{8}$ (Multiplying each term by -1)

Now, we will make the denominators of both rational numbers same by taking their LCM as follows:

LCM of 9 and 8 is 72

Now, $\frac{5}{9} = \frac{5\times8}{9\times8} = \frac{40}{72}$ (Multiplying each term by 8)

$\frac{3}{8} = \frac{3\times9}{8\times9} = \frac{27}{72}$ (Multiplying each term by 9)

Clearly 40 > 27

Thus, $\frac{40}{72} > \frac{27}{40}$

Therefore, $\frac{5}{9} > \frac{3}{8} \Rightarrow \frac{5}{9} > \frac{-3}{-8}$

(viii) $\mathbf{\frac{5}{-8}, \frac{-7}{12}}$

Solution: Firstly we will make the denominator positive in $\frac{5}{-8}$ as follows:

$\frac{5}{-8} = \frac{5\times(-1)}{-8\times(-1)} = \frac{-5}{8}$ (Multiplying each term by -1)

Now, we will make the denominators of both rational numbers same by taking their LCM as follows:

LCM of 12 and 8 is 24

Now, $\frac{-7}{12} = \frac{-7\times2}{12\times2} = \frac{-14}{24}$ (Multiplying each term by 2)

$\frac{-5}{8} = \frac{-5\times3}{8\times3} = \frac{-15}{24}$ (Multiplying each term by 3)

Clearly -14 > -15

Thus, $\frac{-14}{24} > \frac{-15}{24}$

Therefore, $\frac{-7}{12} > \frac{-5}{8} => \frac{-7}{12} > \frac{5}{-8}$

Question 3 – Which of the two rational numbers in each of the following pairs of rational numbers is smaller?

(i) $\frac{-6}{-13}, \frac{7}{13}$

Solution: Firstly we will make the denominator positive in $\frac{-6}{-13}$ as follows:

$\frac{-6}{-13} = \frac{-6\times(-1)}{-13\times(-1)} = \frac{6}{13}$ (Multiplying each term by -1)

Here, we can see that denominators of both the rational numbers are equal that is 13.

Now, we will compare the numerators of both rational numbers to compare them.

Clearly, $6 < 7$

=> $\frac{6}{13} < \frac{7}{13}$

=> $\frac{-6}{-13} < \frac{7}{13}$

(ii) $\frac{16}{-5}, 3$

Solution: Firstly we will make the denominator positive in $\frac{16}{-5}$ as follows:

$\frac{16}{-5} = \frac{16\times(-1)}{-5\times(-1)} = \frac{-16}{5}$ (Multiplying each term by -1)

We can write $3 = \frac{3}{1}$

Now, we will make the denominators of both rational numbers same by taking their LCM as follows:

LCM of 5 and 1 is 5

Now, $\frac{-16}{5} = \frac{-16\times1}{5\times1} = \frac{-16}{5}$ (Multiplying each term by 1)

$\frac{3}{1} = \frac{3\times5}{1\times5} = \frac{15}{5}$ (Multiplying each term by 5)

Clearly, $-16 < 15$

Thus, $\frac{-16}{5} < \frac{15}{5}$

Therefore, $\frac{-16}{5} < \frac{3}{1} \Rightarrow \frac{16}{-5} < 3$

(iii) $\boldsymbol{\frac{-4}{3}, \frac{8}{-7}}$

Solution: Firstly we will make the denominator positive in $\frac{8}{-7}$ as follows:

$\frac{8}{-7} = \frac{8\times(-1)}{-7\times(-1)} = \frac{-8}{7}$ (Multiplying each term by -1)

Now, we will make the denominators of both rational numbers same by taking their LCM as follows:

LCM of 3 and 7 is 21

Now, $\frac{-4}{3} = \frac{-4\times7}{3\times7} = \frac{-28}{21}$ (Multiplying each term by 7)

$\frac{-8}{7} = \frac{-8\times3}{7\times3} = \frac{-24}{21}$ (Multiplying each term by 3)

Clearly, $-28 < -24$

Thus, $\frac{-28}{21} < \frac{-24}{21}$

Therefore, $\frac{-4}{3} < \frac{-8}{7} \Rightarrow \frac{-4}{3} < \frac{8}{-7}$

(iv) $\boldsymbol{\frac{-12}{5}, -3}$

Solution: We can write $-3 = \frac{-3}{1}$

Now, we will make the denominators of both rational numbers same by taking their LCM as follows:

LCM of 5 and 1 is 5

Now, $\frac{-12}{5} = \frac{-12\times1}{5\times1} = \frac{-12}{5}$ (Multiplying each term by 1)

$\frac{-3}{1} = \frac{-3\times5}{1\times5} = \frac{-15}{5}$ (Multiplying each term by 5)

Clearly, $-15 < -12$

Thus, $\frac{-15}{5} < \frac{-12}{5}$

Therefore, $\frac{-3}{1} < \frac{-12}{5}$ => $-3 < \frac{-12}{5}$

Question 4 – Fill in the blanks by the correct symbol of >, =, or <:

(i) $\frac{-6}{7} \ldots\ldots \frac{7}{13}$

Solution: Firstly, we will make the denominators of both rational numbers same by taking their LCM as follows:

LCM of 7 and 13 is 91

Now, $\frac{-6}{7} = \frac{-6\times13}{7\times13} = \frac{-78}{91}$ (Multiplying each term by 13)

$\frac{7}{13} = \frac{7\times7}{13\times7} = \frac{49}{91}$ (Multiplying each term by 7)

Clearly, $-78 < 49$

Thus, $\frac{-78}{91} < \frac{49}{91}$

Therefore, $\frac{-6}{7} < \frac{7}{13}$

(ii) $\frac{-3}{5} \ldots\ldots \frac{-5}{6}$

Solution: Firstly, we will make the denominators of both rational numbers same by taking their LCM as follows:

LCM of 5 and 6 is 30

Now, $\frac{-3}{5} = \frac{-3\times6}{5\times6} = \frac{-18}{30}$ (Multiplying each term by 6)

$\frac{-5}{6} = \frac{-5\times5}{6\times5} = \frac{-25}{30}$ (Multiplying each term by 5)

Clearly, $-25 < -18$

Thus, $\frac{-25}{30} < \frac{-18}{30}$

Therefore, $\frac{-3}{5} > \frac{-5}{6}$

(iii) $\frac{-2}{3} \ldots\ldots \frac{5}{-8}$

Solution: Firstly we will make the denominator positive in $\frac{5}{-8}$ as follows:

$\frac{5}{-8} = \frac{5\times(-1)}{-8\times(-1)} = \frac{-5}{8}$ (Multiplying each term by -1)

Now, we will make the denominators of both rational numbers same by taking their LCM as follows:

LCM of 3 and 8 is 24

Now, $\frac{-2}{3} = \frac{-2\times8}{3\times8} = \frac{-16}{24}$ (Multiplying each term by 8)

$\frac{-5}{8} = \frac{-5\times3}{8\times3} = \frac{-15}{24}$ (Multiplying each term by 3)

Clearly, $-16 < -15$

Thus, $\frac{-16}{24} < \frac{-15}{24}$

Therefore, $\frac{-2}{3} < \frac{5}{-8}$

(iv) 0 $\ldots\ldots \frac{-2}{5}$

Solution: We can write $0 = \frac{0}{1}$

Now $\frac{-2}{5}$ is a negative rational number which is smaller than zero since it comes on the left side of zero on the real number line.

Therefore, $0 > \frac{-2}{5}$

Question 5 – Arrange the following rational numbers in ascending order:

(i) $\frac{3}{5}, \frac{-17}{-30}, \frac{8}{-15}, \frac{-7}{10}$

Solution: Firstly we will make the denominator positive in all the rational numbers as follows:

One number = $\frac{3}{5}$

Second number = $\frac{-17}{-30} = \frac{-17\times(-1)}{-30\times(-1)} = \frac{17}{30}$

Third number = $\frac{8}{-15} = \frac{8\times(-1)}{-15\times(-1)} = \frac{-8}{15}$

Forth number = $\frac{-7}{10}$

Now, we will make the denominators of both rational numbers same by taking their LCM as follows:

LCM of 5, 30, 15 and 10 is 30

Now, $\frac{3}{5} = \frac{3\times6}{5\times6} = \frac{18}{30}$ (Multiplying each term by 6)

$\frac{17}{30} = \frac{17\times1}{30\times1} = \frac{17}{30}$ (Multiplying each term by 1)

$\frac{-8}{15} = \frac{-8\times2}{15\times2} = \frac{-16}{30}$ (Multiplying each term by 2)

$\frac{-7}{10} = \frac{-7\times3}{10\times3} = \frac{-21}{30}$ (Multiplying each term by 3)

Clearly, $-21 < -16 < 17 < 18$

Thus, $\frac{-21}{30} < \frac{-16}{30} < \frac{17}{30} < \frac{18}{30}$

Therefore, $\frac{-7}{10} < \frac{-8}{15} < \frac{17}{30} < \frac{3}{5}$

=> $\frac{7}{10} < \frac{8}{-15} < \frac{17}{-30} < \frac{3}{5}$

(ii) $\boldsymbol{\frac{-4}{9}, \frac{5}{-12}, \frac{7}{-18}, \frac{2}{-3}}$

Solution: Firstly we will make the denominator positive in all the rational numbers as follows:

One number = $\frac{-4}{9}$

Second number = $\frac{5}{-12} = \frac{5\times(-1)}{-12\times(-1)} = \frac{-5}{12}$

Third number = $\frac{7}{-18} = \frac{7\times(-1)}{-18\times(-1)} = \frac{-7}{18}$

Forth number = $\frac{2}{-3} = \frac{2\times(-1)}{-3\times(-1)} = \frac{-2}{3}$

Now, we will make the denominators of both rational numbers same by taking their LCM as follows:

LCM of 9, 12, 18 and 3 is 36

Now, $\frac{-4}{9} = \frac{-4\times4}{9\times4} = \frac{-16}{36}$ (Multiplying each term by 4)

$\frac{-5}{12} = \frac{-5\times3}{12\times3} = \frac{-15}{36}$ (Multiplying each term by 3)

$\frac{-7}{18} = \frac{-7\times2}{18\times2} = \frac{-14}{36}$ (Multiplying each term by 2)

$\frac{-2}{3} = \frac{-2\times12}{3\times12} = \frac{-24}{36}$ (Multiplying each term by 12)

Clearly, $-24 < -16 < -15 < -14$

Thus, $\frac{-24}{36} < \frac{-16}{36} < \frac{-15}{36} < \frac{-14}{36}$

Therefore, $\frac{-2}{3} < \frac{-4}{9} < \frac{-5}{12} < \frac{-7}{18}$

=> $\frac{2}{-3} < \frac{-4}{9} < \frac{5}{-12} < \frac{7}{-18}$

Question 6 – Arrange the following rational numbers in descending order:

(i) $\frac{7}{8}, \frac{64}{16}, \frac{36}{-12}, \frac{5}{-4}, \frac{140}{28}$

Solution: Firstly we will make the denominator positive in all the rational numbers as follows:

One number = $\frac{7}{8}$

Second number = $\frac{64}{16}$

Third number = $\frac{36}{-12} = \frac{36\times(-1)}{-12\times(-1)} = \frac{-36}{12}$

Forth number = $\frac{5}{-4} = \frac{5\times(-1)}{-4\times(-1)} = \frac{-5}{4}$

Fifth number = $\frac{140}{28}$

Now, we will make the denominators of both rational numbers same by taking their LCM as follows:

LCM of 8, 16, 12, 4 and 28 is 336

Now, $\frac{7}{8} = \frac{7\times42}{8\times42} = \frac{294}{336}$ (Multiplying each term by 42)

$\frac{64}{16} = \frac{64\times21}{16\times21} = \frac{1344}{336}$ (Multiplying each term by 21)

$\frac{-36}{12} = \frac{-36\times28}{12\times28} = \frac{-1008}{336}$ (Multiplying each term by 28)

$\frac{-5}{4} = \frac{-5\times84}{4\times84} = \frac{-420}{336}$ (Multiplying each term by 84)

$\frac{140}{28} = \frac{140\times12}{28\times12} = \frac{1680}{336}$ (Multiplying each term by 12)

Clearly, $1680 > 1344 > 294 > -420 > -1008$

Thus, $\frac{1680}{336} > \frac{1344}{336} > \frac{294}{336} > \frac{-420}{336} > \frac{-1008}{336}$

Therefore, $\frac{140}{28} > \frac{64}{16} > \frac{7}{8} > \frac{-5}{4} > \frac{-36}{12}$

=> $\frac{140}{28} > \frac{64}{16} > \frac{7}{8} > \frac{5}{-4} > \frac{36}{-12}$

(ii) $\mathbf{\frac{-3}{10}, \frac{17}{-30}, \frac{7}{-15}, \frac{-11}{20}}$

Solution: Firstly we will make the denominator positive in all the rational numbers as follows:

One number = $\frac{-3}{10}$

Second number = $\frac{17}{-30} = \frac{17\times(-1)}{-30\times(-1)} = \frac{-17}{30}$

Third number = $\frac{7}{-15} = \frac{7\times(-1)}{-15\times(-1)} = \frac{-7}{15}$

Forth number = $\frac{-11}{20}$

Now, we will make the denominators of both rational numbers same by taking their LCM as follows:

LCM of 10, 30, 15 and 20 is 60

Now, $\frac{-3}{10} = \frac{-3\times6}{10\times6} = \frac{-18}{60}$ (Multiplying each term by 6)

$\frac{-17}{30} = \frac{-17\times2}{30\times2} = \frac{-34}{60}$ (Multiplying each term by 2)

$\frac{-7}{15} = \frac{-7\times4}{15\times4} = \frac{-28}{60}$ (Multiplying each term by 4)

$\frac{-11}{20} = \frac{-11\times3}{20\times3} = \frac{-33}{60}$ (Multiplying each term by 3)

Clearly, $-18 > -28 > -33 > -34$

Thus, $\frac{-18}{60} > \frac{-28}{60} > \frac{-33}{60} > \frac{-34}{60}$

Therefore, $\frac{-3}{10} > \frac{-7}{15} > \frac{-11}{20} > \frac{-17}{30}$

=> $\frac{-3}{10} > \frac{7}{-15} > \frac{-11}{20} > \frac{17}{-30}$

Question 7 – Which of the following statements are true:

(i) The rational number $\frac{29}{23}$ lies to the left of zero on the number line.

Solution: False

Reasoning: Since $\frac{29}{23}$ is a positive rational number thus, it lies on the right of zero on the number line.

(ii) The rational number $\frac{-12}{-17}$ lies to the left of zero on the number line

Solution: False

Reasoning: We can write $\frac{-12}{-17} = \frac{-12\times(-1)}{-17\times(-1)} = \frac{12}{17}$ and we know that $\frac{12}{17}$ is a positive rational number thus, it lies on the right of zero on the number line.

(iii) The rational number $\frac{3}{4}$ lies to the right of zero on the number line.

Solution: True

Reasoning: All positive rational numbers lie on the right of zero on the number line.

(iv) The rational numbers $\frac{-12}{-5}$ *and* $\frac{-7}{17}$ are on the opposite side of zero on the number line.

Solution: True

Reasoning: We can write $\frac{-12}{-5} = \frac{-12\times(-1)}{-5\times(-1)} = \frac{12}{5}$ and it is a positive rational number so it will lie on the right side of zero on number line and $\frac{-7}{17}$ is a negative rational number so it will lie on left of zero on number line. Therefore, both rational numbers will lie on the opposite side of zero on number line.

(v) The rational numbers $\frac{-21}{5}$ *and* $\frac{7}{-31}$ are on the opposite side of zero on the number line.

Solution: False

Reasoning: We can write $\frac{7}{-31} = \frac{7\times(-1)}{-31\times(-1)} = \frac{-7}{31}$ and it is a negative rational number so it will lie on the left side of zero on number line and $\frac{-21}{5}$ is also a negative rational number so it will lie on left of zero on number line. Therefore, both rational numbers will lie on the left side of zero on number line.

(vi) The rational numbers $\frac{-3}{-5}$ is on the right of $\frac{-4}{7}$ on the number line.

Solution: True

Reasoning: We can write $\frac{-3}{-5} = \frac{-3\times(-1)}{-5\times(-1)} = \frac{3}{5}$ and it is a positive rational number whereas $\frac{-4}{7}$ is a negative rational number. We know that positive rational numbers always lie on the right side of negative rational numbers on number line. Thus, $\frac{-3}{-5}$ lie on right of $\frac{-4}{7}$ on number line.

Objective Type Questions

Question 1: $\frac{44}{-77}$ in standard form is?

Solution: Firstly, we will make the denominator positive as follows:

$\frac{44}{-77} = \frac{44\times(-1)}{-77\times(-1)} = \frac{-44}{77}$ (Multiplying each term by -1)

Now, 44 = 2 × 2 × 11 and

77 = 7 × 11

We see that 11 is common factor between 44 and 77 thus, HCF (44, 77) is 11

We will divide each term by 11 in order to get the standard form

=> $\frac{-44}{77} = \frac{-44\div11}{77\div11} = \frac{-4}{7}$

Question 2: $-\frac{102}{119}$ in standard form is?

Solution: Firstly, we will take HCF of 102 and 119 as follows:

Now, 102 = 2 × 3 × 17 and 119 = 7 × 17

We see that 17 is common factor between 102 and 119 thus, HCF (102, 119) is 17

We will divide each term by 17 in order to get the standard form

=> $\frac{-102}{119} = \frac{-102\div17}{119\div17} = \frac{-6}{7}$

Question 3 – A rational number equal to $\frac{-2}{3}$ is?

Solution: Since $\frac{10}{-15} = \frac{10\times(-1)}{-15\times(-1)} = \frac{-10}{15}$

Now, 10 = 2 × 5 and 15 = 3 × 5

We see that 5 is common factor between 10 and 15 thus, HCF (10, 15) is 5

We will divide each term by 5 in order to get the standard form

=> $\frac{-10}{15} = \frac{-10\div5}{15\div5} = \frac{-2}{3}$

Thus, $\frac{10}{-15}$ is equal to $\frac{-2}{3}$

Question 4 – If $\frac{-3}{7} = \frac{x}{35}$, then x=?

Solution: We know that: $\frac{a}{b} = \frac{c}{d} <=> a \times d = c \times b$

=> $\frac{-3}{7} = \frac{x}{35}$

=> $(-3) \times (35) = 7x$

=> $-105 = 7x$

=> $x = \frac{-105}{7} = -15$

Question 5 – Which of the following is correct?

- **(a)** $\frac{5}{9} > \frac{-3}{-8}$
- **(b)** $\frac{5}{9} < \frac{-3}{-8}$
- **(c)** $\frac{2}{-3} < \frac{-8}{7}$
- **(d)** $\frac{4}{-3} > \frac{-8}{7}$

Solution - Consider: $\frac{5}{9}\, and\, \frac{-3}{-8}$

We can write $\frac{-3}{-8} = \frac{-3\times(-1)}{-8\times(-1)} = \frac{3}{8}$

Now, we will make the denominators of both rational numbers same by taking their LCM as follows:

LCM of 9 and 8 is 72

Now, $\frac{5}{9} = \frac{5\times8}{9\times8} = \frac{40}{72}$ (Multiplying each term by 8)

$\frac{3}{8} = \frac{3\times9}{8\times9} = \frac{27}{72}$ (Multiplying each term by 9)

Clearly 40 > 27

Thus, $\frac{5}{9} > \frac{3}{8} \Rightarrow \frac{5}{9} > \frac{-3}{-8}$

Therefore, option (a) is correct

Question 6 – If the rational numbers $\frac{-2}{3}$ *and* $\frac{4}{x}$ represents a pair of equivalent rational numbers, then x=?

Solution: It is given that $\frac{-2}{3} = \frac{4}{x}$

Now, we know that: $\frac{a}{b} = \frac{c}{d} \Leftrightarrow a \times d = c \times b$

=> $\frac{-2}{3} = \frac{4}{x}$

=> $(-2) \times (x) = 4 \times 3$

=> $-2x = 12$

=> $x = \frac{-12}{2} = -6$

Question 7 – What is the additive identity element in the set of whole numbers?

Solution: 0 is the additive identity element in the set of whole numbers because by additive identity we mean that $a + 0 = 0 + a = a$ for any value of 'a'.

Question 8 – What is the multiplicative identity element in the set of whole numbers?

Solution: 1 is the multiplicative identity element in the set of whole numbers because by multiplicative identity we mean that $a \times 1 = 1 \times a = a$ for any value of 'a'.

Question 9 – Which of the following is not zero?

(a) 0×0
(b) $\frac{0}{3}$
(c) $\frac{7-7}{3}$
(d) $9 + 0$

Solution: Since $0 \times 0 = 0$

$\frac{0}{3} = 0$ and $\frac{7-7}{3} = \frac{0}{3} = 0$

But, $9 + 0 = 9 \neq 0$

Therefore, option (d) is correct

Question 10 – The whole number nearest to 457 and divisible by 11 is?

Solution: 462 is the whole number nearest to 457 which id divisible by 11.

$$\frac{462}{11} = 42$$

Question 11: If $-\frac{3}{8}$ *and* $\frac{x}{-24}$ are equivalent rational numbers, then x =?

Solution: It is given that $\frac{-3}{8} = \frac{x}{-24}$

Now, we know that: $\frac{a}{b} = \frac{c}{d} <=> a \times d = c \times b$

=> $\frac{-3}{8} = \frac{x}{-24}$

=> $(-3) \times (-24) = 8 \times x$

=> $72 = 8x$

=> $x = \frac{72}{8} = 9$

Question 12: If $\frac{27}{-45}$ is expressed as a rational number with denominator 5, then the numerator is?

Solution: In order to make the denominator as 5, we must divide each term by $\frac{-45}{5} = -9$.

So, we will divide the numerator and denominator by -9 as follows:

=> $\frac{27}{-45} = \frac{27 \div (-9)}{-45 \div (-9)} = \frac{-3}{5}$

Thus, we get numerator = -3

Question 13 – Which of the following pairs of rational numbers are on the opposite sides of the zero on the number line?

(a) $\frac{3}{7}$ *and* $\frac{5}{12}$
(b) $-\frac{3}{7}$ *and* $-\frac{5}{12}$

(c) $\frac{3}{7}$ *and* $-\frac{5}{12}$
(d) None of these

Solution: Option (c) is correct because $\frac{3}{7}$ is a positive rational number and it will lie on the right side of zero whereas $-\frac{5}{12}$ is a negative rational number so it will lie on the left side of zero. Therefore, both rational numbers will lie on the opposite side of zero on the number line.

Question 14 – The rational number equal to $\frac{2}{-3}$ is?

Solution: The rational number $\frac{-6}{9}$ is equal to the $\frac{2}{-3}$ because we can write $\frac{-6}{9} = \frac{6}{-9}$

Now, 6 = 2 × 3 and 9 = 3 × 3

We can see that 3 is common factor between 6 and 9. Thus, we will divide each term by 3

=> $\frac{6}{-9} = \frac{6 \div 3}{-9 \div 3} = \frac{2}{-3}$

Question 15: If $-\frac{3}{4} = \frac{6}{x}$, then x =?

Solution: It is given that $\frac{-3}{4} = \frac{6}{x}$

Now, we know that: $\frac{a}{b} = \frac{c}{d} <=> a \times d = c \times b$

=> $\frac{-3}{4} = \frac{6}{x}$

=> $(-3) \times (x) = 6 \times 4$

=> $-3x = 24$

=> $x = \frac{24}{-3} = -8$