Introduction

Numbers in generalized form: A number is said to be in generalized form if it is expresses as the sum of the products of its digits with their respective place values.

Two digit number: In a two digit number, if tens digit is 'a' and unit digit is 'b' then we can write it as $10a + b$

Three digit number: consider a three digit number having a, b and c as its hundreds digit, tens digit and unit digit respectively.

Then, we can write it as $100a + 10b + c$

Examples:

Example 1 - In a 2-digit number, the units digit is four times the tens digit and the sum of the digits is 10. Find the number.

Solution **-** Let us assume tens digit be x

Then, units digit = 4x

Given that sum of digits = 10

=> $x + 4x = 10$

=> $5x = 10$

=> $x = 2$

Tens digit (a) = 2 and units digit (b) = 4(2) = 8

Number = 10a+b = $10(2) + 8 = 20 + 8 = 28$

Example 2 - The sum of the digits of a two-digit number is 8. The number obtained by interchanging its digits is 18 more than the original number. Find the original number.

Solution **-** Let tens digit be 'a' and unit digit be 'b'

Then, two digit number will be $10a + b$

According to given question,

We have $a + b = 8$ $\longrightarrow$ (I)

On interchanging the digits, number becomes = $10b + a$

Given that $10b + a$ = 18 + $10a + b$

=> $10b - b + a - 10a = 18$

=> $9b - 9a = 18$

=> $b - a = 2$

=> $b = a + 2$ ⟶ (II)

Substituting (II) in (I) we get.

$a + b = 8$

=> $a + a + 2 = 8$

=> $2a + 2 = 8$

=> $2a = 8 - 2 = 6$

=> $a = 3$

Then, $b = a + 2 = 3 + 2 = 5$

Therefore, original number = $10a + b = 10(3) + 5 = 30 + 5 = 35$

Example 3 - In a 3-digit number, the hundreds digit is twice the tens digit while the units digit is thrice the tens digit. Also, the sum of its digits is 18. Find the number.

Solution **-** Let us assume tens digit be x

Then, hundreds digit = 2x

Units digit = 3x

Given that sum of digits = 18

=> $x + 2x + 3x = 18$

=> $6x = 18$

=> $x = 3$

Thus, hundreds digit (a) = 2(3) = 6

Tens digit = 3

Units digit = 3(3) = 9

We know that three digit number $=100a + 10b$+c

=> Number = $100(6) + 10(3) + 9$

= $600 + 30 + 9 = 639$

Exercise 5A

Question 1 - The units digit of a two-digit number is 3 and seven times the sum of the digits is the number itself. Find the number.

Solution - Given that unit digit is 3

Let tens digit be a

Then, two digit number = $10a + 3$

According to given question,'

$7(a + 3) = 10a + 3$

=> $7a + 21 = 10a + 3$

=> $10a - 7a = 21 - 3$

=> $3a = 18$

=> $a = 6$

Thus, number = $10a + 3 = 10(6) + 3 = 60 + 3 = 63$

Question 2 - In a two-digit number, the digits at the units place is double the digit in the tens place. The number exceeds the sum of its digit by 18. Find the number.

Solution - Let tens digit be 'a' and units digit be 'b'

Then, number = $10a + b$

Given that $b = 2a$

Also, $10a + b = a + b + 18$

=> $10a - a = 18$

=> $9a = 18$

=> $a = 2$

Thus, $b = 2a = 2(2) = 4$

Number = $10a + b = 10(2) + 4 = 20 + 4 = 24$

Question 3 - A two-digit number is 3 more than 4 times the sum of its digits. If 18 is added to the number, its digits are reversed. Find the number.

Solution - Let tens digit be ‘a’ and units digit be ‘b’

Then, number = $10a + b$

According to given question,

$10a + b = 3 + 4(a + b)$

=> $10a + b = 3 + 4a + 4b$

=> $10a - 4a + b - 4b = 3$

=>$6a - 3b = 3$

=> $2a - b = 1$ ⟶ (I)

Also given that,

$10a + b + 18 = 10b + a$

=> $10a - a + b - 10b + 18 = 0$

=> $9a - 9b + 18 = 0$

=> $a - b + 2 = 0$

=> $b = a + 2$ ⟶ (II)

Substituting (II) in (I), we get

$2a - b = 1$

=> $2a - (a + 2) = 1$

=> $2a - a - 2 = 1$

=> $a - 2 = 1$

=> $a = 1 + 2 = 3$

Then, b= $a + 2 = 3 + 2 = 5$

Therefore, number = $10a + b = 10(3) + 5 = 30 + 5 = 35$

Question 4 - The sum of the digits of a two-digit number is 15. The number obtained by interchanging its digits exceeds the given number by 9. Find the original number.

Solution - Let tens digit be 'a' and units digit be 'b'

Then, number = $10a + b$

According to given question,

$a + b = 15 \longrightarrow$ (I)

Also $10b + a = 9 + 10a + b$

=> $10b - b + a - 10a = 9$

=> $9b - 9a = 9$

=> $b - a = 1$

=> $b = a + 1 \longrightarrow$ (II)

Substituting (II) in (I), we get

$a + b = 15$

=> $a + a + 1 = 15$

=> $2a + 1 = 15$

=> $2a = 15 - 1 = 14$

=> $a = 7$

Then, $b = a + 1 = 7 + 1 = 8$

Thus, number = $10a + b = 10(7) + 8 = 70 + 8 = 78$

Question 5 - The difference between a 2-digit number and the number obtained by interchanging its digits is 63. What is the difference between the digits of the number?

Solution - Let tens digit be 'a' and units digit be 'b'

Then, number = $10a + b$

On interchanging the digits, number = $10b + a$

According to given question,

$(10a + b) - (10b + a) = 63$

=> $10a + b - 10b - a = 63$

=> $9a - 9b = 63$

=> $a - b = 7$

So, the difference between the digits of the number is 7

Question 6 - In a 3-digit number, the tens digit is thrice the units digit and the hundreds digit is four times the units digit. Also, the sum of its digits is 16. Find the number.

Solution **-** Let the units digit be x

Then tens digit = 3x

Hundreds digit = 4x

Given that sum of digits = 16

=> $x + 3x + 4x = 16$

=> $8x = 16$

=> $x = 2$

Thus, hundreds digit (a) = 4(2) = 8

Tens digit (b) = 3(2) = 6

Units digit (c) = 2

We know that three digit number = $100a + 10b + c$

$= 100(8) + 10(6) + 2 = 800 + 60 + 2 = 862$

Various tests of divisibility

I. Test of divisibility by 2: A given number is divisible by 2 only when its unit's digit is 0, 2, 4, 6 or 8.

Example 1 - Each of the numbers 60, 72, 84, 96, 108 is divisible by 2.

II. Test of divisibility by 3: A given number is divisible by 3 only when the sum of its digits is divisible by 3.

Example 2 - Test the divisibility of each of the following numbers by 3:

(a) 18657

Solution - Sum of digits $=1 + 8 + 6 + 5 + 7 = 27$.

Since 27 is divisible by 3 therefore 18657 is divisible by 3.

(b) 967458

Solution - Sum of digits $=9 + 6 + 7 + 4 + 5 + 8 = 39$.

Since 39 is divisible by 3 therefore 967458 is divisible by 3.

(c) 263705

Solution - Sum of digits $=2 + 6 + 3 + 7 + 0 + 5 = 23$.

Since 23 is not divisible by 3 therefore 263705 is not divisible by 3.

Example 3 - Find all possible values of x for which the 4-digit number 754x is divisible by 3. Also, find each such number.

Solution - We are given that 754x is divisible by 3

Then, $(7 + 5 + 4 + x)$ must be divisible by 3

=> $(16 + x)$ must be divisible by 3

Now, x can take values 2 or 5 or 8

Hence, all possible numbers can be 7542, 7545, 7548

(III) Test of divisibility by 9: A given number is divisible by 9 only when the sum of its digits is divisible by 9

Example 4 - Test the divisibility of each of the following numbers by 9:

(a) 27981

Solution - Sum of digits $=2 + 7 + 9 + 8 + 1 = 27$.

Since 27 is divisible by 9 therefore 27981 is divisible by 9

(b) 869517

Solution - Sum of digits $=8 + 6 + 9 + 5 + 1 + 7 = 36$.

Since 36 is divisible by 9 therefore 869517 is divisible by 9.

(c) 937546

Solution **-** Sum of digits $=9+3+7+5+4+6=34$.

Since 34 is not divisible by 9 therefore 937546 is not divisible by 9.

(d) 336899

Solution **-** Sum of digits $=3+3+6+8+9+9=38$.

Since 38 is not divisible by 9 therefore 336899 is not divisible by 9.

Example 5 - Find all possible values of y for which the 4-digit number 51y3 is divisible by 9. Also, find each such number.

Solution **-** We are given that 51y3 is divisible by 9

Then, $(5+1+y+3)$ must be divisible by 9

=> $(9+y)$ must be divisible by 9

Now, y can take values 0 or 9

Hence, all possible numbers can be 5103, 5193.

Example 6 - Give an example of a number which is divisible by 3 but not divisible by 9.

Solution **-** Consider a number 123

Sum of digits = $1+2+3=6$

Now, since 6 is divisible by 3 therefore 123 is divisible by 3

But 6 is not divisible by 9 so 123 is not divisible by 9

IV Test of divisibility by 5: A given number is divisible by 5 only when its unit digit is 0 or 5.

Example 7 - Each of the numbers 67930 and 89715 is divisible by 5. None of the numbers 146, 278, 513, 684, 341, 482,507 is divisible by 5.

V Test of divisibility by 10: A given number is divisible by 10 only when its unit digit is 0.

Example 8 - Each of the numbers 90, 120, 230, 350, 470, etc., is divisible by 10.

VI Test of divisibility by 4: A given number is divisible by 4 only when the number formed by its last two digits is divisible by 4.

Example 9 - Test the divisibility of each of the following numbers by 5:

(a) 35692

Solution - Last two digits = 92.

Since 92 is divisible by 4 therefore 35692 is divisible by 4.

(b) 41328

Solution - Last two digits = 28.

Since 28 is divisible by 4 therefore 41328 is divisible by 4.

(c) 10874

Solution - Last two digits = 74.

Since 74 is not divisible by 4 therefore 10874 is not divisible by 4.

(d) 154326

Solution - Last two digits = 26.

Since 26 is not divisible by 4 therefore 154326 is not divisible by 4.

VII Test of divisibility by 8: A given number is divisible by 8 only when the number formed by its last three digits is divisible by 8.

Example 10 - Test the divisibility of each of the following numbers by 8:

(a) 49104

Solution - Last three digits = 104

Since 104 is divisible by 8 therefore 49104 is divisible by 8.

(b) 570312

Solution - Last three digits = 312

Since 312 is divisible by 8 therefore 570312 is divisible by 8.

(c) 685218

Solution - Last three digits = 218

Since 218 is not divisible by 8 therefore 685218 is not divisible by 8.

(d) 739514

Solution - Last three digits = 514

Since 514 is not divisible by 8 therefore 739514 is not divisible by 8.

Example 11 - Give an example of a number which is divisible by 4 but not divisible by 8.

Solution - Consider a number 1236

Last two digit = 36 and 36 is divisible by 4 thus 1236 is divisible by 4

Last three digit= 236 and 236 is not divisible by 8 thus 1236 is not divisible by 8.

VIII Test of divisibility by 7:

Step1: Double the unit digit of given number.

Step2: subtract the above number from the number formed by excluding the unit digit of the given number.

Step3: if the number so obtained is divisible by 7 then the given number is divisible by 7.

Example 12 - Test the divisibility of each of the following numbers by 7:

(a) 672

Solution - Unit digit = 2

Now, $67 - (2 \times 2) = 67 - 4$ =63

Since 63 is divisible by 7 thus 672 is divisible by 7.

(b) 5341

Solution - Unit digit = 1

Now, $534 - (2 \times 1) = 534 - 2$ =532

Since 532 is divisible by 7 thus 5341 is divisible by 7

(c) 1067

Solution - Unit digit = 7

Now, $106 - (2 \times 7) = 106 - 14$ =92

Since 92 is not divisible by 7 thus 672 is not divisible by 7.

(d) 7305

Solution - Unit digit = 5

Now, $730 - (2 \times 5) = 730 - 10$ =720

Since 720 is not divisible by 7 thus 7305 is not divisible by 7

IX Test of divisibility by 11: A given number is divisible by 11, if the difference between the sum of its digits at odd places and the sum of its digits at even places, is either 0 or a number divisible by 11

Example 13 - Test the divisibility of each of the following numbers by 11:

(a) 863478

Solution - Sum of digits at odd places = $8 + 3 + 7 = 18$

Sum of digits at even places = $6 + 4 + 8 = 18$

Since $18 - 18 = 0$

Therefore 863478 is divisible by 11

(b) 4832718

Solution - Sum of digits at odd places = $4 + 3 + 7 + 8 = 22$

Sum of digits at even places = $8 + 2 + 1 = 11$

Since$22 - 11 = 11$, this is divisible by 11

Therefore, 4832718 is divisible by 11

(c) 5436708

Solution - Sum of digits at odd places = $5 + 3 + 7 + 8 = 23$

Sum of digits at even places = $4 + 6 + 0 = 10$

Since$23 - 10 = 13$, this is not divisible by 11

Therefore, 5436708 is not divisible by 11

Exercise 5B

Question 1 - Test the divisibility of each of the following numbers by 2:

We know that a given number is divisible by 2 only when its unit's digit is 0, 2, 4, 6 or 8.

(a) 94

Solution - Since unit digit of 94 is 4 therefore 94 is divisible by 2

(b) 570

Solution - Since unit digit of 570 is 0 therefore 570 is divisible by 2

(c) 285

Solution - Since unit digit of 285 is 5 therefore 285 is not divisible by 2

(d) 2398

Solution - Since unit digit of 2398 is 8 therefore 2398 is divisible by 2

(e) 79532

Solution - Since unit digit of 79532 is 2 therefore 79532 is divisible by 2

(f) 13576

Solution - Since unit digit of 13576 is 6 therefore 13576 is divisible by 2

(g) 46821

Solution - Since unit digit of 46821 is 1 therefore 46821 is not divisible by 2

(h) 84663

Solution - Since unit digit of 84663 is 3 therefore 84663 is not divisible by 2

(i) 66669

Solution - Since unit digit of 66669 is 9 therefore 66669 is not divisible by 2

Question 2 - Test the divisibility of each of the following numbers by 5:

We know that a given number is divisible by 5 only when its unit digit is 0 or 5.

(a) 95

Solution - Since unit digit of 95 is 5 therefore 95 is divisible by 5

(b) 470

Solution - Since unit digit of 470 is 0 therefore 470 is divisible by 5

(c) 1056

Solution - Since unit digit of 1056 is 6 therefore 1056 is not divisible by 5

(d) 2735

Solution - Since unit digit of 2735 is 5 therefore 2735 is divisible by 5

(e) 55053

Solution - Since unit digit of 55053 is 3 therefore 55053 is not divisible by 5

(f) 35790

Solution - Since unit digit of 35790 is 0 therefore 35790 is divisible by 5

(g) 98765

Solution - Since unit digit of 98765 is 5 therefore 98765 is divisible by 5

(h) 42658

Solution - Since unit digit of 42658 is 8 therefore 42658 is not divisible by 5

(i) 77990

Solution - Since unit digit of 77990 is 0 therefore 77990 is divisible by 5

Question 3 - Test the divisibility of each of the following numbers by 10:

We know that a given number is divisible by 10 only when its unit digit is 0.

(a) 205

Solution - Since the unit digit of 205 is 5 therefore 205 is not divisible by 10

(b) 90

Solution - Since the unit digit of 90 is 0 therefore 90 is divisible by 10

(c) 1174

Solution - Since the unit digit of 1174 is 4 therefore 1174 is not divisible by 10

(e) 57930

Solution - Since the unit digit of 57930 is 0 therefore 57930 is divisible by 10

(f) 60005

Solution - Since the unit digit of 60005 is 5 therefore 60005 is not divisible by 10

Question 4 - Test the divisibility of each of the following numbers by 3:

We know that a given number is divisible by 3 only when the sum of its digits is divisible by 3.

(a) 83

Solution - Sum of digits $=8 + 3 = 11$.

Since 11 is not divisible by 3 therefore 83 is not divisible by 3.

(b) 378

Solution - Sum of digits $=3 + 7 + 8 = 18$.

Since 18 is divisible by 3 therefore 378 is divisible by 3.

(c) 474

Solution - Sum of digits $=4 + 7 + 4 = 15$.

Since 15 is divisible by 3 therefore 474 is divisible by 3.

(d) 1693

Solution - Sum of digits $=1 + 6 + 9 + 3 = 19$.

Since 19 is not divisible by 3 therefore 1693 is not divisible by 3

(e) 20345

Solution - Sum of digits $=2 + 0 + 3 + 4 + 5 = 14$.

Since 14 is not divisible by 3 therefore 20345 is not divisible by 3.

(f) 67035

Solution - Sum of digits $=6 + 7 + 0 + 3 + 5 = 21$.

Since 21 is divisible by 3 therefore 67035 is divisible by 3.

(g) 591282

Solution - Sum of digits $=5 + 9 + 1 + 2 + 8 + 2 = 27$.

Since 27 is divisible by 3 therefore 591282 is divisible by 3.

(h) 903164

Solution - Sum of digits $=9 + 0 + 3 + 1 + 6 + 4 = 23$.

Since 23 is not divisible by 3 therefore 903164 is not divisible by 3.

(i) 100002

Solution - Sum of digits $=1 + 0 + 0 + 0 + 0 + 2 = 3$.

Since 3 is divisible by 3 therefore 100002 is divisible by 3.

Question 5 - Test the divisibility of each of the following numbers by 9:

We know that a given number is divisible by 9 only when the sum of its digits is divisible by 9

(a) 327

Solution - Sum of digits $=3 + 2 + 7 = 12$.

Since 12 is not divisible by 9 therefore 327 is not divisible by 9.

(b) 7524

Solution - Sum of digits $=7 + 5 + 2 + 4 = 18$.

Since 18 is divisible by 9 therefore 7524 is divisible by 9

(c) 32022

Solution - Sum of digits $=3 + 2 + 0 + 2 + 2 = 9$.

Since 9 is divisible by 9 therefore 32022 is divisible by 9.

(d) 64302

Solution - Sum of digits $=6 + 4 + 3 + 0 + 2 = 15$.

Since 15 is not divisible by 9 therefore 64302 is not divisible by 9.

(e) 89361

Solution - Sum of digits $=8 + 9 + 3 + 6 + 1 = 27$.

Since 27 is divisible by 9 therefore 89361 is divisible by 9.

(f) 14799

Solution - Sum of digits $=1 + 4 + 7 + 9 + 9 = 30$.

Since 30 is not divisible by 9 therefore 14799 is not divisible by 9.

(g) 66888

Solution - Sum of digits $=6 + 6 + 8 + 8 + 8 = 36$.

Since 36 is divisible by 9 therefore 66888 is divisible by 9.

(h) 30006

Solution - Sum of digits $=3 + 0 + 0 + 0 + 6 = 9$.

Since 9 is divisible by 9 therefore 30006 is divisible by 9.

(i) 33333

Solution - Sum of digits $=3 + 3 + 3 + 3 + 3 = 15$.

Since 15 is not divisible by 9 therefore 33333 is not divisible by 9.

Question 6 - Test the divisibility of each of the following numbers by 4:

We know that a given number is divisible by 4 only when the number formed by its last two digits is divisible by 4.

(a) 134

Solution - Last two digits = 34.

Since 34 is not divisible by 4 therefore 134 is not divisible by 4.

(b) 618

Solution - Last two digits = 18.

Since 18 is not divisible by 4 therefore 618 is not divisible by 4.

(c) 3928

Solution - Last two digits = 28.

Since 28 is divisible by 4 therefore 3928 is divisible by 4

(d) 50176

Solution - Last two digits = 76

Since 76 is divisible by 4 therefore 50176 is divisible by 4.

(e) 39392

Solution - Last two digits = 92.

Since 92 is divisible by 4 therefore 39392 is divisible by 4.

(f) 56794

Solution - Last two digits = 94.

Since 94 is not divisible by 4 therefore 56794 is not divisible by 4.

(g) 86102

Solution - Last two digits = 02.

Since 02 is not divisible by 4 therefore 86102 is not divisible by 4.

(h) 66666

Solution - Last two digits = 66.

Since 66 is not divisible by 4 therefore 66666 is not divisible by 4.

(i) 99918

Solution - Last two digits = 18.

Since 18 is not divisible by 4 therefore 99918 is not divisible by 4.

(j) 77736

Solution - Last two digits = 36.

Since 36 is divisible by 4 therefore 77736 is divisible by 4.

Question 7 - Test the divisibility of each of the following numbers by 8:

We know that a given number is divisible by 8 only when the number formed by its last three digits is divisible by 8.

(a) 6132

Solution - Last three digits = 132

Since 132 is not divisible by 8 therefore 6132 is not divisible by 8

(b) 7304

Solution - Last three digits = 304

Since 304 is divisible by 8 therefore 7304 is divisible by 8

(c) 59312

Solution - Last three digits = 312

Since 312 is divisible by 8 therefore 59312 is divisible by 8

(d) 66664

Solution - Last three digits = 664

Since 664 is divisible by 8 therefore 66664 is divisible by 8

(e) 44444

Solution - Last three digits = 444

Since 444 is not divisible by 8 therefore 44444 is not divisible by 8

(f) 154360

Solution - Last three digits = 360

Since 360 is divisible by 8 therefore 154360 is divisible by 8

(g) 998818

Solution - Last three digits = 818

Since 818 is not divisible by 8 therefore 998818 is not divisible by 8

(h) 265472

Solution - Last three digits = 472

Since 472 is divisible by 8 therefore 265472 is divisible by 8

(i) 7350162

Solution - Last three digits = 162

Since 162 is not divisible by 8 therefore 7350162 is not divisible by 8

Question 8 - Test the divisibility of each of the following numbers by 11:

We know that a given number is divisible by 11, if the difference between the sum of its digits at odd places and the sum of its digits at even places, is either 0 or a number divisible by 11

(a) 22222

Solution - Sum of digits at odd places = $2 + 2 + 2 = 8$

Sum of digits at even places = $2 + 2 = 4$

Since,$8 - 4 = 4$ that is not divisible by 11

Therefore, 22222 is not divisible by 11

(b) 444444

Solution - Sum of digits at odd places = $4 + 4 + 4 = 12$

Sum of digits at even places = $4 + 4 + 4 = 12$

Since,$12 - 12 = 0$

Therefore, 444444 is divisible by 11

(c) 379654

Solution - Sum of digits at odd places = $3 + 9 + 5 = 17$

Sum of digits at even places = $7 + 6 + 4 = 17$

Since,$17 - 17 = 0$

Therefore, 379654 is divisible by 11

(d) 1057982

Solution - Sum of digits at odd places = $1 + 5 + 9 + 2 = 17$

Sum of digits at even places = $0 + 7 + 8 = 15$

Since, $17 - 15 = 2$, that is not divisible by 11

Therefore, 1057982 is not divisible by 11

(e) 6543207

Solution - Sum of digits at odd places = $6 + 4 + 2 + 7 = 19$

Sum of digits at even places = $5 + 3 + 0 = 8$

Since, $19 - 8 = 11$

Therefore, 6543207 is divisible by 11

(f) 818532

Solution - Sum of digits at odd places = $8 + 8 + 3 = 19$

Sum of digits at even places = $1 + 5 + 2 = 8$

Since, $19 - 8 = 11$

Therefore, 818532 is divisible by 11

(g) 900163

Solution - Sum of digits at odd places = $9 + 0 + 6 = 15$

Sum of digits at even places = $0 + 1 + 3 = 4$

Since, $15 - 4 = 11$

Therefore, 900163 is divisible by 11

(h) 7531622

Solution - Sum of digits at odd places = $7 + 3 + 6 + 2 = 18$

Sum of digits at even places = $5 + 1 + 2 = 8$

Since, $18 - 8 = 10$, that is not divisible by 11

Therefore, 7531622 is not divisible by 11

Question 9 - Test the divisibility of each of the following numbers by 7:

(a) 693

Solution - Unit digit = 3

Now, $69 - (2 \times 3) = 69 - 6 =63$

Since 63 is divisible by 7 thus 693 is divisible by 7.

(b) 7896

Solution - Unit digit = 6

Now, $789 - (2 \times 6) = 789 - 12 =777$

Since 777 is divisible by 7 thus 7896 is divisible by 7

(c) 3467

Solution - Unit digit = 7

Now, $346 - (2 \times 7) = 346 - 14 =332$

Since 332 is not divisible by 7 thus 3467 is not divisible by 7

(d) 12873

Solution - Unit digit = 3

Now, $1287 - (2 \times 3) = 1287 - 6 =1281$

Since 1281 is divisible by 7 thus 12873 is divisible by 7.

(e) 65436

Solution - Unit digit = 6

Now, $6543 - (2 \times 6) = 6543 - 12 =6531$

Since 6531 is divisible by 7 thus 65436 is divisible by 7.

(f) 54636

Solution - Unit digit = 6

Now, $5463 - (2 \times 6) = 5463 - 12 =5451$

Since 5451 is not divisible by 7 thus 54636 is not divisible by 7.

(g) 98175

Solution - Unit digit = 5

Now, $9817 - (2 \times 5) = 9817 - 10$ =9807

Since 9807 is divisible by 7 thus 98175 is divisible by 7.

(h) 88777

Solution - Unit digit = 7

Now, $8877 - (2 \times 7) = 8877 - 14$ =8863

Since 8863 is not divisible by 7 thus 88777 is not divisible by 7.

Question 10 - Find all possible values of x for which the number 7x3 is divisible by 3. Also, find each such number.

Solution - We are given that 7x3 is divisible by 3

Then, $(7 + x + 3)$ must be divisible by 3

=> $(10 + x)$ must be divisible by 3

Now x can take values 2 or 5 or 8

Hence, all possible numbers can be 723, 753, and 783

Question 11 - Find all possible values of y for which the number 53y1 is divisible by 3. Also, find each such number.

Solution - We are given that 53y1 is divisible by 3

Then, $(5 + 3 + y + 1)$ must be divisible by 3

=> $(9 + x)$ must be divisible by 3

Now x can take values 3 or 6 or 9

Hence, all possible numbers can be 5331, 5361, and 5391

Question 12 - Find the value of x for which the number x806 is divisible by 9. Also, find the number.

Solution - We are given that x806 is divisible by 9

Then, $(x + 8 + 0 + 6)$ must be divisible by 9

=> $(14 + x)$ must be divisible by 9

Now x can take 4 only

Hence, possible number is 4806

Question 13 - Find the value of z for which the number 471z8 is divisible by 9. Also, find the number.

Solution - We are given that 471z8 is divisible by 9

Then, $(4 + 7 + 1 + z + 8)$ must be divisible by 9

=> $(20 + z)$ must be divisible by 9

Now z can take 7 only

Hence, possible number is 47178

Question 14 - Give five examples of numbers, each one of which is divisible by 3 but not divisible by 9

Solution - Consider (a) 231

Sum of digits = $2 + 3 + 1 = 6$

Now 6 is divisible by 3 thus 231 is divisible by 3

But 6 is not divisible by 9 so 231 is not divisible by 9

(b) 678

Sum of digits = $6 + 7 + 8 = 21$

Now 21 is divisible by 3 thus 678 is divisible by 3

But 21 is not divisible by 9 so 678 is not divisible by 9

(c) 708

Sum of digits = $7 + 0 + 8 = 15$

Now 15 is divisible by 3 thus 708 is divisible by 3

But 15 is not divisible by 9 so 708 is not divisible by 9

(d) 3678

Sum of digits = $3 + 6 + 7 + 8 = 24$

Now 24 is divisible by 3 thus 3678 is divisible by 3

But 24 is not divisible by 9 so 3678 is not divisible by 9

(e) 435

Sum of digits = $4 + 3 + 5 = 12$

Now 12 is divisible by 3 thus 435 is divisible by 3

But 12 is not divisible by 9 so 435 is not divisible by 9

Question 15 - Give five examples of numbers, each one of which is divisible by 4 but not divisible by 8

Solution - Consider (a) 1348

Last two digits = 48 which is divisible by 4 thus 1348 is divisible by 4

Last three digits = 348 which is not divisible by 8 thus 1348 is not divisible by 8

(b) 2516

Last two digits = 16 which is divisible by 4 thus 2516 is divisible by 4

Last three digits = 516 which is not divisible by 8 thus 2516 is not divisible by 8

(c) 1332

Last two digits = 32 which is divisible by 4 thus 1332 is divisible by 4

Last three digits = 332 which is not divisible by 8 thus 1332 is not divisible by 8

(d) 10236

Last two digits = 36 which is divisible by 4 thus 10236 is divisible by 4

Last three digits = 236 which is not divisible by 8 thus 10236 is not divisible by 8

(e) 8612

Last two digits = 12 which is divisible by 4 thus 8612 is divisible by 4

Last three digits = 612 which is not divisible by 8 thus 8612 is not divisible by 8

Replacing alphabets by suitable numerals

Let us understand this by examples

Examples:

Example 1 - Replace A, B, C, D by suitable numerals in the following:

6 A B 5

+ D 5 8 C

9 3 5 1

Solution - We can see that C = 6

Now$5 + 6 = 11$, so 1 is carried over

$(1 + 6 + 8) = 15$, so 1 is carried over => B = 6

$(1 + 7 + 5) = 13$, so 1 is carried over => A = 7

$(1 + 6 + 2) = 9$ => D = 2

Therefore, $A = 7, B = 6, C = 6, D = 2$

Example 2 - Find the values of A, B and C in the following:

3 5 A

- C B 8

1 8 3

Solution - We know that$(11 - 8) = 3$, so A must be 1 and 1 is taken away from 5

Now $(14 - B) = 8$ =>B must be 6 and 1 is taken away from 3

Also, $(2 - C) = 1$=> C must be 1

Therefore, A = 1, B = 6 and C = 1

Ex3 find the values of A and B in the following:

4 A

$\times$ 6

2 B 4

Solution - Since $6 \times 4 = 24$ so A must be 4. Carry over 2

Now $6 \times 4 + 2 = 24 + 2 = 26$ so B must be 6

Therefore, A = 4 and B = 6

Example 4 - Find the values of A, B, C; when

$$\begin{array}{r} A\ B \\ \underline{\times\ B\ A} \\ \underline{B\ C\ B} \end{array}$$

Solution - We can see from above that $B \times A = B$=> A = 1

Thus, the given problem becomes:

$$\begin{array}{r} 1\ B \\ \underline{\times\ B\ 1} \\ 1\ B \\ \underline{B\ B^2 \times\ } \\ \underline{B\ (1 + B^2)\ B} \end{array}$$

Now $1 + B^2$ is a one digit number

And B $\neq A = 1$

So if B= 2 then it will be one digit number

Also $C = 1 + B^2 = 1 + 2^2 = 1 + 4 = 5$

Therefore, A = 1, B = 2, C = 5

Example 5 - Find the values of A, B, C in the following:

$$\begin{array}{l} 9\ )\ 4\ A\ B\ (\ 5\ C \\ \quad\ \ \underline{-4\ 5} \\ \qquad\ 3\ B \\ \qquad\ \underline{-3\ 6} \\ \qquad\quad 0 \end{array}$$

Solution - We can see that $(A - 5) = 3$=> A = 8

Also $9 \times 4 = 36$ => C = 4 and B = 6

Therefore, A = 8, B = 6 and C = 4

Example 6 - Find the values of A, B, C, D, E, F, G in the following:

.

```
A B ) 4 C D E ( 1 F G
      -2 8
      ------
       1 5 D
      -1 4 0
      --------
         1 6 E
        -1 6 E
      ---------
           0
```

Solution - We can see that A must be 2 and B must be 8

Now $(13 - 8) = 5$=> C must be 3

Now $28 \times 5 = 140$=> F must be 5

Also, $(D - 0) = 6$=> D = 6

Also,$28 \times 6 = 168$ thus E must be 8 and G is 6

Therefore, A = 2, B = 8, C = 3, D = 6, E = 8, F = 5 and G= 6

Exercise 5C

Replace A, B, C by suitable numerals.

Question 1

```
  5 A
+ 8 7
 C B 3
```

Solution - Since $(6 + 7) = 13$=> A must be 6 and 1 is carried over

Now $1 + 5 + 8 = 14$=> C is 1 and B is 4

Therefore, A = 6, B = 4, C = 1

Question 2

```
  4 C B 6
+ 3 6 9 A
  8 1 7 3
```

Solution - Since $(6 + 7) = 13$=> A must be 7 and 1 is carried over.

Now $(1 + 7 + 9) = 17$=> B must be 7 and 1 is carried over

Also, $(1 + 4 + 6) = 11$=> C must be 4 and 1 is carried over.

$(1 + 4 + 3) = 8$

Therefore, A = 7, B = 7, C = 4

Question 3

```
  A
 +A
 +A
 B A
```

Solution - Since $A + A + A$ is a two digit number

So if we take A=4

Then, $4 + 4 + 4 = 12$ which is not of the form BA

Now take A =5

Then, $5 + 5 + 5 = 15$ which is of the form BA

Therefore, A =5 and B = 1

Question 4

$$\begin{array}{r} 6\,A \\ -\,A\,B \\ \hline 3\,7 \end{array}$$

Solution - Starting from left $6 - A = 3$

Thus, A must be less than or equal to 3

A≤ 3

Now next column: $A - B = 7$ and borrowing is involved so 1 is taken from 6

=> $6 - 1 - A = 3$=> $5 - A = 3$=> A=2

Now we know that $(12 - 5) = 7$

Then, B must be 5

Therefore, A = 2 and B = 5

Question 5

$$\begin{array}{r} C\,B\,5 \\ -\ 2\,8\,A \\ \hline 2\,5\,9 \end{array}$$

Solution - Since $(15 - 6) = 9$=> A must be 6 and 1 is taken away from B

Now $(B - 1 - 8) = 5$ =>$B - 9 = 5$

Since $14 - 9 = 5$=> B must be 4 and 1 is taken away from C

=> $C - 1 - 2 = 2$

=> $C = 5$

Therefore, A = 6, B = 4, C = 5

Question 6

$$\begin{array}{r} A\ B \\ \underline{\times\ 3} \\ \underline{C\ A\ B} \end{array}$$

Solution - Since $B \times 3 = B$=> B can be 0 or 5

If B=5

Then $5 \times 3 = 15$ and 1 is carried over.

Now, $3 \times A + 1 = A$ is not possible for any number

Therefore, B must be 0

Now, $3 \times A + 0 = A$

=> $3 \times A = A$

It is possible either for 0 or 5

If A=0, then all numbers will become zero which is not possible

If A=5 then $3 \times 5 = 15$ and 1 is carried over.

Then, C must be 1.

Therefore, A=5, B=0, C=1

Question 7

$$\begin{array}{r} A\ B \\ \underline{\times\ B\ A} \\ \underline{(B+1)\ C\ B} \end{array}$$

Solution - Since $B \times A = B$

=> A = 1

Thus the given problem becomes:

1 B

$\times$ B 1

1 B

B B^2 $\times$

B $(1+B^2)$ B

Now $1+B^2$ is a one digit number.

And B $\neq A = 1$

Here, first digit is B+1

Thus, 1 will be carried from B^2+1

So it becomes $(B+1)(B^2-9)B$

=> $C = B^2 - 9$

Now All of B, B+1, B^2-9 are one digit number which is only possible when B = 3

Thus, $C = 9^2 - 9 = 0$

Therefore, A = 1, B = 3 and C = 0

Question 8

6) 5 A B (9 C

-5 4

3 B

-3 6

0

Solution - We can see that $(A-4) = 3$=> A = 7

Also, $6 \times 6 = 36$ => C = 6 and B = 6

Therefore, A = 7, B = 6 and C = 6

Question 9 - Find two numbers whose product is a 1-digit number and the sum is a 2-digit number.

Solution - If we take one number be 1

And another number be 9

Then, product of these two numbers $= 1 \times 9 = 9$, which is a one digit number

Sum of these two numbers $= 1 + 9 = 10$, which is a two digit number

Question 10 - Find three whole numbers whose product and sum are equal

Solution - If we take numbers 1, 2 and 3

Then, product = $1 \times 2 \times 3 = 6$

Sum = $1 + 2 + 3 = 6$

Thus, both sum and product are equal.

Question 11 - Complete the magic square given below, so that the sum of the numbers in each row or in each column or along each diagonal is 15.

6	**1**	**a**
b	**5**	**c**
d	**e**	**f**

Solution - Since it is given that sum of numbers in each row or column or along diagonal = 15

Now we can see that $6 + 1 + a = 15 => 7 + a = 15 => a = 15 - 7 => a = 8$

Also, $a + 5 + d = 15 => 8 + 5 + d = 15 => 13 + d = 15 => d = 15 - 13 = 2$

$6 + b + d = 15 => 6 + b + 2 = 15 => 8 + b = 15 => b = 15 - 8 = 7$

$b + 5 + c = 15 => 7 + 5 + c = 15 => 12 + c = 15 => c = 15 - 12 = 3$

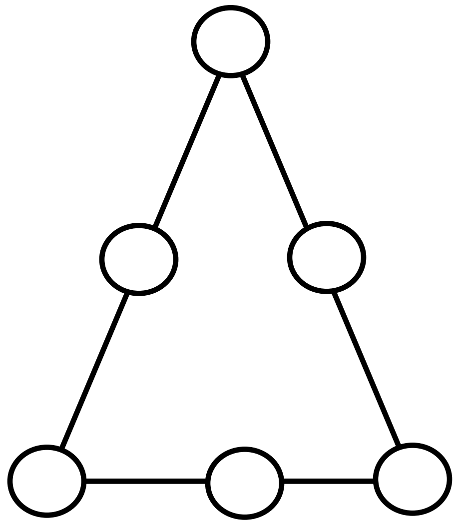
$a + c + f = 15 => 8 + 3 + f = 15 => 11 + f = 15 => f = 15 - 11 = 4$

$1 + 5 + e = 15 => 6 + e = 15 => e = 15 - 6 = 9$

Thus, magic square becomes:

6	**1**	**8**
7	**5**	**3**
2	**9**	**4**

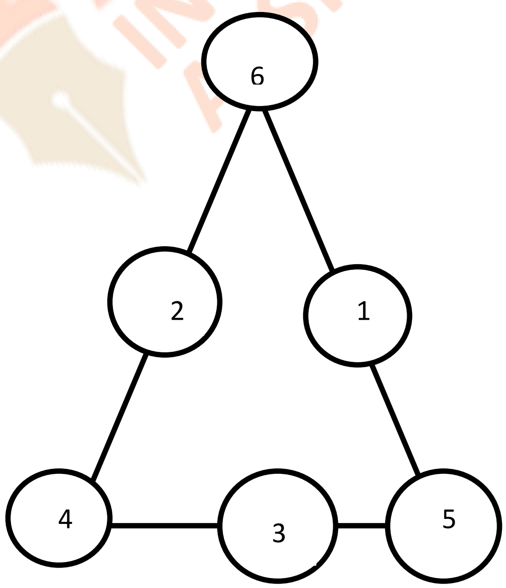
Question 12 - Fill in the numbers from 1 to 6 without repetition, so that each side of the triangle adds up to 12.



Solution - Since $6 + 1 + 5 = 12$

$6 + 2 + 4 = 12$

$4 + 5 + 3 = 12$



Question 13 - Fibonacci numbers: Take 10 numbers as shown below:

a, b, (a+b), (a+2b), (2a+3b), (3a+5b), (5a+8b), (8a+13b), (3a+21b), and (21a+34b)

Sum of all these numbers = 11(5a+8b)= 11 ×7th number

Taking a= 8, b= 13: write 10 Fibonacci numbers and verify that sum of all these numbers= 11 × 7th number.

Solution - Fibonacci numbers are: I, II, (I+II), (II+III), (III+IV), (IV+V), (V+VI), (VI+VII), (VII+VIII), (VIII+IX), (IX+X)

We are given a = 8 and b = 13

Numbers are: 8, 13, 21, 34, 55, 89, 144, 233, 377, 610.

To verify: Sum of all these numbers = 11(5a+8b) = 11 ×7th number

LHS: Sum of all numbers= $8 + 13 + 21 + 34 + 55 + 89 + 144 + 233 + 377 + 610$ =1584

RHS: 11×7th number = $11 \times 144 = 1584$

Thus, LHS=RHS

Question 14 - Complete the magic square:

a	**14**	**b**	**0**
8	**c**	**6**	**11**
4	**d**	**e**	**7**
f	**2**	**1**	**12**

Solution - Since we know that in square sum of each row = Sum of each column = sum of each diagonal

We see that in last column $0 + 11 + 7 + 12 = 30$

In 4th row $f + 2 + 1 + 12 = 30 => f + 15 = 30 => f = 15$

In diagonal $0 + 6 + d + f = 30 => 6 + d + 15 = 30 => 21 + d = 30 => d = 9$

In 2nd row $8 + c + 6 + 11 = 30 => 25 + c = 30 => c = 5$

In 3rd row $4 + 9 + e + 7 = 30 => 20 + e = 30 => e = 10$

In 1st column $a + 8 + 4 + 15 = 30 => a + 27 = 30 => a = 3$

In 3rd column $b + 6 + 10 + 1 = 30 => b + 17 = 30 => b = 13$

Therefore, magic square becomes

3	14	13	0
8	5	6	11
4	9	10	7
15	2	1	12

Exercise 5D

Question 1 - If 5x6 is exactly divisible by 3, then the least value of x is

Solution - It is given that 5x6 is exactly divisible by 3

Then, $5 + x + 6$ must be divisible by 3

=> $11 + x$ must be divisible by 3

Thus, x can take values 1, 4 or 7

So, least value of x is 1

Question 2 - If 64y8 is exactly divisible by 3, then the least value of y is

Solution - It is given that 64y8 is exactly divisible by 3

Then, $6 + 4 + y + 8$ must be divisible by 3

=> $18 + y$ must be divisible by 3

Thus, y can take values 0, 3, 6 or 9

So, least value of y is 0

Question 3 - If 7x8 is exactly divisible by 9, then the least value of x is

Solution - It is given that 7x8 is exactly divisible by 9

Then, $7 + x + 8$ must be divisible by 9

=> $15 + x$ must be divisible by 9

Thus, x can take value 3 only

So, least value of x is 3

Question 4 - If 37y4 is exactly divisible by 9, then the least value of y is

Solution - It is given that 37y4 is exactly divisible by 9

Then, $3 + 7 + y + 4$ must be divisible by 9

=> $14 + y$ must be divisible by 9

Thus, y can take value 4 only

So, least value of x is 4

Question 5 - If 4xy7 is exactly divisible by 3, then the least value of $x + y$ is

Solution - It is given that 4xy7 is exactly divisible by 3

Then, $4 + x + y + 7$ must be divisible by 3

=> $11 + x + y$ must be divisible by 3

Thus, (x + y) can take values 1, 4 or 7

So, least value of (x+y) is 1

Question 6 - If x7y5 is exactly divisible by 3, then the least value of $x + y$ is

Solution - It is given that x7y5 is exactly divisible by 3

Then, $x + 7 + y + 5$ must be divisible by 3

=> $12 + x + y$ must be divisible by 3

Thus, (x + y) can take values 0, 3, 6 or 9

But $x + y$ cannot be 0 (if $x + y = 0$ then both x and y must be 0 which is not possible because x is the first digit)

So, least value of (x+y) is 3

Question 7 - If x4y5z is exactly divisible by 9, then the least value of $x + y + z$ is

Solution -It is given that x4y5z is exactly divisible by 9

Then, $x + 4 + y + 5 + z$ must be divisible by 9

=> $9 + x + y + z$ must be divisible by 9

Thus, $x + y + z$ can take value 0 or 9

But $x + y + z$ cannot be 0 (if $x + y + z = 0$ then all x, y, z must be 0 which is not possible because x is the first digit)

So, least value of $x + y + z$ is 9

Question 8 - If 1A2B5 is exactly divisible by 9, then the least value of $(A + B)$ is

Solution - It is given that 1A2B5 is exactly divisible by 9

Then, $1 + A + 2 + B + 5$ must be divisible by 9

=> $8 + A + B$ must be divisible by 9

Thus, $A + B$ can take value 1 only

So, least value of $A + B$ is 1

Question 9 - If the 4-digit number x27y is exactly divisible by 9, then the least value of $x + y$ is

Solution - It is given that x27y is exactly divisible by 9

Then, $x + 2 + 7 + y$ must be divisible by 9

=> $9 + x + y$ must be divisible by 9

Thus, $x + y$ can take value 0 or 9 only

But $x + y$ cannot be 0 (if $x + y = 0$ then all x, y must be 0 which is not possible because x is the first digit)

So, least value of $x + y$ is 9