Chapter 6

Exponents

Exponents are a way to indicate the continued product of a number multiplied with itself a number of times.

It can be represented as: $(\frac{a}{b})^n = \frac{a^n}{b^n}$ where $\frac{a}{b}$ is any rational number.

Examples:

Example 1 – Find the value of each of the following:

(i) $\mathbf{11^2}$

Solution: We can write $11^2 = 11 \times 11 = 121$

(ii) $\mathbf{9^3}$

Solution: We can write $9^3 = \underline{9 \times 9} \times 9 = 81 \times = 729$

(iii) $\mathbf{5^4}$

Solution: We can write $5^4 = \underline{5 \times 5} \times \underline{5 \times 5} = 25 \times 25 = 625$

Example 2 – Find the value of each of the following:

(i) $(\mathbf{-3})^2$

Solution: We can write $(-3)^2 = (-3) \times (-3) = 9$

(ii) $(\mathbf{-4})^3$

Solution: We can write $(-4)^3 = (\underline{-4) \times (-4}) \times (-4) = (16) \times (-4) = -64$

(iii) $(\mathbf{-5})^4$

Solution: We can write $(-5)^4 = (\underline{-5) \times (-5}) \times (\underline{-5) \times (-5}) = (25) \times (25) = 625$

Example 3 – Simplify:

(i) 2×10^3

Solution: We can write $2 \times 10^3 = 2 \times 10 \times 10 \times 10 = 2 \times 1000 = 2000$

(ii) $7^2 \times 2^2$

Solution: We can write $\mathbf{7^2} \times \mathbf{2^2} = \underline{7 \times 7} \times \underline{2 \times 2} = 49 \times 4 = 196$

(iii) $2^3 \times 5$

Solution: We can write $\mathbf{2^3} \times \mathbf{5} = \underline{2 \times 2} \times \underline{2 \times 5} = 4 \times 10 = 40$

(iv) 0×10^2

Solution: We can write $0 \times 10^2 = 0 \times \underline{10 \times 10} = 0 \times 100 = 0$

Example 4 – Simplify:

(i) $5^2 \times 3^3$

Solution: We can write $5^2 \times 3^3 = \underline{5 \times 5} \times \underline{3 \times 3} \times 3 = 25 \times \underline{9 \times 3} = 25 \times 27 = 675$

(ii) $2^4 \times 3^2$

Solution: We can write $2^4 \times 3^2 = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{3 \times 3} = \underline{4 \times 4} \times 9 = 16 \times 9 = 144$

(iii) $3^2 \times 10^4$

Solution: We can write $3^2 \times 10^4 = \underline{3 \times 3} \times \underline{10 \times 10} \times \underline{10 \times 10} = 9 \times \underline{100 \times 100} = 9 \times 10000 = 90000$

(iv) $5^3 \times 2^4$

Solution: We can write $5^3 \times 2^4 = \underline{5 \times 5} \times 5 \times \underline{2 \times 2} \times \underline{2 \times 2} = 25 \times 5 \times \underline{4 \times 4} = 125 \times 16 = 2000$

Example 5 – Simplify:

(i) $(-\mathbf{3}) \times (-\mathbf{2})^{\mathbf{3}}$

Solution: We can write $(-3) \times (-2)^3 = \underline{(-3) \times (-2)} \times \underline{(-2) \times (-2)} = (6) \times (4) = 24$

(ii) $(-\mathbf{3})^{\mathbf{2}} \times (-\mathbf{5})^{\mathbf{2}}$

Solution: We can write $(-3)^2 \times (-5)^2 = \underline{(-3) \times (-3)} \times \underline{(-5) \times (-5)} = (9) \times (25) = 225$

(iii) $(-\mathbf{2})^{\mathbf{3}} \times (-\mathbf{10})^{\mathbf{3}}$

Solution: We can write $(-2)^3 \times (-10)^3 = \underline{(-2) \times (-2)} \times \underline{(-2) \times (-10)} \times \underline{(-10) \times (-10)} = 4 \times 20 \times 100 = 80 \times 100 = 8000$

(iv) $(-\mathbf{2})^{\mathbf{4}} \times (-\mathbf{5})^{\mathbf{2}}$

Solution: We can write $(-2)^4 \times (-5)^2 = \underline{(-2) \times (-2)} \times \underline{(-2) \times (-2)} \times \underline{(-5) \times (-5)} = 4 \times 4 \times 25 = 16 \times 25 = 400$

Example 6 – Simplify:

(i) $(\mathbf{1})^{\mathbf{5}}$

Solution: We can write $(1)^5 = 1 \times 1 \times 1 \times 1 \times 1 = 1$

(ii) $(-\mathbf{1})^{\mathbf{5}}$

Solution: We can write $(-1)^5 = \underline{(-1) \times (-1)} \times \underline{(-1) \times (-1)} \times (-1) = \underline{(1) \times (1)} \times (-1) = 1 \times (-1) = -1$

(iii) $(-\mathbf{1})^{\mathbf{6}}$

Solution: We can write $(-1)^6 = \underline{(-1) \times (-1)} \times \underline{(-1) \times (-1)} \times \underline{(-1) \times (-1)} = (1) \times (1) \times (1) = 1$

(iv) $(-\mathbf{1})^{\mathbf{7}}$

Solution: We can write $(-1)^7 = \underline{(-1) \times (-1)} \times \underline{(-1) \times (-1)} \times \underline{(-1) \times (-1)} \times (1) = (1) \times (1) \times (1) \times (-1) = -1$

Example 7 – Express each of the following in the form $\frac{p}{q}$:

(i) $(\frac{2}{3})^2$

Solution: We can write $(\frac{2}{3})^2 = \frac{2}{3} \times \frac{2}{3} = \frac{2\times2}{3\times3} = \frac{4}{9}$

(ii) $(\frac{-3}{4})^3$

Solution: We can write $(\frac{-3}{4})^3 = \frac{-3}{4} \times \frac{-3}{4} \times \frac{-3}{4} = \frac{(-3)\times(-3)\times(-3)}{4\times4\times4} = \frac{9\times(-3)}{16\times4} = \frac{-27}{64}$

(iii) $(\frac{-2}{5})^4$

Solution: We can write $(\frac{-2}{5})^4 = \frac{-2}{5} \times \frac{-2}{5} \times \frac{-2}{5} \times \frac{-2}{5} = \frac{(-2)\times(-2)\times(-2)\times(-2)}{5\times5\times5\times5} = \frac{4\times4}{25\times25} = \frac{16}{625}$

Example 8 – Identify the greater number in each of the following:

(i) $5^3 or\ 3^5$

Solution: We can write $5^3 = 5 \times 5 \times 5 = 25 \times 5 = 125$ and $3^5 = 3 \times 3 \times 3 \times 3 \times 3 = 9 \times 9 \times 3 = 81 \times 3 = 243$

Clearly, $243 > 125$

=> $3^5 > 5^3$

(ii) $2^8 or\ 8^2$

Solution: We can write $2^8 = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} = 4 \times 4 \times 4 \times 4 = 16 \times 16 = 256$ and $8^2 = 8 \times 8 = 64$

Clearly, $256 > 64$

=> $2^8 > 8^2$

(iii) $2^{10} or\ 10^2$

Solution: We can write $2^{10} = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} = 4 \times 4 \times 4 \times 4 \times 4 = 16 \times 16 \times 4 = 256 \times 4 = 1024$ and $10^2 = 10 \times 10 = 100$

Clearly, $1024 > 100$

=> $2^{10} > 10^2$

(iv) $2^{100} or\ 100^2$

Solution: We can see that $2^{10} > 10^2$ and $2^8 > 8^2$ in the above examples

Therefore, we can say that $2^{100} > 100^2$

Example 9 – Find the product of the cube of $\frac{-2}{3}$ and the square of $\frac{4}{-5}$

Solution: We will find $(\frac{-2}{3})^3 \times (\frac{4}{-5})^2$

=> $\frac{-2}{3} \times \frac{-2}{3} \times \frac{-2}{3} \times \frac{4}{-5} \times \frac{4}{-5}$

=> $\frac{(-2)\times(-2)\times(-2)}{3\times3\times3} \times \frac{4\times4}{(-5)\times(-5)}$

=> $\frac{-8}{27} \times \frac{16}{25} = \frac{(-8)\times16}{27\times25} = \frac{-128}{675}$

Example 10 – Express the following as a rational number:

$$\left(\frac{1}{2}\right)^3 \times \left(\frac{-3}{5}\right)^3 \times \left(\frac{-4}{9}\right)^2$$

Solution: We have $(\frac{1}{2})^3 \times (\frac{-3}{5})^3 \times (\frac{-4}{9})^2$

=> $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{-3}{5} \times \frac{-3}{5} \times \frac{-3}{5} \times \frac{-4}{9} \times \frac{-4}{9}$

=> $\frac{1}{8} \times \frac{(-3)\times(-3)\times(-3)}{5\times5\times5} \times \frac{(-4)\times(-4)}{9\times9}$

=> $\frac{1}{8} \times \frac{-27}{125} \times \frac{16}{81}$

=> $\frac{-\overset{-1}{\cancel{27}}\times\overset{2}{\cancel{16}}}{\cancel{8}\times125\times\underset{3}{\cancel{81}}}$

=> $\frac{-1\times2}{125\times3}$

=> $\frac{-2}{375}$

Question 11 – Simplify:

(i) $(-\mathbf{3})^{\mathbf{2}} \times \left(\frac{-\mathbf{5}}{\mathbf{12}}\right)^{\mathbf{2}}$

Solution: We have $(-3)^2 \times \left(\frac{-5}{12}\right)^2$

=> $(-3) \times (-3) \times \frac{-5}{12} \times \frac{-5}{12}$

=> $9 \times \frac{(-5)\times(-5)}{12\times12}$

=> $9 \times \frac{25}{144}$

=> $\frac{\cancel{9}\times25}{\underset{16}{\cancel{144}}}$

=> $\frac{25}{16}$

(ii) $(\frac{-\mathbf{2}}{\mathbf{5}})^{\mathbf{3}} \div (\frac{-\mathbf{3}}{\mathbf{10}})^{\mathbf{4}}$

Solution: Firstly we will solve $\left(\frac{-\mathbf{2}}{\mathbf{5}}\right)^{\mathbf{3}}$

=> $\frac{-2}{5} \times \frac{-2}{5} \times \frac{-2}{5}$

=> $\frac{(-2)\times(-2)\times(-2)}{5\times5\times5} = \frac{-8}{125}$

Now, $(\frac{-3}{10})^4$

=> $\frac{-3}{10} \times \frac{-3}{10} \times \frac{-3}{10} \times \frac{-3}{10}$

=> $\frac{(-3)\times(-3)\times(-3)\times(-3)}{10\times10\times10\times10} = \frac{81}{10000}$

Thus, we have $\left(\frac{-2}{5}\right)^3 \div (\frac{-3}{10})^4$

=> $\frac{-8}{125} \div \frac{81}{10000}$

=> $\frac{-8}{\cancel{125}} \times \frac{\overset{80}{\cancel{10000}}}{81}$

=> $\frac{-8\times80}{81} = \frac{-640}{81}$

Example 12 – Simplify:

(i) $\boldsymbol{\{\left(\frac{1}{2}\right)^2 - (\frac{1}{4})^3\} \times 2^3}$

Solution: We have $\{\left(\frac{1}{2}\right)^2 - (\frac{1}{4})^3\} \times 2^3$

=> $\left\{\left(\frac{1}{2}\times\frac{1}{2}\right) - \left(\frac{1}{4}\times\frac{1}{4}\times\frac{1}{4}\right)\right\} \times 2^3$

=> $\left\{\frac{1}{4} - \frac{1}{64}\right\} \times 2^3$

Now, LCM of (4 and 64) is 64

Thus, $\frac{1}{4} = \frac{1\times16}{4\times16} = \frac{16}{64}$ (Multiplying each term by 16)

Thus, $\left\{\frac{1}{4} - \frac{1}{64}\right\} \times 2^3 = \left\{\frac{16}{64} - \frac{1}{64}\right\} \times 2^3$

=> $\frac{(16-1)}{64} \times 2^3$

=> $\frac{15}{64} \times 2^3$

=> $\frac{15}{64} \times 8 = \frac{15\times\cancel{8}}{\underset{8}{\cancel{64}}}$

=> $\frac{15}{8}$

(ii) $\{(\mathbf{3^2-2^2})\div(\frac{\mathbf{1}}{\mathbf{5}})^{\mathbf{2}}\}$

Solution: We have $\{(\mathbf{3^2-2^2})\div(\frac{\mathbf{1}}{\mathbf{5}})^{\mathbf{2}}\}$

=> $\left\{(3\times 3-2\times 2)\div\left(\frac{1}{5}\times\frac{1}{5}\right)\right\}$

=> $\left\{(9-4)\div\left(\frac{1}{25}\right)\right\}$

=> $\left\{5\div\frac{1}{25}\right\}$

=> $5\times\frac{25}{1}=5\times 25=125$

Example 13 – If a = 2 and b = 3, then find the values of each of the following:

(i) $\boldsymbol{a^a+b^b}$

Solution: Since $a=2\ and\ b=3$

Thus, $a^a+b^b=\ (2)^2+(3)^3$

=> $(2\times 2)+(3\times 3\times 3)$

=> $4+27=31$

(ii) $\boldsymbol{a^b+b^a}$

Solution: Since $a=2\ and\ b=3$

Thus, $a^b+b^a=\ (2)^3+(3)^2$

=> $(2\times 2\times 2)+(3\times 3)$

=> $8+9=17$

(iii) $\left(\boldsymbol{\frac{a}{b}}\right)^{\boldsymbol{a}}$

Solution: Since $a = 2\ and\ b = 3$

Thus, $\left(\boldsymbol{\frac{a}{b}}\right)^{\boldsymbol{a}} = \left(\boldsymbol{\frac{2}{3}}\right)^{\boldsymbol{2}}$

=> $\frac{2}{3} \times \frac{2}{3} = \frac{2\times2}{3\times3} = \frac{4}{9}$

(iv) $\left(\boldsymbol{\frac{1}{a} + \frac{1}{b}}\right)^{\boldsymbol{a}}$

Solution: Since $a = 2\ and\ b = 3$

Thus, $\left(\frac{1}{a} + \frac{1}{b}\right)^{a} = \left(\frac{1}{2} + \frac{1}{3}\right)^{2}$

Now, LCM of 2 and 3 is 63

=> $\frac{1}{2} = \frac{1\times3}{2\times3} = \frac{3}{6}$ (Multiplying each term by 3) and

$\frac{1}{3} = \frac{1\times2}{3\times2} = \frac{2}{6}$(Multiplying each term by 2)

Thus, $\left(\frac{1}{2} + \frac{1}{3}\right)^{2} = \left(\frac{3}{6} + \frac{2}{6}\right)^{2}$

=> $\left(\frac{3+2}{6}\right)^{2} = \left(\frac{5}{6}\right)^{2}$

=> $\frac{5}{6} \times \frac{5}{6} = \frac{5\times5}{6\times6} = \frac{25}{36}$

Example 14 – Simplify and express each of the following as power of a rational number:

(i) $(\boldsymbol{\frac{2}{3}})^{\boldsymbol{3}} \times \left(-\boldsymbol{\frac{6}{7}}\right)^{\boldsymbol{2}} \times \left(-\boldsymbol{\frac{7}{4}}\right) \times \boldsymbol{\frac{3}{2}}$

Solution: We have $(\boldsymbol{\frac{2}{3}})^{\boldsymbol{3}} \times \left(-\boldsymbol{\frac{6}{7}}\right)^{\boldsymbol{2}} \times \left(-\boldsymbol{\frac{7}{4}}\right) \times \boldsymbol{\frac{3}{2}}$

=> $\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{-6}{7} \times \frac{-6}{7} \times \frac{-7}{4} \times \frac{3}{2}$

=> $\left(\frac{2\times2\times2}{3\times3\times3}\right)\times\frac{(-6)\times(-6)}{7\times7}\times\frac{-7}{4}\times\frac{3}{2}$

=> $\frac{8}{27}\times\frac{36}{49}\times\frac{-7}{4}\times\frac{3}{2}$

4 9 -1

=> $\frac{\cancel{8}\times\cancel{36}\times(\cancel{-7})\times\cancel{3}}{\cancel{27}\times\cancel{49}\times\cancel{4}\times\cancel{2}}$

7

=> $\frac{4\times-1}{7}=\frac{-4}{7}=(\frac{-4}{7})^1$

(ii) $-\left(\frac{2}{5}\right)^2\times\left(\frac{5}{7}\right)^2\times\frac{49}{5}+\left(-\frac{4}{5}\right)^3\times\frac{5}{4}\times\frac{3}{4}$

Solution: We have $-\left(\frac{2}{5}\right)^2\times\left(\frac{5}{7}\right)^2\times\frac{49}{5}+\left(-\frac{4}{5}\right)^3\times\frac{5}{4}\times\frac{3}{4}$

=> $-\left(\frac{2}{5}\times\frac{2}{5}\right)\times\left(\frac{5}{7}\times\frac{5}{7}\right)\times\frac{49}{5}+\left(-\frac{4}{5}\times-\frac{4}{5}\times-\frac{4}{5}\right)\times\frac{5}{4}\times\frac{3}{4}$

=> $-\left(\frac{2\times2}{5\times5}\right)\times\left(\frac{5\times5}{7\times7}\right)\times\frac{49}{5}+\left(\frac{(-4)\times(-4)\times(-4)}{5\times5\times5}\right)\times\frac{5}{4}\times\frac{3}{4}$

=> $-\frac{4}{25}\times\frac{25}{49}\times\frac{49}{5}+(-\frac{64}{125})\times\frac{5}{4}\times\frac{3}{4}$

4

=> $\frac{-4\times\cancel{25}\times\cancel{49}}{\cancel{25}\times\cancel{49}\times5}+\frac{-\cancel{64}\times\cancel{5}\times3}{\cancel{125}\times\cancel{4}\times\cancel{4}}$

25

=> $\frac{-4}{5}+(\frac{-4\times3}{25})$

=> $\frac{-4}{5}+(-\frac{12}{25})$

=> $\frac{-4}{5}-\frac{12}{25}$ (Since (+, - = -))

Now, LCM of (5 and 25) is 25

Thus, $\frac{-4}{5}=\frac{-4\times5}{5\times5}=\frac{-20}{25}$ (Multiplying each term by 5)

$\frac{12}{25}=\frac{12\times1}{25\times1}=\frac{12}{25}$ (Multiplying each term by 1)

=> $\frac{-4}{5}-\frac{12}{25}=\frac{-20}{25}-\frac{12}{25}=\frac{-20-12}{25}=\frac{-32}{25}=(\frac{-32}{25})^1$

Example 15 – Express each of the following in exponential form:

(i) $(-4) \times (-4) \times (-4) \times (-4) \times (-4) \times (-4)$

Solution: We can see that number of times (-4) is occurring = 6

Thus, $(-4) \times (-4) \times (-4) \times (-4) \times (-4) \times (-4) = (-4)^6$

(ii) $\frac{3}{5} \times \frac{3}{5} \times \frac{3}{5} \times \frac{3}{5}$

Solution: We can see that number of times $\frac{3}{5}$ is occurring = 4

Thus, $\frac{3}{5} \times \frac{3}{5} \times \frac{3}{5} \times \frac{3}{5} = (\frac{3}{5})^4$

Example 16 – Express each of the following in exponential form:

(i) $2 \times 2 \times 2 \times a \times a$

Solution: We can see that number of times 2 is occurring = 3

Number of times ‘a’ is occurring = 2

Thus, $2 \times 2 \times 2 \times a \times a = (2)^3 \times a^2$

(ii) $a \times a \times a \times a \times b \times b \times c \times c \times c \times c \times c$

Solution: We can see that number of times ‘a’ is occurring = 4

Number of times ‘b’ is occurring = 2

Number of times ‘c’ is occurring = 5

Thus, $a \times a \times a \times a \times b \times b \times c \times c \times c \times c \times c = a^4 \times b^2 \times c^5 = a^4b^2c^5$

(iii) $a \times a \times a \times (\frac{-2}{3}) \times (\frac{-2}{3})$

Solution: We can see that number of times ‘a’ is occurring = 3

Number of times $\frac{-2}{3}$ is occurring = 2

Thus, $a \times a \times a \times (\frac{-2}{3}) \times (\frac{-2}{3}) = (a)^3 \times (\frac{-2}{3})^2$

Example 17 – Express each of the following numbers in exponential form:

(i) 128

Solution: Here, we will first factorise 128 as follows:

=> $128 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

We observe that 2 is occurring 7 times.

Thus, $128 = 2^7$

2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

(ii) 243

Solution: Here, we will first factorise 243 as follows:

=> $128 = 3 \times 3 \times 3 \times 3 \times 3$

We observe that 3 is occurring 5 times.

Thus, $243 = 3^5$

3	243
3	81
3	27
3	9
3	3
	1

(iii) 3125

Solution: Here, we will first factorise 128 as follows:

5	3125
5	625
5	125
5	25
5	5
	1

=> $3125 = 5 \times 5 \times 5 \times 5 \times 5$

We observe that 5 is occurring 5 times.

Thus, $3125 = 5^5$

Example 18 – Express each of the following numbers as a product of powers of their prime factors:

(i) 432

Solution: Here, we will first factorise 432 into prime factors as

follows:

2	432
2	216
2	108
2	54
3	27
3	9
3	3
	1

=> $432 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3$

We observe that number of times 2 is occurring = 4

Number of times 3 is occurring = 3

Thus, $432 = 2^4 \times 3^3$

(ii) 648

Solution: Here, we will first factorise 648 into prime factors as follows:

2	648
2	324
2	162
3	81
3	27
3	9
3	3
	1

=> $648 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3$

We observe that number of times 2 is occurring = 3

Number of times 3 is occurring = 4

Thus, $648 = 2^3 \times 3^4$

(iii) 540

Solution: Here, we will first factorise 540 into prime factors as follows:

2	540
2	270
3	135
3	45
3	15
5	5
	1

=> $540 = 2 \times 2 \times 3 \times 3 \times 3 \times 5$

We observe that number of times 2 is occurring = 2

Number of times 3 is occurring = 3

Number of times 5 is occurring = 1

Thus, $540 = 2^2 \times 3^3 \times 5$

Example 19 – Express the following numbers as product of powers of their prime factors:

(i) 1000

Solution: Here, we will first factorise 1000 into prime factors as follows:

2	1000
2	500
2	250
5	125
5	25
5	5
	1

=> $1000 = 2 \times 2 \times 2 \times 5 \times 5 \times 5$

We observe that number of times 2 is occurring = 3

Number of times 5 is occurring = 3

Thus, $1000 = 2^3 \times 5^3$

(ii) 16000

Solution: Here, we will first factorise 16000 into prime factors as follows:

2	16000
2	8000
2	4000
2	2000
2	1000
2	500
2	250
5	125
5	25
5	5
	1

=> $16000 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 5$

We observe that number of times 2 is occurring = 7

Number of times 5 is occurring = 3

Thus, $16000 = 2^7 \times 5^3$

(iii) 3600

Solution: Here, we will first factorise 3600 into prime factors as follows:

2	3600
2	1800
2	900
2	450
3	225
3	75
5	25
5	5
	1

=> $3600 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5$

We observe that number of times 2 is occurring = 4

Number of times 3 is occurring = 2

Number of times 5 is occurring = 2

Thus, $3600 = 2^4 \times 3^2 \times 5^2$

Example 20 – Express each of the following rational numbers in exponential form:

(i) $\frac{27}{64}$

Solution: Here, we will first factorise 27 and 64 into prime factors as follows:

3	27
3	9
3	3
	1

2	64
2	32
2	16
2	8
2	4
2	2
	1

=> 27 = 3 × 3 × 3

=>$64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 4 \times 4 \times 4$

In 27, we observe that number of times 3 is occurring = 3

In 64, number of times 4 is occurring = 3

Thus, $27 = 3^3$ and $64 = 4^3$

Therefore, $\frac{27}{64} = \frac{3^3}{4^3} = (\frac{3}{4})^3$ (Using the formula $(\frac{a}{b})^n = \frac{a^n}{b^n}$)

(ii) $\frac{-27}{125}$

Solution: Here, we will first factorise 27 and 125 into prime factors as follows:

3	27
3	9
3	3
	1

5	125
5	25
5	5
	1

=> 27 = 3 × 3 × 3

=>125 = 5 × 5 × 5

In 27, we observe that number of times 3 is occurring = 3

In 125, number of times 5 is occurring = 3

Thus, $-27 = (-3)^3$ and $125 = 5^3$

Therefore, $\frac{-27}{125} = \frac{(-3)^3}{5^3} = (\frac{-3}{5})^3$ (Using the formula $(\frac{a}{b})^n = \frac{a^n}{b^n}$)

(iii) $\frac{-1}{243}$

Solution: Here, we will first factorise 243 into prime factors as follows:

3	243
3	81
3	27
3	9
3	3
	1

=> 243 = 3 × 3 × 3 × 3 × 3

In 243, we observe that number of times 3 is occurring = 5

Thus, $-1 = (-1)^5$ and $243 = 3^5$

Therefore, $\frac{-1}{243} = \frac{(-1)^5}{3^5} = (\frac{-1}{3})^5$ (Using the formula $(\frac{a}{b})^n = \frac{a^n}{b^n}$)

Exercise 6.1

Question 1 – Find the value of each of the following:

(i) $\mathbf{13^2}$

Solution: We can write $13^2 = 13 \times 13 = 169$

(ii) $\mathbf{7^3}$

Solution: We can write $7^3 = 7 \times 7 \times 7 = 343$

(iii) $\mathbf{3^4}$

Solution: We can write $3^4 = 3 \times 3 \times 3 \times 3 = 81$

Question 2 – Find the value of each of the following:

(i) $(-7)^2$

Solution: We can write $(-7)^2 = (-7) \times (-7) = 49$

(ii) $(-3)^4$

Solution: We can write $\left(-3\right)^4 = \underline{(-3) \times (-3)} \times \underline{(-3) \times (-3)} = 9 \times 9 = 81$

(iii) $(-5)^5$

Solution: We can write $\left(-5\right)^5 = \underline{(-5) \times (-5)} \times \underline{(-5) \times (-5)} \times (-5) = 25 \times 25 \times (-5) = 625 \times (-5) = -3125$

Question 3 – Simplify:

(i) $\mathbf{3 \times 10^2}$

Solution: We can write $3 \times 10^2 = 3 \times 10 \times 10 = 3 \times 100 = 300$

(ii) $2^2 \times 5^3$

Solution: We can write $2^2 \times 5^3 = 2 \times 2 \times 5 \times 5 \times 5 = 4 \times 25 \times 5 = 100 \times 5 = 500$

(iii) $3^3 \times 5^2$

Solution: We can write $3^3 \times 5^2 = 3 \times 3 \times 3 \times 5 \times 5 = 9 \times 3 \times 25 = 27 \times 25 = 675$

Question 4 – Simplify:

(i) $3^2 \times 10^4$

Solution: We can write $3^2 \times 10^4 = 3 \times 3 \times 10 \times 10 \times 10 \times 10 = 9 \times 100 \times 100 = 9 \times 10000 = 90000$

(ii) $2^4 \times 3^2$

Solution: We can write $2^4 \times 3^2 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 = 4 \times 4 \times 9 = 16 \times 9 = 144$

(iii) $5^2 \times 3^4$

Solution: We can write $5^2 \times 3^4 = 5 \times 5 \times 3 \times 3 \times 3 \times 3 = 25 \times 9 \times 9 = 225 \times 9 = 2025$

Question 5 – Simplify:

(i) $(-2) \times (-3)^3$

Solution: We can write $(-2) \times (-3)^3 = (-2) \times (-3) \times (-3) \times (-3) = 6 \times 9 = 54$

(ii) $(-3)^2 \times (-5)^3$

Solution: We can write $(-3)^2 \times (-5)^3 = (-3) \times (-3) \times (-5) \times (-5) \times (-5) = 9 \times 25 \times (-5) = 225 \times (-5) = -1125$

(iii) $(-2)^5 \times (-10)^2$

Solution: We can write $(-2)^5 \times (-10)^2 = (-2) \times (-2) \times (-2) \times (-2) \times (-2) \times (-10) \times (-10) = 4 \times 4 \times (-2) \times 100 = 16 \times (-2) \times 100 = -32 \times 100 = -3200$

Question 6 – Simplify:

(i) $(\frac{3}{4})^2$

Solution: We can write $(\frac{3}{4})^2 = \frac{3}{4} \times \frac{3}{4} = \frac{3\times3}{4\times4} = \frac{9}{16}$

(ii) $\left(\frac{-2}{3}\right)^4$

Solution: We can write $\left(\frac{-2}{3}\right)^4 = \frac{-2}{3} \times \frac{-2}{3} \times \frac{-2}{3} \times \frac{-2}{3} = \frac{(-2)\times(-2)\times(-2)\times(-2)}{3\times3\times3\times3} = \frac{4\times4}{9\times9} = \frac{16}{81}$

(iii) $\left(\frac{-4}{5}\right)^5$

Solution: We can write $\left(\frac{-4}{5}\right)^5 = \frac{-4}{5} \times \frac{-4}{5} \times \frac{-4}{5} \times \frac{-4}{5} \times \frac{-4}{5} = \frac{(-4)\times(-4)\times(-4)\times(-4)\times(-4)}{5\times5\times5\times5\times5} =$ $\frac{16\times16\times(-4)}{25\times25\times5} = \frac{256\times(-4)}{625\times5} = \frac{-1024}{3125}$

Question 7 – Identify the greater number in each of the following:

(i) 2^5 *or* 5^2

Solution: We can write $2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 4 \times 4 \times 2 = 16 \times 2 = 32$ and $5^2 = 5 \times 5 \times$ $= 25$

Clearly, $32 > 25$

=> $2^5 > 5^2$

(ii) 3^4 *or* 4^3

Solution: We can write $3^4 = 3 \times 3 \times 3 \times 3 = 9 \times 9 = 81$ and $4^3 = 4 \times 4 \times 4 = 16 \times 4 = 64$

Clearly, $81 > 64$

=> $3^4 > 4^3$

(iii) 3^5 *or* 5^3

Solution: We can write $3^5 = 3 \times 3 \times 3 \times 3 \times 3 = 9 \times 9 \times 3 = 81 \times 3 = 243$ and $5^3 = 5 \times 5 \times 5 = 25 \times 5 = 125$

Clearly, $243 > 125$

=> $3^5 > 5^3$

Question 8 – Express each of the following in exponential form:

(i) $(-\mathbf{5}) \times (-\mathbf{5}) \times (-\mathbf{5})$

Solution: We can see that number of times (-5) is occurring = 3

Thus, $(-5) \times (-5) \times (-5) = (-5)^3$

(ii) $(\frac{-\mathbf{5}}{\mathbf{7}}) \times (\frac{-\mathbf{5}}{\mathbf{7}}) \times (\frac{-\mathbf{5}}{\mathbf{7}}) \times (\frac{-\mathbf{5}}{\mathbf{7}})$

Solution: We can see that number of times $\frac{-5}{7}$ is occurring = 4

Thus, $(\frac{-5}{7}) \times (\frac{-5}{7}) \times (\frac{-5}{7}) \times (\frac{-5}{7}) = (\frac{-5}{7})^4$

(iii) $\frac{\mathbf{4}}{\mathbf{3}} \times \frac{\mathbf{4}}{\mathbf{3}} \times \frac{\mathbf{4}}{\mathbf{3}} \times \frac{\mathbf{4}}{\mathbf{3}} \times \frac{\mathbf{4}}{\mathbf{3}}$

Solution: We can see that number of times $\frac{4}{3}$ is occurring = 5

Thus, $\frac{4}{3} \times \frac{4}{3} \times \frac{4}{3} \times \frac{4}{3} \times \frac{4}{3} = (\frac{4}{3})^5$

Question 9 – Express each of the following in exponential form:

(i) $\boldsymbol{x \times x \times x \times x \times a \times a \times b \times b \times b}$

Solution: We can see that number of times 'x' is occurring = 4

Number of times 'a' is occurring = 2

Number of times 'b' is occurring = 3

Thus, $x \times x \times x \times x \times a \times a \times b \times b \times b = x^4 \times a^2 \times b^3 = x^4 a^2 b^3$

(ii) $(-\mathbf{2}) \times (-\mathbf{2}) \times (-\mathbf{2}) \times (-\mathbf{2}) \times \boldsymbol{a} \times \boldsymbol{a} \times \boldsymbol{a}$

Solution: We can see that number of times (-2) is occurring = 4

Number of times 'a' is occurring = 3

Thus, $(-2) \times (-2) \times (-2) \times (-2) \times a \times a \times a = (-2)^4 \times a^3$

(iii) $\left(\frac{-2}{3}\right) \times \left(\frac{-2}{3}\right) \times x \times x \times x$

Solution: We can see that number of times $\frac{-2}{3}$ is occurring = 2

Number of times ‘x’ is occurring = 3

Thus, $\left(\frac{-2}{3}\right) \times \left(\frac{-2}{3}\right) \times x \times x \times x = (\frac{-2}{3})^2 \times x^3$

Question 10 – Express each of the following numbers in exponential form:

(i) 512

Solution: Here, we will first factorise 512 as follows:

=> $512 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

We observe that 2 is occurring 9 times.

Thus, $512 = 2^9$

2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

(ii) 625

Solution: Here, we will first factorise 625 as follows:

=> $625 = 5 \times 5 \times 5 \times 5$

We observe that 5 is occurring 4 times.

Thus, $625 = 5^4$

5	625
5	125
5	25
5	5
	1

(iii) 729

Solution: Here, we will first factorise 729 as follows:

3	729
3	243
3	81
3	27
3	9
3	3
	1

=> $729 = 3 \times 3 \times 3 \times 3 \times 3 \times 3$

We observe that 3 is occurring 6 times.

Thus, $729 = 3^6$

Question 11 – Express each of the following numbers as a product of powers of their prime factors:

(i) 36

Solution: Here, we will first factorise 432 into prime factors as follows:

2	36
2	18
3	9
3	3
	1

=> $36 = 2 \times 2 \times 3 \times 3$

We observe that number of times 2 is occurring = 2

Number of times 3 is occurring = 2

Thus, $36 = 2^2 \times 3^2$

(ii) 675

Solution: Here, we will first factorise 675 into prime factors as follows:

3	675
3	225
3	75
5	25
5	5
	1

=> $675 = 3 \times 3 \times 3 \times 5 \times 5$

We observe that number of times 3 is occurring = 3

Number of times 5 is occurring = 2

Thus, $675 = 3^3 \times 5^2$

(iii) 392

Solution: Here, we will first factorise 392 into prime factors as follows:

2	392
2	196
2	98
7	49
7	7
	1

=> $392 = 2 \times 2 \times 2 \times 7 \times 7$

We observe that number of times 2 is occurring = 3

Number of times 7 is occurring = 2

Thus, $392 = 2^3 \times 7^2$

Question 12 – Express each of the following numbers as a product of powers of their prime factors:

(i) 450

Solution: Here, we will first factorise 450 into prime factors as follows:

3	450
3	150
2	50
5	25
5	5
	1

=> $450 = 3 \times 3 \times 2 \times 5 \times 5$

We observe that number of times 2 is occurring = 1

Number of times 3 is occurring = 2

Number of times 5 is occurring = 2

Thus, $450 = 2 \times 3^2 \times 5^2$

(ii) 2800

Solution: Here, we will first factorise 2800 into prime factors as follows:

2	2800
2	1400
2	700
2	350
5	175
5	35
7	7
	1

=> $2800 = 2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 7$

We observe that number of times 2 is occurring = 4

Number of times 5 is occurring = 2

Number of times 7 is occurring = 1

Thus, $2800 = 2^4 \times 5^2 \times 7$

(iii) 24000

Solution: Here, we will first factorise 24000 into prime factors as follows:

2	24000
2	12000
2	6000
2	3000
2	1500
2	750
3	375
5	125
5	25
5	5
	1

=> $24000 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 5 \times 5 \times 5$

We observe that number of times 2 is occurring = 6

Number of times 3 is occurring = 1

Number of times 5 is occurring = 3

Thus, $24000 = 2^6 \times 5^3 \times 3$

Question 13 – Express each of the following as a rational number of the form $\frac{p}{q}$:

(i) $(\frac{3}{7})^2$

Solution: We can write $\left(\frac{3}{7}\right)^2 = \frac{3}{7} \times \frac{3}{7} = \frac{3\times3}{7\times7} = \frac{9}{49}$

(ii) $(\frac{7}{9})^3$

Solution: We can write $\left(\frac{7}{9}\right)^3 = \frac{7}{9} \times \frac{7}{9} \times \frac{7}{9} = \frac{7\times7\times7}{9\times9\times9} = \frac{343}{729}$

(iii) $(\frac{-2}{3})^4$

Solution: We can write $\left(\frac{-2}{3}\right)^4 = \frac{-2}{3} \times \frac{-2}{3} \times \frac{-2}{3} \times \frac{-2}{3} \times = \frac{(-2)\times(-2)\times(-2)\times(-2)}{3\times3\times3\times3} = \frac{4\times4}{9\times9} = \frac{16}{81}$

Question 14 – Express each of the following rational numbers in power notation:

(i) $\frac{49}{64}$

Solution: Here, we will first factorise 49 and 64 into prime factors as follows:

7	49
7	7
	1

2	64
2	32
2	16
2	8
2	4
2	2
	1

=> 49 = 7 × 7

=>$64 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} = 8 \times 8$

In 49, we observe that number of times 7 is occurring = 2

In 64, number of times 8 is occurring = 2

Thus, $49 = 7^2$ *and* $64 = 8^2$

Therefore, $\frac{49}{64} = \frac{7^2}{8^2} = (\frac{7}{8})^2$ (Using the formula $(\frac{a}{b})^n = \frac{a^n}{b^n}$)

(ii) $\mathbf{\frac{-64}{125}}$

Solution: Here, we will first factorise 64 and 125 into prime factors as follows:

2	64
2	32
2	16
2	8
2	4
2	2
	1

5	125
5	25
5	5
	1

=> 64 = $2 \times 2 \times 2 \times 2 \times 2 \times 2 = 4 \times 4 \times 4$

=>$125 = 5 \times 5 \times 5$

In 64, we observe that number of times 4 is occurring = 3

In 125, number of times 5 is occurring = 3

Thus, $-64 = (-4)^3$ *and* $125 = 5^3$

Therefore, $\frac{-64}{125} = \frac{(-4)^3}{5^3} = (\frac{-4}{5})^3$ (Using the formula $(\frac{a}{b})^n = \frac{a^n}{b^n}$)

(iii) $\frac{-1}{216}$

Solution: Here, we will first factorise 216 into prime factors as follows:

2	216
2	108
2	54
3	27
3	9
3	3
	1

=> $216 = 2 \times 2 \times 2 \times 3 \times 3 \times 3$

In 216, we observe that number of times 2 is occurring = 3 and number of times 3 is occurring = 3

Thus, $-1 = (-1)^3$ *and* $216 = 2^3 \times 3^3 = (2 \times 3)^3 = 6^3$

Therefore, $\frac{-1}{216} = \frac{(-1)^3}{6^3} = (\frac{-1}{6})^3$ (Using the formula $(\frac{a}{b})^n = \frac{a^n}{b^n}$)

Question 15 – Find the value of each of the following:

(i) $\left(\frac{-1}{2}\right)^2 \times 2^3 \times \left(\frac{3}{4}\right)^2$

Solution: We have $(\frac{-1}{2})^2 \times (2)^3 \times (\frac{3}{4})^2$

=> $\frac{-1}{2} \times \frac{-1}{2} \times 2 \times 2 \times 2 \times \frac{3}{4} \times \frac{3}{4}$

=> $\frac{1}{2\times2} \times 8 \times \frac{3\times3}{4\times4}$

=> $\frac{1}{4} \times 8 \times \frac{9}{16}$

=> $\frac{\cancel{8}\times9}{4\times\cancel{16}}$

2

=> $\frac{1\times9}{4\times2}$

=> $\frac{9}{8}$

(ii) $\left(\frac{-3}{5}\right)^4 \times (\frac{4}{9})^4 \times \left(\frac{-15}{18}\right)^2$

Solution: We have $\left(\frac{-3}{5}\right)^4 \times (\frac{4}{9})^4 \times \left(\frac{-15}{18}\right)^2$

=> $\frac{-3}{5} \times \frac{-3}{5} \times \frac{-3}{5} \times \frac{-3}{5} \times \frac{4}{9} \times \frac{4}{9} \times \frac{4}{9} \times \frac{4}{9} \times \frac{-15}{18} \times \frac{-15}{18}$

=> $\frac{(-3)\times(-3)\times(-3)\times(-3)}{5\times5\times5\times5} \times \frac{4\times4\times4\times4}{9\times9\times9\times9} \times \frac{(-15)\times(-15)}{18\times18}$

=> $\frac{\cancel{81}}{\underset{25}{\cancel{625}}} \times \frac{\overset{64}{\cancel{256}}}{\underset{\underset{9}{\cancel{81}}}{\cancel{6561}}} \times \frac{\overset{\cancel{9}}{\cancel{225}}}{\underset{81}{\cancel{324}}}$

=> $\frac{64}{25\times9\times81} = \frac{64}{18225}$

Question 16: If a = 2 and b = 3, then find the values of each of the following:

(i) $(\boldsymbol{a}+\boldsymbol{b})^{\boldsymbol{a}}$

Solution: Since $a = 2\ and\ b = 3$

Thus, $(a+b)^a = (2+3)^2$

=> $(5)^2$

=> $5 \times 5 = 25$

(ii) $(\boldsymbol{ab})^{\boldsymbol{b}}$

Solution: Since $a = 2\ and\ b = 3$

Thus, $(ab)^b = (2 \times 3)^3$

=> $(6)^3$

=> $6 \times 6 \times 6 = 216$

(iii) $(\frac{\boldsymbol{b}}{\boldsymbol{a}})^{\boldsymbol{b}}$

Solution: Since $a = 2\ and\ b = 3$

Thus, $(\frac{b}{a})^b = \left(\frac{3}{2}\right)^3$

=> $\frac{3}{2} \times \frac{3}{2} \times \frac{3}{2} = \frac{3 \times 3 \times 3}{2 \times 2 \times 2}$

=> $\frac{27}{8}$

(iv) $(\frac{a}{b} + \frac{b}{a})^{a}$

Solution: Since $a = 2\ and\ b = 3$

Thus, $(\frac{a}{b} + \frac{b}{a})^{a} = \left(\frac{2}{3} + \frac{3}{2}\right)^{2}$

Now, LCM of 3 and 2 is 6

=> $\frac{2}{3} = \frac{2 \times 2}{3 \times 2} = \frac{4}{6}$ (Multiplying each term by 2) and

$\frac{3}{2} = \frac{3 \times 3}{2 \times 3} = \frac{9}{6}$ (Multiplying each term by 3)

Thus, $\left(\frac{2}{3} + \frac{3}{2}\right)^{2} = \left(\frac{4}{6} + \frac{9}{6}\right)^{2}$

=> $\left(\frac{4+9}{6}\right)^{2} = \left(\frac{13}{6}\right)^{2} = \frac{13 \times 13}{6 \times 6}$

=> $\frac{169}{36}$

Laws of Exponents

First law: It states that for any non-zero rational number ‘a’ and natural numbers ‘m’ and ‘n’, we have $a^{m} \times a^{n} = a^{m+n}$. It means that while multiplying two exponents having same base and different powers, powers get added and base remains same.

Second law: It states that for any non-zero rational number ‘a’ and natural numbers ‘m’ and ‘n’ such that m > n, we have $a^{m} \div a^{n} = \frac{a^{m}}{a^{n}} = a^{m-n}$. It means that while dividing two exponents having same base and different powers, powers get subtracted and base remains same.

Third law: It states that for any non-zero rational number ‘a’ and natural numbers ‘m’ and ‘n’, we have$(\ a^{m})^{n} = a^{m \times n} = (\ a^{n})^{m}$.

Fourth law: It states that for any non-zero rational numbers 'a' and 'b' and natural number 'n', we have $a^n \times b^n = (ab)^n$.

Fifth law: It states that for any non-zero rational numbers 'a' and 'b' and natural number 'n', we have $\frac{a^n}{b^n} = (\frac{a}{b})^n$.

Examples:

Example 1 – Using laws of exponents simplify and write the answer in exponential form:

(i) $2^5 \times 2^3$

Solution: Here we can see that in multiplication of two exponents, base is same and powers are different. Thus, we will use the law as follows: $a^m \times a^n = a^{m+n}$

=> $2^5 \times 2^3 = 2^{(5+3)} = 2^8$

(ii) $3^2 \times 3^4 \times 3^8$

Solution: Here we can see that in multiplication of three exponents, base is same and powers are different. Thus, we will use the law as follows: $a^m \times a^n \times a^p = a^{m+n+p}$

=> $3^2 \times 3^4 \times 3^8 = 3^{(2+4+8)} = 3^{(6+8)} = 3^{14}$

(iii) $6^{15} \div 6^{10}$

Solution: Here we can see that in division of two exponents, base is same and powers are different. Thus, we will use the law as follows: $a^m \div a^n = \frac{a^m}{a^n} = a^{m-n}$

=> $6^{15} \div 6^{10} = 6^{(15-10)} = 6^5$

(iv) $(5^3)^2$

Solution: Here, we will use the law as follows: $(a^m)^n = a^{m \times n} = (a^n)^m$

=> $(5^3)^2 = 5^{(3\times2)} = 5^6$

(v) $(7^2)^3 \div 7^3$

Solution: Firstly we will use the law as follows: $(a^m)^n = a^{m\times n} = (a^n)^m$

=> $(7^2)^3 = 7^{(2\times 3)} = 7^6$

Now, we have $(7^2)^3 \div 7^3$

=> $7^6 \div 7^3$

Now, by using $a^m \div a^n = \frac{a^m}{a^n} = a^{m-n}$

=> $7^6 \div 7^3 = 7^{(6-3)} = 7^3$

(vi) $2^5 \times 3^5$

Solution: Here we can see that in multiplication of two exponents, base is different and power is same. Thus, we will use the law as follows: $a^n \times b^n = (ab)^n$

=> $2^5 \times 3^5 = (2\times 3)^5 = 6^5$

(vii) $a^4 \times b^4$

Solution: Here we can see that in multiplication of two exponents, base is different and power is same. Thus, we will use the law as follows: $a^n \times b^n = (ab)^n$

=> $a^4 \times b^4 = (a\times b)^4 = (ab)^4$

(viii) $(2^{20} \div 2^{15}) \times 2^3$

Solution: Firstly, we can see that in division of two exponents, base is same and powers are different. Thus, we will use the law as follows: $a^m \div a^n = \frac{a^m}{a^n} = a^{m-n}$

=> $(2^{20} \div 2^{15}) = 2^{(20-15)} = 2^5$

Now, we have $2^5 \times 2^3$

Using $a^m \times a^n = a^{m+n}$

=> $2^5 \times 2^3 = 2^{(5+3)} = 2^8$

Example 2 – Simplify and express each of the following in exponential form:

(i) $\frac{\mathbf{3^7}}{\mathbf{3^4 \times 3^3}}$

Solution: Firstly we will use the law $a^m \times a^n = a^{m+n}$

=> $\frac{\mathbf{3^7}}{\mathbf{3^4 \times 3^3}} = \frac{\mathbf{3^7}}{\mathbf{3^{(4+3)}}}$

=> $\frac{3^7}{3^7}$

Now, by using $a^m \div a^n = \frac{a^m}{a^n} = a^{m-n}$, we have

=> $\frac{3^7}{3^7} = 3^{(7-7)} = 3^0 = 1$

(ii) $\{(\mathbf{5^2})^{\mathbf{3}} \times \mathbf{5^4}\} \div \mathbf{5^7}$

Solution: Firstly we will use the law $(a^m)^n = a^{m\times n} = (a^n)^m$

=> $\{(5^2)^3 \times 5^4\} \div 5^7 = \{(5)^{2\times 3} \times 5^4\} \div 5^7 = \{5^6 \times 5^4\} \div 5^7$

Now, using $a^m \times a^n = a^{m+n}$, we have

$\{5^6 \times 5^4\} \div 5^7 = (5)^{6+4} \div 5^7 = 5^{10} \div 5^7$

Now, using $a^m \div a^n = \frac{a^m}{a^n} = a^{m-n}$, we have

$5^{10} \div 5^7 = 5^{10-7} = 5^3$

(iii) $(\frac{\mathbf{3^7}}{\mathbf{3^2}}) \times \mathbf{3^5}$

Solution: Firstly, by using $a^m \div a^n = \frac{a^m}{a^n} = a^{m-n}$, we have

=> $\left(\frac{3^7}{3^2}\right) \times 3^5 = (3)^{7-2} \times 3^5 = 3^5 \times 3^5$

Now, by using $a^m \times a^n = a^{m+n}$, we have

$3^5 \times 3^5 = 3^{(5+5)} = 3^{10}$

(iv) $\frac{4^5 \times a^8 b^3}{4^5 \times a^5 b^2}$

Solution: We can write $\frac{4^5 \times a^8 b^3}{4^5 \times a^5 b^2}$

=> $\frac{4^5}{4^5} \times \frac{a^8}{a^5} \times \frac{b^3}{b^2}$

Now, using $a^m \div a^n = \frac{a^m}{a^n} = a^{m-n}$, we have

=> $(4)^{5-5} \times (a)^{8-5} \times (b)^{3-2}$

=> $4^0 \times a^3 \times b$

=> $1 \times a^3 \times b = a^3 b$

Example 3 – Simplify and express each of the following in exponential form:

(i) $2^5 \times 5^5$

Solution: By using $a^n \times b^n = (ab)^n$, we have

$2^5 \times 5^5 = (2 \times 5)^5 = 10^5$

(ii) $2^3 \times 2^2 \times 5^5$

Solution: Firstly by using $a^m \times a^n = a^{m+n}$, we have

$2^3 \times 2^2 \times 5^5 = (2)^{(3+2)} \times 5^5 = 2^5 \times 5^5$

Now, by using $a^n \times b^n = (ab)^n$, we have

$2^5 \times 5^5 = (2 \times 5)^5 = 10^5$

(iii) $\{(\mathbf{2^2})^{\mathbf{3}} \times \mathbf{3^6}\} \times \mathbf{5^6}$

Solution: Firstly by using $(a^m)^n = a^{m \times n} = (a^n)^m$, we have

$\{(2^2)^3 \times 3^6\} \times 5^6 = \{(2)^{(2\times3)} \times 3^6\} \times 5^6 = \{2^6 \times 3^6\} \times 5^6$

Now, by using $a^n \times b^n = (ab)^n$, we have

$\{2^6 \times 3^6\} \times 5^6 = (2 \times 3)^6 \times 5^6 = 6^6 \times 5^6$

Again, by using $a^n \times b^n = (ab)^n$, we have

$6^6 \times 5^6 = (6 \times 5)^6 = 30^6$

(iv) $(\boldsymbol{\frac{a}{b}})^{\mathbf{5}} \times \boldsymbol{b^{10}}$

Solution: We can write $(\frac{a}{b})^5 \times b^{10} = \frac{a^5}{b^5} \times b^{10}$

=> $a^5 \times \frac{b^{10}}{b^5}$

Now, using $a^m \div a^n = \frac{a^m}{a^n} = a^{m-n}$, we have

=> $a^5 \times (b)^{10-5}$

=> $a^5 \times b^5$

Now, using $a^n \times b^n = (ab)^n$, we have

=> $a^5 \times b^5 = (a \times b)^5 = ab^5$

Example 4 – Write exponential form for $\mathbf{8 \times 8 \times 8 \times 8}$ taking base as 2.

Solution: Firstly, we can see that 8 is occurring 4 times.

Thus we have, $8 \times 8 \times 8 \times 8 = 8^4$

And $8 = 2 \times 2 \times 2$

=> $8 \times 8 \times 8 \times 8 = 8^4 = (2^3)^4$

Now, by using $(a^m)^n = a^{m \times n} = (a^n)^m$, we have

=> $(2^3)^4 = (2)^{3\times4} = 2^{12}$

Example 5 – Simplify and write each of the following in exponential form:

(i) $8^2 \div 2^3$

Solution: Since $8 = 2 \times 2 \times 2 = 2^3$

=> $8^2 \div 2^3 = \frac{8^2}{2^3} = \frac{(2^3)^2}{2^3}$

Now, using $(a^m)^n = a^{m \times n} = (a^n)^m$, we have

=> $\frac{(2^3)^2}{2^3} = \frac{(2)^6}{2^3}$

Now, by using $a^m \div a^n = \frac{a^m}{a^n} = a^{m-n}$, we have

$$\frac{(2)^6}{2^3} = (2)^{6-3} = 2^3$$

(ii) $25^4 \div 5^3$

Solution: Since $25 = 5 \times 5 = 5^2$

=> $25^4 \div 5^3 = \frac{25^4}{5^3} = \frac{(5^2)^4}{5^3}$

Now, using $(a^m)^n = a^{m \times n} = (a^n)^m$, we have

=> $\frac{(5^2)^4}{5^3} = \frac{(5)^8}{5^3}$

Now, by using $a^m \div a^n = \frac{a^m}{a^n} = a^{m-n}$, we have

$$\frac{(5)^8}{5^3} = (5)^{8-3} = 5^5$$

(iii) $\frac{\mathbf{2^8 \times a^5}}{\mathbf{4^3 \times a^3}}$

Solution: Since $4 = 2 \times 2 = 2^2$

Thus, we can write $\frac{2^8 \times a^5}{4^3 \times a^3} = \frac{2^8 \times a^5}{(2^2)^3 \times a^3}$

Now, using$(a^m)^n = a^{m \times n} = (a^n)^m$, we have

=> $\frac{2^8 \times a^5}{(2^2)^3 \times a^3} = \frac{2^8 \times a^5}{(2)^{2 \times 3} \times a^3} = \frac{2^8 \times a^5}{(2)^6 \times a^3}$

=> $\frac{2^8}{2^6} \times \frac{a^5}{a^3}$

Now, by using $a^m \div a^n = \frac{a^m}{a^n} = a^{m-n}$, we have

=> $(2)^{8-6} \times (a)^{5-3}$

=> $2^2 \times a^2$

=> $(2 \times a)^2 = (2a)^2$

(iv) $\frac{\mathbf{2^3 \times 3^4 \times 4}}{\mathbf{3 \times 32}}$

Solution: Since $4 = 2 \times 2 = 2^2$ and $32 = 2 \times 2 \times 2 \times 2 \times 2 = 2^5$

Thus, $\frac{2^3 \times 3^4 \times 4}{3 \times 32} = \frac{2^3 \times 3^4 \times 2^2}{3 \times 2^5}$

=> $\frac{2^3 \times 2^2 \times 3^4}{3 \times 2^5}$

Using $a^m \times a^n = a^{m+n}$, we have

=> $\frac{(2)^{3+2} \times 3^4}{3 \times 2^5} = \frac{(2)^5 \times 3^4}{3 \times 2^5}$

=> $\frac{2^5}{2^5} \times \frac{3^4}{3}$

By using $a^m \div a^n = \frac{a^m}{a^n} = a^{m-n}$, we have

$=> (2)^{5-5} \times (3)^{4-1}$

$=> 2^0 \times 3^3 = 1 \times 3^3 = 3^3$

Example 6 – Simplify:

(i) $\mathbf{2^{55} \times 2^{60} - 2^{97} \times 2^{18}}$

Solution: Using $a^m \times a^n = a^{m+n}$, we have

$=> (2)^{55+60} - (2)^{97+18}$

$=> 2^{115} - 2^{115} = 0$

(ii) $\mathbf{2^3 \times a^3 \times 5a^4}$

Solution: We can write $2^3 \times a^3 \times 5a^4 = 2^3 \times 5 \times a^3 \times a^4$

$=> 8 \times 5 \times a^3 \times a^4$

$=> 40 \times a^3 \times a^4$

Now, using $a^m \times a^n = a^{m+n}$, we have

$=> 40 \times (a)^{3+4}$

$=> 40 \times a^7 = 40a^7$

(iii) $\mathbf{\frac{3^n+3^{n+1}}{3^{n+1}-3^n}}$, where n is a natural number.

Solution: Since $3^{n+1} = 3^n \times 3^1$

Thus, $\frac{3^n+3^{n+1}}{3^{n+1}-3^n} = \frac{3^n+3^n \times 3^1}{3^n \times 3^1 - 3^n}$

Taking out common factor (3n)

$=> \frac{3^n(1+3^1)}{3^n(3^1-1)}$

$=> \frac{3^n(4)}{3^n(2)} = \frac{3^n}{3^n} \times \frac{2^2}{2}$

=> $3^{n-n} \times 2^{2-1}$

=> $3^0 \times 2^1$

=> $1 \times 2 = 2$

Example 7 – Simplify:

(i) $\frac{\mathbf{12^4 \times 9^3 \times 4}}{\mathbf{6^3 \times 8^2 \times 27}}$

Solution: We will first convert the numbers into prime factors as follows:

$$\frac{12^4 \times 9^3 \times 4}{6^3 \times 8^2 \times 27} = \frac{(2 \times 2 \times 3)^4 \times (3 \times 3)^3 \times (2 \times 2)}{(2 \times 3)^3 \times (2 \times 2 \times 2)^2 \times (3 \times 3 \times 3)}$$

$$\frac{(2^2 \times 3)^4 \times (3^2)^3 \times (2^2)}{(2)^3 \times (3)^3 \times (2^3)^2 \times (3^3)} = \frac{(2^2)^4 \times (3)^4 \times (3^2)^3 \times (2^2)}{(2)^3 \times (3)^3 \times (2^3)^2 \times (3^3)}$$

Since $(\,a^m)^n = a^{m \times n} = (\,a^n)^m$

$$\frac{(2)^8 \times (3)^4 \times (3)^6 \times (2^2)}{(2)^3 \times (3)^3 \times (2)^6 \times (3^3)}$$

$$\frac{((2)^8 \times (2)^2) \times ((3)^4 \times (3^6))}{((2)^3 \times (2)^6) \times ((3)^3 \times (3^3))}$$

Using $a^m \times a^n = a^{m+n}$, we have

$$\frac{2^{8+2} \times 3^{4+6}}{2^{3+6} \times 3^{3+3}} = \frac{2^{10} \times 3^{10}}{2^9 \times 3^6}$$

$$\frac{2^{10}}{2^9} \times \frac{3^{10}}{3^6}$$

By using $a^m \div a^n = \frac{a^m}{a^n} = a^{m-n}$, we have

$2^{10-9} \times 3^{10-6} = 2^1 \times 3^4 = 2 \times 81 = 162$

(ii) $\frac{\mathbf{25\times5^2\times t^8}}{\mathbf{10^3\times t^4}}$

Solution: We will first convert the numbers into prime factors as follows:

$$\frac{25\times5^2\times t^8}{10^3\times t^4}=\frac{(5\times5)\times(5)^2\times(t)^8}{(2\times5)^3\times(t)^4}$$

$\frac{(5)^2\times(5)^2\times(t)^8}{(2)^3\times(5)^3\times(t)^4}=\frac{(5)^{2+2}\times(t)^8}{(2)^3\times(5)^3\times(t)^4}$ (Using $a^m\times a^n=a^{m+n}$)

$\frac{(5)^4\times(t)^8}{(2)^3\times(5)^3\times(t)^4}=\frac{(5)^{4-3}\times(t)^{8-4}}{(2)^3}$ $(Using\ a^m\div a^n=\frac{a^m}{a^n}=a^{m-n})$

$$\frac{5\times(t)^4}{(2)^3}=\frac{5\times(t)^4}{8}=\frac{5t^4}{8}$$

(iii) $\frac{\mathbf{3^5\times10^5\times25}}{\mathbf{5^7\times6^5}}$

Solution: We will first convert the numbers into prime factors as follows:

$$\frac{3^5\times10^5\times25}{5^7\times6^5}=\frac{3^5\times(2\times5)^5\times(5)^2}{(5)^7\times(2\times3)^4}$$

$$\frac{3^5\times(2)^5\times5^5\times(5)^2}{(5)^7\times(2)^4\times3^4}$$

Using $a^m\times a^n=a^{m+n}\ and\ a^m\div a^n=\frac{a^m}{a^n}=a^{m-n}$, we have

$$\frac{3^5\times(2)^5\times5^{5+2}}{(5)^7\times(2)^4\times3^4}=\frac{3^5\times(2)^5\times5^7}{(5)^7\times(2)^4\times3^4}$$

$$2^{5-4}\times3^{5-4}\times5^{7-7}$$

$$2^1\times3^1\times5^0=2\times3\times1=6$$

Example 8 – Express each of the following as a product of prime factors only in exponential form:

(i) 108×192

Solution: Using prime factorisation of 108 and 192, we have

2	108
2	54
3	27
3	9
3	3
	1

2	192
2	96
2	48
2	24
2	12
2	6
3	3
	1

We can see that $108 = 2 \times 2 \times 3 \times 3 \times 3 = 2^2 \times 3^3$ and

$192 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 = 2^6 \times 3$

Thus, $108 \times 192 = (2^2 \times 3^3) \times (2^6 \times 3)$

$(2^2 \times 2^6) \times (3^3 \times 3)$

Using $a^m \times a^n = a^{m+n}$, we have

$2^{2+6} \times 3^{3+1} = 2^8 \times 3^4$

(ii) 729×64

Solution: Using prime factorisation of 729 and 64, we have

2	64
2	32
2	16
2	8
2	4
2	2
	1

3	729
3	243
3	81
3	27
3	9
3	3
	1

We can see that $64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^6$ and

$729 = 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 3^6$

Thus, $729 \times 64 = 3^6 \times 2^6$

Example 9 – Compare the following numbers:

(i) 2.7×10^{12} and 1.5×10^8

Solution: We know that $2.7 = \frac{27}{10}$ and $1.5 = \frac{15}{10}$

Thus, $2.7 \times 10^{12} = \frac{27}{10} \times 10^{12} = 27 \times 10^{12-1} = 27 \times 10^{11}$

$1.5 \times 10^8 = \frac{15}{10} \times 10^8 = 15 \times 10^{8-1} = 15 \times 10^7$

=> $(27 \times 10^{11}) - (15 \times 10^7)$

=> $(27 \times 10^7 \times 10^4)$ - (15×10^7)

=> $3 \times 10^7((9 \times 10^4) - 5))$

=> $3 \times 10^7(90000 - 5)$

=> $3 \times 10^7(89995)$ is clearly greater than zero

Thus, $2.7 \times 10^{12} > 1.5 \times 10^8$

(ii) 4×10^{14} *and* 3×10^7

Solution: $(4 \times 10^{14}) - (3 \times 10^7)$

=> $(4 \times 10^7 \times 10^7)$ - (3×10^7)

=> $10^7(4 \times 10^7 - 3)$is clearly greater than zero

Thus, $4 \times 10^{14} > 3 \times 10^7$

Example 10 – Find the values of ‘n’ in each of the following:

(i) $(2^2)^n = (2^3)^4$

Solution: By using$(a^m)^n = a^{m\times n} = (a^n)^m$, we have

$(2^2)^n = (2^3)^4$

=> $2^{2n} = 2^{12}$

Since the base is same, we can equate the powers as follows:

2n = 12

=> n = $\frac{12}{2} = 6$

(ii) $2^{5n} \div 2^n = 2^4$

Solution: $Using\ a^m \div a^n = \frac{a^m}{a^n} = a^{m-n}$ we have

$2^{5n} \div 2^n = 2^4$

$2^{5n-n} = 2^4$

$2^{4n} = 2^4$

Since the base is same, we can equate the powers as follows:

4n = 4

=> n = $\frac{4}{4} = 1$

(iii) $\mathbf{2^{n-5} \times 5^{n-4} = 5}$

Solution: We can write it as: $\frac{2^n}{2^5} \times \frac{5^n}{5^4} = 5$

=> $\frac{2^n \times 5^n}{2^5 \times 5^4} = 5$

By cross multiplication, we get

$2^n \times 5^n = 5 \times 2^5 \times 5^4$

$2^n \times 5^n = 2^5 \times 5^{4+1}$

$2^n \times 5^n = 2^5 \times 5^5$

$(2 \times 5)^n = (2 \times 5)^5$

$10^n = 10^5$

Since the base is same, we can equate the powers as follows:

n = 5

(iv) $\mathbf{2^{n-7} \times 5^{n-4} = 1250}$

Solution: We can write it as follows:

$$\frac{2^n}{2^7} \times \frac{5^n}{5^4} = 1250$$

$$\frac{2^n \times 5^n}{2^7 \times 5^4} = 1250$$

Since, $1250 = 2 \times 5 \times 5 \times 5 \times 5 = 2 \times 5^4$

Thus, $\frac{2^n \times 5^n}{2^7 \times 5^4} = 2 \times 5^4$

By cross multiplication, we have

$2^n \times 5^n = 2 \times 5^4 \times 2^7 \times 5^4$

$2^n \times 5^n = 2^{1+7} \times 5^{4+4}$

$2^n \times 5^n = 2^8 \times 5^8$

$(2 \times 5)^n = (2 \times 5)^8$

$10^n = 10^8$

Since the base is same, we can equate the powers as follows:

Thus, $n = 8$

(v) $\mathbf{5^{n-2} \times 3^{2n-3} = 135}$

Solution: We can write it as follows:

$$\frac{5^n}{5^2} \times \frac{3^{2n}}{3^3} = 135$$

$$\frac{5^n \times 3^{2n}}{5^2 \times 3^3} = 135$$

Since, $135 = 3 \times 3 \times 3 \times 5 = 5 \times 3^3$

Thus, $\frac{5^n \times 3^{2n}}{5^2 \times 3^3} = 5 \times 3^3$

By cross multiplication, we have

$5^n \times 3^{2n} = 5 \times 3^3 \times 5^2 \times 3^3$

$5^n \times 3^{2n} = 5^{1+2} \times 3^{3+3}$

$5^n \times 3^{2n} = 5^3 \times 3^6$

Now, $3^{2n} = (3^2)^n = 9^n$ and $3^6 = (3^2)^3 = 9^3$

Thus, $5^n \times 9^n = 5^3 \times 9^3$

$(5 \times 9)^n = (5 \times 9)^3$

$45^n = 45^3$

Since the base is same, we can equate the powers as follows:

Thus, $n = 3$

Example 11 – If $\mathbf{25^{n-1} + 100 = 5^{2n-1}}$, find the value of n.

Solution: We can write it as follows:

$100 = 5^{2n-1} - 25^{n-1}$

Now, $25 = 5 \times 5 = 5^2$ and $100 = 2 \times 2 \times 5 \times 5 = 2^2 \times 5^2$

Thus, $2^2 \times 5^2 = 5^{2n-1} - (5^2)^{n-1}$

$2^2 \times 5^2 = 5^{2n-1} - (5)^{2n-2}$

$2^2 \times 5^2 = \frac{5^{2n}}{5} - \frac{(5)^{2n}}{5^2}$

$2^2 \times 5^2 = \frac{5^{2n}}{5} - \frac{(5)^{2n}}{25}$

$2^2 \times 5^2 = \frac{5 \times 5^{2n} - (5)^{2n}}{25}$

$2^2 \times 5^2 = \frac{5^{2n}(5-1)}{25}$

By cross multiplication, we have

$2^2 \times 5^2 \times 25 = 5^{2n} \times 4$

$2^2 \times 5^2 \times 5^2 = 5^{2n} \times 2^2$

$\frac{2^2 \times 5^2 \times 5^2}{2^2} = 5^{2n}$

$2^{2-2} \times 5^{2+2} = 5^{2n}$

$2^0 \times 5^4 = 5^{2n}$

$1 \times 5^4 = 5^{2n}$

$5^4 = 5^{2n}$

Since the base is same, we can equate the powers as follows:

Thus, $2n = 4$

=> $n = \frac{4}{2} = 2$

Example 12 – Find 'n' such that

(i) $\boldsymbol{(\frac{2}{3})^3 \times (\frac{2}{3})^5 = (\frac{2}{3})^{n-2}}$

Solution: Firstly by using $a^m \times a^n = a^{m+n}$, we have

$(\frac{2}{3})^{(3+5)} = (\frac{2}{3})^{n-2}$

$(\frac{2}{3})^{(8)} = (\frac{2}{3})^{n-2}$

Since the base is same so we can equate the powers as follows:

$8 = n - 2$

=> $8 + 2 = \text{n}$

=> $n = 10$

(ii) $\boldsymbol{(\frac{125}{8})^5 \times (\frac{125}{8})^n = (\frac{5}{2})^{18}}$

Solution: Firstly by using $a^m \times a^n = a^{m+n}$, we have

$(\frac{125}{8})^{(5+n)} = (\frac{5}{2})^{18}$

Now, $\frac{125}{8} = \frac{5\times5\times5}{2\times2\times2} = \frac{5^3}{2^3} = (\frac{5}{2})^3$

Thus, $(\frac{5}{2})^3)^{(5+n)} = (\frac{5}{2})^{18}$

$(\frac{5}{2})^{(15+3n)} = (\frac{5}{2})^{18}$

Since the base is same so we can equate the powers as follows:

$15 + 3n = 18$

3n = 18 – 15

3n = 3

=> n = $\frac{3}{3} = 1$

Example 13 – If $\frac{p}{q} = \left(\frac{2}{3}\right)^3 \div \left(\frac{6}{7}\right)^0$, find the value of $(\frac{q}{p})^3$

Solution: Since $a^0 = 1$ for any 'a'

Thus, $\frac{p}{q} = \left(\frac{2}{3}\right)^2 \div 1$

Now, $\frac{p}{q} = (\frac{2\times2}{3\times3}) \div 1$

=> $\frac{p}{q} = \frac{4}{9} \div 1$

=> $\frac{p}{q} = \frac{4}{9} \times \frac{1}{1} = \frac{4}{9}$

=> $\frac{q}{p} = \frac{9}{4}$

=> $\left(\frac{q}{p}\right)^3 = \left(\frac{9}{4}\right)^3$

=> $\left(\frac{q}{p}\right)^3 = \frac{9\times9\times9}{4\times4\times4} = \frac{729}{64}$

Example 14 – Find the value of 'm' so that

$\boldsymbol{(-3)^{m+1} \times (-3)^5 = (-3)^7}$

Solution: Firstly, by using $a^m \times a^n = a^{m+n}$, we have

$(-3)^{m+1+5} = (-3)^7$

$(-3)^{m+6} = (-3)^7$

Now, since the base is same so we can equate the powers

$m + 6 = 7$

$m = 7 – 6$

$m = 1$

Exercise 6.2

Question 1 – Using laws of exponents simplify and write the answer in exponential form:

(i) $\boldsymbol{2^3 \times 2^4 \times 2^5}$

Solution: Here we can see that in multiplication of three exponents, base is same and powers are different. Thus, we will use the law as follows: $a^m \times a^n \times a^p = a^{m+n+p}$

=> $2^3 \times 2^4 \times 2^5 = 2^{(3+4+5)} = 2^{(7+5)} = 2^{12}$

(ii) $\boldsymbol{5^{12} \div 5^3}$

Solution: Here we can see that in division of two exponents, base is same and powers are different. Thus, we will use the law as follows: $a^m \div a^n = \frac{a^m}{a^n} = a^{m-n}$

=> $5^{12} \div 5^3 = 5^{(12-3)} = 5^9$

(iii) $(\mathbf{7^2})^{\mathbf{3}}$

Solution: Here, we will use the law as follows: $(a^m)^n = a^{m\times n} = (a^n)^m$

=> $(7^2)^3 = 7^{(2\times 3)} = 7^6$

(iv) $(\mathbf{3^2})^{\mathbf{5}} \div \mathbf{3^4}$

Solution: Firstly we will use the law as follows: $(a^m)^n = a^{m\times n} = (a^n)^m$

=> $(3^2)^5 = 3^{(2\times 5)} = 3^{10}$

Now, we have $(3^2)^5 \div 3^4$

=> $3^{10} \div 3^4$

Now, by using $a^m \div a^n = \frac{a^m}{a^n} = a^{m-n}$

=> $3^{10} \div 3^4 = 3^{(10-4)} = 3^6$

(v) $\mathbf{3^7} \times \mathbf{2^7}$

Solution: Here we can see that in multiplication of two exponents, base is different and power is same. Thus, we will use the law as follows: $a^n \times b^n = (ab)^n$

=> $3^7 \times 2^7 = (3\times 2)^7 = 6^7$

(vi) $(\mathbf{5^{21}} \div \mathbf{5^{13}}) \times \mathbf{5^7}$

Solution: Firstly, we can see that in division of two exponents, base is same and powers are different. Thus, we will use the law as follows: $a^m \div a^n = \frac{a^m}{a^n} = a^{m-n}$

=> $(5^{21} \div 5^{13}) = 5^{(21-13)} = 5^8$

Now, we have $5^8 \times 5^7$

Using $a^m \times a^n = a^{m+n}$

=> $5^8 \times 5^7 = 5^{(8+7)} = 5^{15}$

Question 2 – Simplify and express each of the following in exponential form:

(i) $\{(2^3)^4 \times 2^8\} \div 2^{12}$

Solution: Firstly we will use the law $(a^m)^n = a^{m\times n} = (a^n)^m$

=> $\{(2^3)^4 \times 2^8\} \div 2^{12} = \{(2)^{3\times 4} \times 2^8\} \div 2^{12} = \{2^{12} \times 2^8\} \div 2^{12}$

Now, using $a^m \times a^n = a^{m+n}$, we have

$\{2^{12} \times 2^8\} \div 2^{12} = (2)^{12+8} \div 2^{12} = 2^{20} \div 2^{12}$

Now, using $a^m \div a^n = \frac{a^m}{a^n} = a^{m-n}$, we have

$2^{20} \div 2^{12} = 2^{20-12} = 2^8$

(ii) $\{8^2 \times 8^4\} \div 8^3$

Solution: Firstly we will use the law $a^m \times a^n = a^{m+n}$, we have

$\{8^2 \times 8^4\} \div 8^3 = (8)^{2+4} \div 8^3 = 8^6 \div 8^3$

Now, using $a^m \div a^n = \frac{a^m}{a^n} = a^{m-n}$, we have

$8^6 \div 8^3 = 8^{6-3} = 8^3$

Now, $8 = 2 \times 2 \times 2$

Thus, $8^3 = (2^3)^3 = (2)^{3\times 3} = 2^9$

(iii) $\left(\frac{5^7}{5^2}\right) \times 5^3$

Solution: Firstly, using $a^m \div a^n = \frac{a^m}{a^n} = a^{m-n}$, we have

$$\left(\frac{5^7}{5^2}\right) \times 5^3 = 5^{7-2} \times 5^3$$

$= 5^5 \times 5^3$

Now, using the law $a^m \times a^n = a^{m+n}$, we have

$5^5 \times 5^3 = 5^{5+3} = 5^8$

(iv) $\frac{\mathbf{5^4 \times x^{10} y^5}}{\mathbf{5^4 \times x^7 y^4}}$

Solution: We can write $\frac{5^4 \times x^{10} y^5}{5^4 \times x^7 y^4}$

=> $\frac{5^4}{5^4} \times \frac{x^{10}}{x^7} \times \frac{y^5}{y^4}$

Now, using $a^m \div a^n = \frac{a^m}{a^n} = a^{m-n}$, we have

=> $(5)^{4-4} \times (x)^{10-7} \times (y)^{5-4}$

=> $5^0 \times x^3 \times y$

=> $1 \times x^3 \times y = x^3 y$

Question 3 – Simplify and express each of the following in exponential form:

(i) $\{(\mathbf{3^2})^{\mathbf{3}} \times \mathbf{2^6}\} \times \mathbf{5^6}$

Solution: Firstly by using$(a^m)^n = a^{m \times n} = (a^n)^m$, we have

$\{(3^2)^3 \times 2^6\} \times 5^6 = \{(3)^{(2\times3)} \times 2^6\} \times 5^6 = \{3^6 \times 2^6\} \times 5^6$

Now, by using $a^n \times b^n = (ab)^n$, we have

$\{3^6 \times 2^6\} \times 5^6 = (3 \times 2)^6 \times 5^6 = 6^6 \times 5^6$

Again, by using $a^n \times b^n = (ab)^n$, we have

$6^6 \times 5^6 = (6 \times 5)^6 = 30^6$

(ii) $\left(\frac{\mathbf{x}}{\mathbf{y}}\right)^{\mathbf{12}} \times \mathbf{y^{24}} \times (\mathbf{2^3})^{\mathbf{4}}$

Solution: Using the law $(a^m)^n = a^{m \times n} = (a^n)^m$

We have $\frac{x^{12}}{y^{12}} \times y^{24} \times (2)^{3\times4}$

=> $\frac{x^{12}}{y^{12}} \times y^{24} \times 2^{12}$

Now, using $a^m \div a^n = \frac{a^m}{a^n} = a^{m-n}$, we have

=> $x^{12} \times (y)^{24-12} \times 2^{12}$

=> $x^{12} \times y^{12} \times 2^{12}$

Now, using $a^n \times b^n \times c^n = (abc)^n$, we have

=> $(2xy)^{12}$

(iii) $\left(\frac{5}{2}\right)^6 \times \left(\frac{5}{2}\right)^2$

Solution: By using $a^m \times a^n = a^{m+n}$, we have

$\left(\frac{5}{2}\right)^6 \times \left(\frac{5}{2}\right)^2 = \left(\frac{5}{2}\right)^{6+2} = \left(\frac{5}{2}\right)^8$

(iv) $\left(\frac{2}{3}\right)^5 \times \left(\frac{3}{5}\right)^5$

Solution: By using $a^m \times b^m = (ab)^m$, we have

$\left(\frac{2}{3}\right)^5 \times \left(\frac{3}{5}\right)^5 = \left(\frac{2}{3} \times \frac{3}{5}\right)^5 = \left(\frac{2}{5}\right)^5$

Question 4 – Write 9 × 9 × 9 × 9 × 9 in exponential form with base 3.

Solution: Firstly, we can see that 9 is occurring 5 times.

Thus we have, $9 \times 9 \times 9 \times 9 \times 9 = 9^5$

And $9 = 3 \times 3 = 3^2$

=> $9 \times 9 \times 9 \times 9 \times 9 = 9^5 = (3^2)^5$

Now, by using $(a^m)^n = a^{m\times n} = (a^n)^m$, we have

=> $(3^2)^5 = (3)^{2\times5} = 3^{10}$

Question 5 – Simplify and write each of the following in exponential form:

(i) $(25)^3 \div 5^3$

Solution: Since $25 = 5\times5 = 5^2$

=> $(25)^3 \div 5^3 = \frac{25^3}{5^3} = \frac{(5^2)^3}{5^3}$

Now, using $(a^m)^n = a^{m\times n} = (a^n)^m$, we have

=> $\frac{(5^2)^3}{5^3} = \frac{(5)^6}{5^3}$

Now, by using $a^m \div a^n = \frac{a^m}{a^n} = a^{m-n}$, we have

$$\frac{(5)^6}{5^3} = (5)^{6-3} = 5^3$$

(ii) $(81)^5 \div (3^2)^5$

Solution: Using $(a^m)^n = a^{m\times n} = (a^n)^m$, we have

$(81)^5 \div (3^2)^5 = (81)^5 \div (3)^{2\times5}$

$= (81)^5 \div 3^{10}$

Now, since $81 = 3\times3\times3\times3 = 3^4$

Thus, $(81)^5 \div 3^{10} = (3^4)^5 \div 3^{10}$

Again, by using $(a^m)^n = a^{m\times n} = (a^n)^m$, we have

$(3^4)^5 \div 3^{10} = (3)^{4\times5} \div 3^{10} = 3^{20} \div 3^{10}$

Now, by using $a^m \div a^n = \frac{a^m}{a^n} = a^{m-n}$, we have

$$\frac{(3)^{20}}{3^{10}} = (3)^{20-10} = 3^{10}$$

(iii) $\frac{\mathbf{9^8 \times (x^2)^5}}{\mathbf{(27)^4 \times (x^3)^2}}$

Solution: Since $9 = 3 \times 3 = 3^2$ and $27 = 3 \times 3 \times 3 = 3^3$

Thus, $\frac{9^8 \times (x^2)^5}{(27)^4 \times (x^3)^2} = \frac{(3^2)^3 \times (x^2)^5}{(3^3)^4 \times (x^3)^2}$

Now, by using $(a^m)^n = a^{m \times n} = (a^n)^m$, we have

$\frac{(3)^{2\times 8} \times (x)^{2\times 5}}{(3)^{3\times 4} \times (x)^{3\times 2}}$

=> $\frac{3^{16} \times x^{10}}{3^{12} \times x^6}$

Using $a^m \div a^n = \frac{a^m}{a^n} = a^{m-n}$, we have

=> $3^{16-12} \times x^{10-6}$

=> $3^4 \times x^4$

By using $a^m \times b^m = (ab)^m$, we have

=> $(3x)^4$

(iv) $\frac{\mathbf{3^2 \times 7^8 \times 13^6}}{\mathbf{21^2 \times 91^3}}$

Solution: We will first convert the numbers into prime factors as follows:

$$\frac{3^2 \times 7^8 \times 13^6}{21^2 \times 91^3} = \frac{3^2 \times 7^8 \times 13^6}{(3 \times 7)^2 \times (13 \times 7)^3}$$

$$\frac{3^2 \times 7^8 \times 13^6}{(3)^2 \times 7^2 \times (13)^3 \times 7^3}$$

Using $a^m \times a^n = a^{m+n}$, we have

$\frac{3^2\times7^8\times13^6}{(3)^2\times7^{2+3}\times(13)^3} = \frac{3^2\times7^8\times13^6}{(3)^2\times7^5\times(13)^3}$

By using $a^m \div a^n = \frac{a^m}{a^n} = a^{m-n}$, we have

$3^{2-2} \times 7^{8-5} \times 13^{6-3} = 3^0 \times 7^3 \times 13^3 = 1 \times 7^3 \times 13^3 = 7^3 \times 13^3$

Now, by using $a^m \times b^m = (ab)^m$, we have

$7^3 \times 13^3 = (7 \times 13)^3 = 91^3$

Question 6 – Simplify:

(i) $(\mathbf{3^5})^{\mathbf{11}} \times (\mathbf{3^{15}})^{\mathbf{4}} - (\mathbf{3^5})^{\mathbf{18}} \times (\mathbf{3^5})^{\mathbf{5}}$

Solution: Firstly, by using $(a^m)^n = a^{m\times n} = (a^n)^m$, we have

=> $(3)^{5\times11} \times (3)^{15\times4} - (3)^{5\times18} \times (3)^{5\times5}$

=> $3^{55} \times 3^{60} - 3^{90} \times 3^{25}$

Now, by using $a^m \times a^n = a^{m+n}$, we have

=> $(3)^{55+60} - (3)^{90+25}$

=> $3^{115} - 3^{115}$

= 0

(ii) $\frac{\mathbf{16\times2^{n+1}-4\times2^n}}{\mathbf{16\times2^{n+2}-2\times2^{n+2}}}$

Solution: Since 16 = 2×2×2×2 = $2^4$and 4 = 2×2 = 2^2

Thus, we can write it as follows

$$\frac{2^4 \times 2^{n+1} - 2^2 \times 2^n}{2^4 \times 2^{n+2} - 2 \times 2^{n+2}}$$

$$\frac{2^4 \times 2^n \times 2 - 2^2 \times 2^n}{2^4 \times 2^n \times 2^2 - 2 \times 2^n \times 2^2}$$

$$\frac{2^5 \times 2^n - 2^2 \times 2^n}{2^6 \times 2^n - 2^n \times 2^3}$$

$\frac{2^2 \times 2^n (2^3-1)}{2^3 \times 2^n (2^3-1)} = \frac{2^2 \times 2^n (8-1)}{2^3 \times 2^n (8-1)} = \frac{2^2 \times 2^n (7)}{2^3 \times 2^n (7)} = \frac{2^2 \times 2^n}{2^3 \times 2^n}$

Now, by using $a^m \div a^n = \frac{a^m}{a^n} = a^{m-n}$, we have

$\frac{2^2 \times 2^n}{2^3 \times 2^n} = \frac{2^{n-n}}{2^{3-2}} = \frac{2^0}{2^1}$

$= \frac{1}{2}$

(iii) $\frac{\mathbf{10 \times 5^{n+1} + 25 \times 5^n}}{\mathbf{3 \times 5^{n+2} + 10 \times 5^{n+1}}}$

Solution: Since 10 = 2×5 and 25 = 5×5 = 5^2

Thus, we can write it as follows

$$\frac{2 \times 5 \times 5^{n+1} + 5^2 \times 5^n}{3 \times 5^{n+2} + 2 \times 5 \times 5^{n+1}}$$

$$\frac{2 \times 5 \times 5^n \times 5 + 5^2 \times 5^n}{3 \times 5^n \times 5^2 + 2 \times 5 \times 5^n \times 5}$$

$$\frac{2 \times 5^2 \times 5^n + 5^2 \times 5^n}{3 \times 5^n \times 5^2 + 2 \times 5^2 \times 5^n}$$

$\frac{5^2 \times 5^n (2+1)}{5^2 \times 5^n (3+2)} = \frac{5^2 \times 5^n (3)}{5^2 \times 5^n (5)}$

Now, by using $a^m \div a^n = \frac{a^m}{a^n} = a^{m-n}$, we have

$\frac{5^2 \times 5^n (3)}{5^2 \times 5^n (5)} = \frac{3 \times 5^{n-n} \times 5^{2-2}}{5} = \frac{3}{5}$

(iv) $\frac{(\mathbf{16})^7\times(\mathbf{25})^5\times(\mathbf{81})^3}{(\mathbf{15})^7\times(\mathbf{24})^5\times(\mathbf{80})^3}$

Solution: We will first convert the numbers into prime factors as follows:

$$\frac{(16)^7\times(25)^5\times(81)^3}{(15)^7\times(24)^5\times(80)^3}=\frac{(2\times2\times2\times2)^7\times(5\times5)^5\times(3\times3\times3\times3)^3}{(3\times5)^7\times(2\times2\times2\times3)^5\times(2\times2\times2\times2\times5)^3}$$

$$\frac{(2^4)^7\times(5^2)^5\times(3^4)^3}{(3)^7\times5^7\times(2^3\times3)^5\times(2^4\times5)^3}$$

Using$(\,a^m)^n=a^{m\times n}=(\,a^n)^m$, we have

$$\frac{2^{4\times7}\times5^{2\times5}\times3^{4\times3}}{(3)^7\times5^7\times(2^3)^5\times3^5\times(2^4)^3\times5^3}$$

$$\frac{2^{28}\times5^{10}\times3^{12}}{(3)^7\times5^7\times(2)^{15}\times3^5\times(2)^{12}\times5^3}$$

By using $a^m\times a^n=a^{m+n}$, we have

$$\frac{2^{28}\times5^{10}\times3^{12}}{(3)^{7+5}\times5^{7+3}\times2^{15+12}}$$

$$\frac{2^{28}\times5^{10}\times3^{12}}{(3)^{12}\times5^{10}\times2^{27}}$$

By using $a^m\div a^n=\frac{a^m}{a^n}=a^{m-n}$, we have

$2^{28-27}\times3^{12-12}\times5^{10-10}=2\times3^0\times5^0=2\times1=2$

Question 7 – Find the values of ‘n’ in each of the following:

(i) $\mathbf{5^{2n}\times5^3=5^{11}}$

Solution: Using $a^m\times a^n=a^{m+n}$ we have

$5^{2n}\times5^3=5^{11}$

$5^{2n+3}=5^{11}$

Since the base is same, we can equate the powers as follows:

$2n + 3 = 11$

=> $2n = 11 - 3$

=> $2n = 8$

=> $n = \frac{8}{2} = 4$

(ii) $9 \times 3^n = 3^7$

Solution: Since 9 = 3 × 3

Thus, we can write it as follows:

$3^2 \times 3^n = 3^7$

Now, by using $a^m \times a^n = a^{m+n}$ we have

$3^{2+n} = 3^7$

Since the base is same so we can equate the powers as follows:

$n + 2 = 7$

$n = 7 - 2 = 5$

(iii) $8 \times 2^{n+2} = 32$

Solution: Since 8 = 2 × 2 × 2 = 2^3 and 32 = 2 × 2 × 2 × 2 × 2 = 2^5

Thus, we can write it as follows:

$2^3 \times 2^{n+2} = 2^5$

Now, by using $a^m \times a^n = a^{m+n}$ we have

$2^{3+n+2} = 2^5$

$2^{n+5} = 2^5$

Since the base is same so we can equate the powers as follows:

$n + 5 = 5$

$n = 5 - 5 = 0$

(iv) $\mathbf{7^{2n+1} \div 49 = 7^3}$

Solution: Since 49 = 7 × 7

Thus, we can write it as follows:

$7^{2n+1} \div 7^2 = 7^3$

By using $a^m \div a^n = \frac{a^m}{a^n} = a^{m-n}$, we have

$7^{2n+1-2} = 7^3$

$7^{2n-1} = 7^3$

Since the base is same so we can equate the powers as follows:

$2n - 1 = 3$

$2n = 3 + 1 = 4$

$n = \frac{4}{2} = 2$

(v) $\mathbf{(\frac{3}{2})^4 \times (\frac{3}{2})^5 = (\frac{3}{2})^{2n+1}}$

Solution: By using $a^m \times a^n = a^{m+n}$ we have

$(\frac{3}{2})^{4+5} = (\frac{3}{2})^{2n+1}$

$(\frac{3}{2})^9 = (\frac{3}{2})^{2n+1}$

Since the base is same so we can equate the powers as follows:

$9 = 2n + 1$

9 -1 = 2n

8 = 2n

$\frac{8}{2} = n$

$n = 4$

(vi) $\left(\frac{2}{3}\right)^{10} \times \left\{\left(\frac{3}{2}\right)^{2}\right\}^{5} = \left(\frac{2}{3}\right)^{2n-2}$

Solution: By using $(a^m)^n = a^{m\times n} = (a^n)^m$, we have

$\left(\frac{2}{3}\right)^{10} \times \left(\frac{3}{2}\right)^{2\times 5} = \left(\frac{2}{3}\right)^{2n-2}$

$\left(\frac{2}{3}\right)^{10} \times \left(\frac{3}{2}\right)^{10} = \left(\frac{2}{3}\right)^{2n-2}$

Now, by using $a^m \times b^m = (ab)^m$, we have

$\left(\frac{2}{3} \times \frac{3}{2}\right)^{10} = \left(\frac{2}{3}\right)^{2n-2}$

$1^{10} = \left(\frac{2}{3}\right)^{2n-2}$

$1 = \left(\frac{2}{3}\right)^{2n-2}$

We can write 1 = $\left(\frac{2}{3}\right)^{0}$

Thus, $\left(\frac{2}{3}\right)^{0} = \left(\frac{2}{3}\right)^{2n-2}$

Since the base is same so we can equate the powers as follows:

0 = 2n – 2

$2n = 2$

$n = \frac{2}{2} = 1$

Question 8 – If $\frac{9^n \times 3^2 \times 3^n - (27)^n}{(3^3)^5 \times 2^3} = \frac{1}{27}$, find the value of n.

Solution: Since 9 = 3 × 3 and 27 = 3 × 3 × 3

$$\frac{(3^2)^n \times 3^2 \times 3^n - (3^3)^n}{(3^3)^5 \times 2^3} = \frac{1}{27}$$

$$\frac{(3)^{2n} \times 3^2 \times 3^n - 3^{3n}}{(3)^{15} \times 2^3} = \frac{1}{27}$$

By using $a^m \times a^n = a^{m+n}$ we have

$$\frac{(3)^{2n+2+n} - 3^{3n}}{(3)^{15} \times 2^3} = \frac{1}{27}$$

$$\frac{(3)^{3n+2} - 3^{3n}}{(3)^{15} \times 2^3} = \frac{1}{27}$$

$$\frac{(3)^{3n} \times 3^2 - 3^{3n}}{(3)^{15} \times 2^3} = \frac{1}{27}$$

$$\frac{(3)^{3n}(3^2 - 1)}{(3)^{15} \times 2^3} = \frac{1}{27}$$

$$\frac{(3)^{3n}(9 - 1)}{(3)^{15} \times 2^3} = \frac{1}{27}$$

$$\frac{(3)^{3n} \times 8}{(3)^{15} \times 2 \times 2 \times 2} = \frac{1}{27}$$

$$\frac{(3)^{3n} \times 8}{(3)^{15} \times 8} = \frac{1}{27}$$

$$\frac{(3)^{3n}}{(3)^{15}} = \frac{1}{27}$$

By using $a^m \div a^n = \frac{a^m}{a^n} = a^{m-n}$, we have

$$\frac{1}{(3)^{15-3n}} = \frac{1}{3^3}$$

=> $(3)^{15-3n} = 3^3$

Since the base is same so we can equate the powers as follows:

=> $15 - 3n = 3$

=> $15 - 3 = 3n$

=> $12 = 3n$

=> $n = \frac{12}{3} = 4$

Use of exponents in expressing large numbers in standard form:

When a number is expressed as the product of a number between 1 and 10 and a positive power of 10, then we can say that number is in its standard form. It is also known as scientific notation.

We will follow a procedure in order to write large numbers in its standard form

Step 1: Firstly, we obtain the number and move the decimal point to the left till we get only one digit to the left of decimal.

Step 2: secondly, write the given number as the product of number so obtained and power of 10 i.e.10^n where n is the number of places the decimal point has been moved to left.

Let us understand this by examples

Example 1 – Express the following numbers in the standard form:

(i) 390878

Solution: In order to write it in its standard form, we will first have to move the decimal point to the left as follows:

$390878 = 390878.0$

We can see that there are total of 6 places to left of decimal point. So, we will move it up to 5 places so that only one digit is there to the left of decimal point and we will write it as a product of number obtained and 10^5

Thus, 390878.0 = 3.90878×10^5

(ii) 3186500000

Solution: In order to write it in its standard form, we will first have to move the decimal point to the left as follows:

$3186500000 = 3186500000.0$

We can see that there are total of 10 places to left of decimal point. So, we will move it up to 9 places so that only one digit is there to the left of decimal point and we will write it as a product of number obtained and 10^9

Thus, $3186500000.0 = 3.186500000 \times 10^9$

$= 3.1865 \times 10^9$

(iii) 65950000

Solution: In order to write it in its standard form, we will first have to move the decimal point to the left as follows:

$65950000 = 65950000.0$

We can see that there are total of 8 places to left of decimal point. So, we will move it up to 7 places so that only one digit is there to the left of decimal point and we will write it as a product of number obtained and 10^7

Thus, $65950000.0 = 6.5950000 \times 10^7$

$= 6.595 \times 10^7$

Example 2 – Write the following numbers in the usual form:

(i) 7.54×10^6

Solution: We will convert it from its standard form to usual form as follows:

Firstly, we will shift the decimal point to the right and simplifying it as shown below

$7.54 \times 10^6 = \frac{754}{\cancel{1}00} \times 100000\cancel{0} = 7{,}540{,}000$

(ii) 9.325×10^{12}

Solution: We will convert it from its standard form to usual form as follows:

Firstly, we will shift the decimal point to the right and simplifying it as shown below

$9.325 \times 10^{12} = \frac{9325}{1000} \times 10^{12} = \frac{9325}{10^3} \div 10^{12}$ = $9325 \times 10^{12-3}$

= 9325×10^9 = 9,325,000,000,000

(iii) 8.4×10^2

Solution: We will convert it from its standard form to usual form as follows:

Firstly, we will shift the decimal point to the right and simplifying it as shown below

$8.4 \times 10^2 = \frac{84}{10} \times 100 = 840$

Exercise 6.3

Question 1 – Express the following numbers in the standard form:

(i) 3908.78

Solution: In order to write it in its standard form, we will first have to move the decimal point to the left as follows:

We have 3908.78

We can see that there are total of 4 places to left of decimal point. So, we will move it up to 3 places so that only one digit is there to the left of decimal point and we will write it as a product of number obtained and 10^3

Thus, 3908.78 = 3.90878×10^3

(ii) $5,00,00,000$

Solution: In order to write it in its standard form, we will first have to move the decimal point to the left as follows:

$50000000 = 50000000.0$

We can see that there are total of 8 places to left of decimal point. So, we will move it up to 7 places so that only one digit is there to the left of decimal point and we will write it as a product of number obtained and 10^7

Thus, $50000000.0 = 5.0000000 \times 10^7$

$= 5 \times 10^7$

(iii) $\mathbf{3186500000}$

Solution: In order to write it in its standard form, we will first have to move the decimal point to the left as follows:

$3186500000 = 3186500000.0$

We can see that there are total of 10 places to left of decimal point. So, we will move it up to 9 places so that only one digit is there to the left of decimal point and we will write it as a product of number obtained and 10^9

Thus, $3186500000.0 = 3.186500000 \times 10^9$

$= 3.1865 \times 10^9$

(iv) $\mathbf{846 \times 10^7}$

Solution: In order to write it in its standard form, we will first have to move the decimal point to the left as follows:

We have $846 \times 10^7 = 846.0 \times 10^7$

We can see that there are total of 3 places to left of decimal point. So, we will move it up to 2 places so that only one digit is there to the left of decimal point and we will write it as a product of number obtained and 10^2

Thus, $846.0 \times 10^7 = 8.46 \times 10^2 \times 10^7$

$= 8.46 \times 10^{2+7} = 8.46 \times 10^9$

(v) 723×10^9

Solution: In order to write it in its standard form, we will first have to move the decimal point to the left as follows:

We have $723 \times 10^9 = 723.0 \times 10^9$

We can see that there are total of 3 places to left of decimal point. So, we will move it up to 2 places so that only one digit is there to the left of decimal point and we will write it as a product of number obtained and 10^2

Thus, $723.0 \times 10^9 = 7.23 \times 10^2 \times 10^9$

$= 7.23 \times 10^{2+9} = 7.23 \times 10^{11}$

Question 2 – Write the following numbers in the usual form:

(i) 4.83×10^7

Solution: We will convert it from its standard form to usual form as follows:

Firstly, we will shift the decimal point to the right and simplifying it as shown below

$4.83 \times 10^7 = \frac{483}{100} \times 10^7 = \frac{483}{10^2} \times 10^7$

$= 483 \times 10^{7-2} = 483 \times 10^5$

$= 4,83,00,000$

(ii) 3.21×10^5

Solution: We will convert it from its standard form to usual form as follows:

Firstly, we will shift the decimal point to the right and simplifying it as shown below

$3.21 \times 10^5 = \frac{321}{100} \times 10^5 = \frac{321}{10^2} \times 10^5$

$= 321 \times 10^{5-2} = 321 \times 10^3$

$= 3{,}21{,}000$

(iii) $\mathbf{3.5 \times 10^3}$

Solution: We will convert it from its standard form to usual form as follows:

Firstly, we will shift the decimal point to the right and simplifying it as shown below

$3.5 \times 10^3 = \frac{35}{10} \times 10^3$

$= 35 \times 10^{3-1} = 35 \times 10^2$

$= 3{,}500$

Question 3 – express the numbers appearing in the following statements in the standard form:

(i) The distance between the Earth and the Moon is $\mathbf{384{,}000{,}000}$ metres.

Solution: In order to write it in its standard form, we will first have to move the decimal point to the left as follows:

$384000000 = 384000000.0$

We can see that there are total of 9 places to left of decimal point. So, we will move it up to 8 places so that only one digit is there to the left of decimal point and we will write it as a product of number obtained and 10^8

Thus, $384000000.0 = 3.84000000 \times 10^8$

$= 3.84 \times 10^8$

Therefore, the distance between the Earth and the Moon is 3.84×10^8 metres.

(ii) Diameter of the Earth is $\mathbf{1{,}27{,}56{,}000}$ metres.

Solution: In order to write it in its standard form, we will first have to move the decimal point to the left as follows:

$12756000 = 12756000.0$

We can see that there are total of 8 places to left of decimal point. So, we will move it up to 7 places so that only one digit is there to the left of decimal point and we will write it as a product of number obtained and 10^7

Thus, $12756000.0 = 1.2756000 \times 10^7$

$= 1.2756 \times 10^7$

Therefore, Diameter of the Earth is 1.2756×10^7metres.

(iii) Diameter of the Sun is 1,400,000,000 metres.

Solution: In order to write it in its standard form, we will first have to move the decimal point to the left as follows:

$1400000000 = 1400000000.0$

We can see that there are total of 10 places to left of decimal point. So, we will move it up to 9 places so that only one digit is there to the left of decimal point and we will write it as a product of number obtained and 10^9

Thus, $1400000000.0 = 1.400000000 \times 10^9$

$= 1.4 \times 10^9$

Therefore, Diameter of the Sun is 1.4×10^9metres.

(iv) The universe is estimated to be about $12{,}000{,}000{,}000$ years old.

Solution: In order to write it in its standard form, we will first have to move the decimal point to the left as follows:

$12000000000 = 12000000000.0$

We can see that there are total of 11 places to left of decimal point. So, we will move it up to 10 places so that only one digit is there to the left of decimal point and we will write it as a product of number obtained and 10^{10}

Thus, $12000000000.0 = 1.2000000000 \times 10^{10}$

$= 1.2 \times 10^{10}$

Therefore, the universe is estimated to be about 1.2×10^{10} years old.

Decimal Number System

We can expand a number and express in terms of powers of 10 by using the following:

$10^0 = 1,$

$10^1 = 10$

$10^2 = 100$

$10^3 = 1000 \textit{ and so on}$

Let us understand this through exercise

Exercise 6.4

Question 1 – Write the following numbers in the expanded exponential forms:

(i) 20068

Solution: We can write it as follows:

$20068 = 2 \times 10000 + 0 \times 1000 + 0 \times 100 + 6 \times 10 + 8 \times 1$

$= 2 \times 10^4 + 0 \times 10^3 + 0 \times 10^2 + 6 \times 10^1 + 8 \times 10^0$

(ii) 420719

Solution: We can write it as follows:

$420719 = 4 \times 100000 + 2 \times 10000 + 0 \times 1000 + 7 \times 100 + 1 \times 10 + 9 \times 1$

$= 4 \times 10^5 + 2 \times 10^4 + 0 \times 10^3 + 7 \times 10^2 + 1 \times 10^1 + 9 \times 10^0$

(iii) 7805192

Solution: We can write it as follows:

$7805192 = 7 \times 1000000 + 8 \times 100000 + 0 \times 10000 + 5 \times 1000 + 1 \times 100 + 9 \times 10 + 2 \times 1$

$= 7 \times 10^6 + 8 \times 10^5 + 0 \times 10^4 + 5 \times 10^3 + 1 \times 10^2 + 9 \times 10^1 + 2 \times 10^0$

(iv) 5004132

Solution: We can write it as follows:

$5004132 = 5 \times 1000000 + 0 \times 100000 + 0 \times 10000 + 4 \times 1000 + 1 \times 100 + 3 \times 10 + 2 \times 1$

$= 5 \times 10^6 + 0 \times 10^5 + 0 \times 10^4 + 4 \times 10^3 + 1 \times 10^2 + 3 \times 10^1 + 2 \times 10^0$

(v) 927303

Solution: We can write it as follows:

$927303 = 9 \times 100000 + 2 \times 10000 + 7 \times 1000 + 3 \times 100 + 0 \times 10 + 3 \times 1$

$= 9 \times 10^5 + 2 \times 10^4 + 7 \times 10^3 + 3 \times 10^2 + 0 \times 10^1 + 3 \times 10^0$

Question 2 – Find the number from each of the following expanded forms:

(i) $7 \times 10^4 + 6 \times 10^3 + 0 \times 10^2 + 4 \times 10^1 + 5 \times 10^0$

Solution: We can write it in expanded form as follows:

=> $7 \times 10000 + 6 \times 1000 + 0 \times 100 + 4 \times 10 + 5 \times 1$

=> $70000 + 6000 + 0 + 40 + 5$

$= 76045$

(ii) $5 \times 10^5 + 4 \times 10^4 + 2 \times 10^3 + 3 \times 10^0$

Solution: Firstly we will write it as $5 \times 10^5 + 4 \times 10^4 + 2 \times 10^3 + 0 \times 10^2 + 0 \times 10^1 + 3 \times 10^0$

Now, we can write it in expanded form as follows:

=> $5 \times 100000 + 4 \times 10000 + 2 \times 1000 + 0 \times 100 + 0 \times 10 + 3 \times 1$

=> $500000 + 40000 + 2000 + 0 + 0 + 3$

= 542003

(iii) $9 \times 10^5 + 5 \times 10^2 + 3 \times 10^1$

Solution: Firstly we will write it as $9 \times 10^5 + 0 \times 10^4 + 0 \times 10^3 + 5 \times 10^2 + 3 \times 10^1 + 0 \times 10^0$

Now, we can write it in expanded form as follows:

=> $9 \times 100000 + 0 \times 10000 + 0 \times 1000 + 5 \times 100 + 3 \times 10 + 0 \times 1$

=> $900000 + 0 + 0 + 500 + 30 + 0$

= 900530

(iv) $3 \times 10^4 + 4 \times 10^2 + 5 \times 10^0$

Solution: Firstly we will write it as $3 \times 10^4 + 0 \times 10^3 + 4 \times 10^2 + 0 \times 10^1 + 5 \times 10^0$

Now, we can write it in expanded form as follows:

=> $3 \times 10000 + 0 \times 1000 + 4 \times 100 + 0 \times 10 + 5 \times 1$

=> $30000 + 0 + 400 + 0 + 5$

=30405

Objective type questions

Question 1: $(\mathbf{6^{-1} - 8^{-1}})^{-1}$ **=?**

Solution: Since $a^{-1} = \frac{1}{a}$

Thus, $(6^{-1} - 8^{-1})^{-1}$ = $(\frac{1}{6} - \frac{1}{8})^{-1}$

Now, LCM of (6 and 8) is 24

Thus, $(\frac{1}{6} - \frac{1}{8})^{-1}$ = $(\frac{1\times4-1\times3}{24})^{-1}$ = $(\frac{4-3}{24})^{-1}$

= $(\frac{1}{24})^{-1} = \frac{24}{1} = 24$

Question 2: $\mathbf{2^{3^2}}$ =?

Solution: Since 3^2 = 9

Thus, 2^{3^2} = 2^9 = 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2 = 512

Question 3: $(\mathbf{3^{-1} \times 5^{-1}})^{-1}$ **=?**

Solution: Since $a^{-1} = \frac{1}{a}$

Thus, $(3^{-1} \times 5^{-1})^{-1}$= $(\frac{1}{3} \times \frac{1}{5})^{-1}$

= $(\frac{1}{3\times5})^{-1} = (\frac{1}{15})^{-1} = \frac{15}{1} = 15$

Question 4: $(-\mathbf{\frac{3}{5}})^{-1}$

Solution: Since $(\frac{a}{b})^{-1}$ = $(\frac{b}{a})$

Thus, $(-\frac{3}{5})^{-1} = -\frac{5}{3}$

Question 5: $(\mathbf{-1})^{301} + (\mathbf{-1})^{302} + (\mathbf{-1})^{303} + \cdots + (\mathbf{-1})^{400}$

Solution: Since $(-1)^n$ = 1 if n is even and -1 if n id odd

Thus, $(-1)^{301}$ = -1, $(-1)^{302}$ = 1 and so on

Therefore, $(-1)^{301} + (-1)^{302} + (-1)^{303} + \cdots + (-1)^{400}$

= $-1 + 1 - 1 + 1 - \cdots + 1$

= 0

Question 6 – If a = 25, then $a^{25^0} + a^{0^{25}}$ =?

Solution: Given that a = 25

Thus, $a^{25^0} + a^{0^{25}} = (25)^{25^0} + (25)^{0^{25}}$

Now, since $a^0 = 1$

Thus, $(25)^{25^0} + (25)^{0^{25}} = (25)^1 + (25)^0$

$= 25 + 1 = 26$

Question 7: $\{\left(\frac{1}{3}\right)^{-3} - \left(\frac{1}{2}\right)^{-3}\} \div \left(\frac{1}{4}\right)^{-3}$ =?

Solution: Since $(\frac{a}{b})^{-1} = (\frac{b}{a})$

=> $\{\left(\frac{1}{3}\right)^{-3} - \left(\frac{1}{2}\right)^{-3}\} \div \left(\frac{1}{4}\right)^{-3}$

=> $\{\left(\frac{3}{1}\right)^{3} - \left(\frac{2}{1}\right)^{3}\} \div \left(\frac{4}{1}\right)^{3}$

=> $\{(3 \times 3 \times 3) - (2 \times 2 \times 2)\} \div (4 \times 4 \times 4)$

=> $\{27 - 8\} \div 64$

=> $19 \div 64$

=> $\frac{19}{64}$

Question 8: $(\mathbf{25^2 - 15^2})^{\frac{3}{2}}$ =?

Solution: Since $25^2 = 25 \times 25 = 625$ and $15^2 = 15 \times 15 = 225$

Thus, $(25^2 - 15^2)^{\frac{3}{2}} = (625 - 225)^{\frac{3}{2}} = (400)^{\frac{3}{2}} = (400^{\frac{1}{2}})^3$

Now, since $400^{\frac{1}{2}} = (20 \times 20)^{\frac{1}{2}} = 20$

Thus, $(400^{\frac{1}{2}})^3 = 20^3 = 20 \times 20 \times 20 = 8000$

Question 9: If $(\frac{5}{3})^{-5} \times (\frac{5}{3})^{11} = (\frac{5}{3})^{8x}$, then x =?

Solution: By using $a^m \times a^n = a^{m+n}$ we have

$(\frac{5}{3})^{-5} \times (\frac{5}{3})^{11} = (\frac{5}{3})^{8x}$

=> $(\frac{5}{3})^{-5+11} = (\frac{5}{3})^{8x}$

=> $(\frac{5}{3})^{6} = (\frac{5}{3})^{8x}$

Now, since the base is same so we can equate the powers as follows:

=> $6 = 8x$

=> $x = \frac{6}{8} = \frac{3}{4}$

Question 10: $[\{(-\frac{1}{3})^2\}^{-2}]^{-1}$

Solution: $[\{(-\frac{1}{3} \times -\frac{1}{3}\}^{-2}]^{-1}$

=> $[\{(\frac{-1\times-1}{3\times3})\}^{-2}]^{-1}$

=> $[\{\frac{1}{9}\}^{-2}]^{-1}$

Now, since $(\frac{a}{b})^{-1} = (\frac{b}{a})$

=> $[\{\frac{9}{1}\}^{2}]^{-1} = [\{9 \times 9\}]^{-1}$

= $[81]^{-1} = \frac{1}{81}$

Question 11: $\frac{(144)^{\frac{1}{2}}+(256)^{\frac{1}{2}}}{3^2-2}$ =?

Solution: Since $(144)^{\frac{1}{2}} = (12 \times 12)^{\frac{1}{2}} = 12$ and $256^{\frac{1}{2}} = (16 \times 16)^{\frac{1}{2}} = 16$

Thus, $\frac{(144)^{\frac{1}{2}}+(256)^{\frac{1}{2}}}{3^2-2} = \frac{12+16}{(3\times3)-2} = \frac{28}{9-2} = \frac{28}{7} = 4$

Question 12: $(\mathbf{1+3+5+7+9+11})^{\frac{3}{2}}$ =?

Solution: Since $1 + 3 + 5 + 7 + 9 + 11 = 36$

Thus, $(1 + 3 + 5 + 7 + 9 + 11)^{\frac{3}{2}} = (36)^{\frac{3}{2}} = (36^{\frac{1}{2}})^3$

Now, $36^{\frac{1}{2}} = (6 \times 6)^{\frac{1}{2}} = 6$

=> $(36^{\frac{1}{2}})^3 = 6^3 = 6 \times 6 \times 6 = 216$

Question 13: If abc = 0, then $\frac{\{(x^a)^b\}^c}{\{(x^b)^c\}^a}$ =?

Solution: We can write $\{(x^a)^b\}^c = \{x^{ab}\}^c = x^{abc}$ and $\{(x^b)^c\}^a = \{x^{bc}\}^a = x^{abc}$

Thus, $\frac{\{(x^a)^b\}^c}{\{(x^b)^c\}^a} = \frac{x^{abc}}{x^{abc}} = \frac{x^0}{x^0} = \frac{1}{1} = 1$ (given that $abc = 0$)

Question 14: $(\mathbf{2^3})^{\mathbf{4}}$ =?

Solution: We know that $(a^m)^n = a^{m\times n}$

Thus, $(2^3)^4 = 2^{3\times4} = 2^{12} = 2^{4\times3} = (2^4)^3$

Question 15: $\{(\mathbf{33})^{\mathbf{2}} - (\mathbf{31})^{\mathbf{2}}\}^{\frac{5}{7}}$ =?

Solution: Since $(33)^2 = 33 \times 33 = 1089$ and $(31)^2 = 31 \times 31 = 961$

Thus, we have $\{(33)^2 - (31)^2\}^{\frac{5}{7}} = \{1089 - 961\}^{\frac{5}{7}} = \{128\}^{\frac{5}{7}} = \{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2\}^{\frac{5}{7}}$

$= \{(2)^7\}^{\frac{5}{7}} = 2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$

Question 16: If $abc = 0$, then find the value of $\{(x^a)^b\}^c$

Solution: We can write $\{(x^a)^b\}^c = \{x^{ab}\}^c = x^{abc}$

Now, since $abc = 0$ thus, $x^{abc} = x^0 = 1$

Question 17: If $a = 3^{-3} - 3^3$ and $b = 3^3 - 3^{-3}$, then $\frac{a}{b} - \frac{b}{a}$ =?

Solution: Given that $a = 3^{-3} - 3^3$ and $b = 3^3 - 3^{-3}$

Thus, $\frac{a}{b} = \frac{3^{-3} - 3^3}{3^3 - 3^{-3}} = \frac{-(3^3 - 3^{-3})}{3^3 - 3^{-3}}$ = -1

Similarly, $\frac{b}{a} = \frac{3^3 - 3^{-3}}{3^{-3} - 3^3} = \frac{-(3^{-3} - 3^3)}{3^{-3} - 3^3}$ = -1

Therefore, $\frac{a}{b} - \frac{b}{a} = (-1) - (-1) = -1 + 1 = 0$

Question 18: What should be multiplied to 6^{-2} so that the product may be equal to 216?

Solution: Let the required number be 'x'

Then according to given question, we have

$6^{-2} \times x = 216$

Now, $216 = 6 \times 6 \times 6$

Thus, $6^{-2} \times x = 6^3$

=> $x = 6^3 \div 6^{-2}$

=> $x = 6^{3-(-2)} = 6^{3+2} = 6^5$

Question 19: If $xyz = 0$, then find the value of $(a^x)^{yz} + (a^y)^{zx} + (a^z)^{xy}$ =?

Solution: We can write $(a^x)^{yz} = (a)^{xyz}$, $(a^y)^{zx} = (a)^{xyz}$ and $(a^z)^{xy} = (a)^{xyz}$

Since, $xyz = 0$, thus we have $(a^x)^{yz} + (a^y)^{zx} + (a^z)^{xy}$ = $(a)^{xyz} + (a)^{xyz} + (a)^{xyz}$

= $a^0 + a^0 + a^0 = 1 + 1 + 1 = 3$

Question 20: If 2^n = 4096, then 2^{n-5} =?

Solution: Firstly, we will break 4096 into prime factors as follows:

2	4096
2	2048
2	1024
2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

Now, $4096 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^{12}$

Thus, 2^n = 4096

=> $2^n = 2^{12}$

Since the base is same so we can equate the powers as follows:

$n = 12$

Therefore, $2^{n-5} = 2^{12-5} = 2^7 = 128$

Question 21 – The number $4,70,394$ in standard form is written as?

Solution: In order to write it in its standard form, we will first have to move the decimal point to the left as follows:

$470394 = 470394.0$

We can see that there are total of 6 places to left of decimal point. So, we will move it up to 5 places so that only one digit is there to the left of decimal point and we will write it as a product of number obtained and 10^5

Thus, $470394.0 = 4.70394 \times 10^5$

Question 22: The number 2.35×10^4 in the usual form is written as?

Solution: We will convert it from its standard form to usual form as follows:

Firstly, we will shift the decimal point to the right and simplifying it as shown below

$2.35 \times 10^4 = \frac{235}{100} \times 10^4 = \frac{235}{10^2} \times 10^4$

$= 235 \times 10^{4-2} = 235 \times 10^2$

$= 23500$

Question 23: If $3^x = 6561$, then 3^{x-3} =?

Solution: Firstly, we will break 6561 into prime factors as follows:

3	6561
3	2187
3	729
3	243
3	81
3	27
3	9
3	3
	1

Now, $6561 = 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 3^8$

Thus, $3^x = 6561$

=> $3^x = 3^8$

Since the base is same so we can equate the powers as follows:

$x = 8$

Therefore, $\mathbf{3^{x-3}} = 3^{8-3} = 3^5 = 243$

Question 24: If $2^{n} = 1024$, then $2^{\frac{n}{2}+2} =$?

Solution: Firstly, we will break 1024 into prime factors as follows:

2	1024
2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

Now, $1024 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^{10}$

Thus, $2^{n} = 1024$

=> $2^{n} = 2^{10}$

Since the base is same so we can equate the powers as follows:

$n = 10$

Therefore, $2^{\frac{n}{2}+2} = 2^{\frac{10}{2}+2} = 2^{5+2} = 2^{7} = 128$

Question 25: $(8^{4} + 8^{2})^{\frac{1}{2}}$

Solution: We have $(8^{4} + 8^{2})^{\frac{1}{2}}$

= $((8 \times 8 \times 8 \times 8) + (8 \times 8))^{\frac{1}{2}}$

= $(4096 + 64)^{\frac{1}{2}}$

= $(4160)^{\frac{1}{2}}$

Now, we will break 4160 into prime factors as follows:

2	4160
2	2080
2	1040
2	520
2	260
2	130
5	65
13	13
	1

Now, $4160 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 13$

Thus, $(4160)^{\frac{1}{2}} = (2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 13)^{\frac{1}{2}}$

$= 2 \times 2 \times 2\sqrt{5 \times 13} = 8\sqrt{65}$