Introduction

Algebraic Expression: In simple terms, an algebraic expression is any expression containing constants, variables and the operations like addition, subtraction, multiplication etc.

For example: $5x - 2, 4xy + 5$

Addition of algebraic expressions: First we collect like terms and then add them. We write the terms of the given expressions in the same order in form of rows with like terms below each other and adding column wise.

Subtraction of algebraic expressions:

Step1: Arrange the terms of the given expressions in the same order.

Step2: write the given expressions in two rows in such a way that the like terms occur one below the other, keeping the expression to be subtracted in the second row.

Step3: Change the sign of each term in the lower row from + to – and from – to +.

Step4: with new signs of the terms of lower row, add column wise.

Examples:

Example 1 - Add: $6a + 8b - 5c, 2b + c - 4a \text{ and } a - 3b - 2c$

Solution **-** We will write all the expressions in the rows in the manner that like terms occur one below the other and then add them column wise.

$$
\begin{array}{r}
6a + 8b - 5c \\
-4a + 2b + c \\
a - 3b - 2c \\
\hline
3a + 7b - 6c \\
\hline
\end{array}
$$

Example 2 - Add: $5x^2 + 3x - 8,\ 4x + 7 - 2x^2\ and\ 6 - 5x + 4x^2$

Solution - We will write all the expressions in the rows in the manner that like terms occur one below the other and then add them column wise.

$$\begin{array}{c} 5x^2 + 3x - 8 \\ -2x^2 + 4x + 7 \\ 4x^2 - 5x + 6 \\ \hline 7x^2 + 2x + 5 \\ \hline \end{array}$$

Example 3 - Add: $8x^2 - 5xy + 3y^2,\ 2xy - 6y^2 + 3x^2\ and\ y^2 + xy - 6x^2$

Solution – We will write all the expressions in the rows in the manner that like terms occur one below the other and then add them column wise.

$$\begin{array}{c} 8x^2 - 5xy + 3y^2 \\ 3x^2 + 2xy - 6y^2 \\ -6x^2 + xy + y^2 \\ \hline 5x^2 - 2xy - 2y^2 \\ \hline \end{array}$$

Example 4 - Subtract $4a + 5b - 3c$ from $6a - 3b + c$

Solution -

$$\begin{array}{c} 6a - 3b + c \\ 4a + 5b - 3c \\ - \quad - \quad + \\ \hline 2a - 8b + 4c \\ \hline \end{array}$$

Example 5 - Subtract $3x^2 - 6x - 4$ from $5 + x - 2x^2$

Solution -

$$\begin{array}{c} -2x^2 + x + 5 \\ 3x^2 - 6x - 4 \\ - \quad + \quad + \\ \hline -5x^2 + 7x + 9 \end{array}$$

Exercise 6A

Add:

Question 1 - $\boldsymbol{8ab, -5ab, 3ab, -ab}$

Solution - We have, $8ab + (-5ab) + 3ab + (-ab)$

=> $8ab - 5ab + 3ab - ab = 11ab - 6ab = 5ab$

Question 2 - $\boldsymbol{7x, -3x, 5x, -x, -2x}$

Solution - We have, $7x + (-3x) + 5x + (-x) + (-2x)$

=> $7x - 3x + 5x - x - 2x = 12x - 6x = 6x$

Question 3 - $\boldsymbol{3a - 4b + 4c, 2a + 3b - 8c, a - 6b + c}$

Solution -

$$3a - 4b + 4c$$

$$2a + 3b - 8c$$

$$a - 6b + c$$

$$6a - 7b - 3c$$

Question 4 - $\boldsymbol{5x - 8y + 2z,\ 3z - 4y - 2x, 6y - z - x\ \ and\ 3x - 2z - 3y}$

Solution - We will write all the expressions in the rows in the manner that like terms occur one below the other and then add them column wise

$$5x - 8y + 2z$$

$$-2x - 4y + 3z$$

$$-x + 6y - z$$

$$3x - 3y - 2z$$

$$5x - 9y + 2z$$

Question 5 - $\boldsymbol{6ax - 2by + 3cz, 6by - 11ax - cz \text{ and } 10cz - 2ax - 3by}$

Solution **-** We will write all the expressions in the rows in the manner that like terms occur one below the other and then add them column wise.

$$6ax - 2by + 3cz$$

$$-11ax + 6by - cz$$

$$-2ax - 3by + 10cz$$

$$-7ax + by + 12cz$$

Question 6 - $\boldsymbol{2x^3 - 9x^2 + 8, 3x^2 - 6x - 5, 7x^3 - 10x + 1 \text{ and } 3 + 2x - 5x^2 - 4x^3}$

Solution **-**We will write all the expressions in the rows in the manner that like terms occur one below the other and then add them column wise

$$2x^3 - 9x^2 + 0x + 8$$

$$0x^3 + 3x^2 - 6x - 5$$

$$7x^3 + 0x^2 - 10x + 1$$

$$-4x^3 - 5x^2 + 2x + 3$$

$$5x^3 - 11x^2 - 14x + 7$$

Question 7 - $\boldsymbol{6p + 4q - r + 3, 2r - 5p - 6, 11q - 7p + 2r - 1 \text{ and } 2q - 3r + 4}$

Solution **-** We will write all the expressions in the rows in the manner that like terms occur one below the other and then add them column wise

$$6p + 4q - r + 3$$

$$-5p + 0q + 2r - 6$$

$$-7p + 11q + 2r - 1$$

$$0p + 2q - 3r + 4$$

$$-6p + 17q$$

Question 8 - $\boldsymbol{4x^2 - 7xy + 4y^2 - 3, 5 + 6y^2 - 8xy + x^2}$ ***and*** $\boldsymbol{6 - 2xy + 2x^2 - 5y^2}$

Solution -We will write all the expressions in the rows in the manner that like terms occur one below the other and then add them column wise.

$$4x^2 - 7xy + 4y^2 - 3$$

$$x^2 - 8xy + 6y^2 + 5$$

$$2x^2 - 2xy - 5y^2 + 6$$

$$7x^2 - 17xy + 5y^2 + 8$$

Subtract:

Question 9: 3$\boldsymbol{a^2b}$ from -5$\boldsymbol{a^2b}$

Solution -We have $(-5a^2b - 3a^2b)$

= -8a^2b

Question 10: -8pq from 6pq

Solution - We have ($6pq -$ (-8pq))

= (6pq+8pq)

= 14pq

Question 11: -2abc from -8abc

Solution -We have$(-8abc) - (-2abc)$)

= $-8abc + 2abc$

= $-6abc$

Question 12: -16p from -11p

Solution -We have ($11p -$ (-16p))

= (11p+16p)

= 27p

Question 13: $2a - 5b + 2c - 9$ from $3a - 4b - c + 6$

Solution -

$$\begin{array}{r} 3a - 4b - c + 6 \\ 2a - 5b + 2c - 9 \\ - \quad + \quad - \quad + \\ \hline a + b - 3c + 15 \\ \hline \end{array}$$

Question 14: $-6p + q + 3r + 8$ from $p - 2q - 5r - 8$

Solution -

$$\begin{array}{r} p - 2q - 5r - 8 \\ -6p + q + 3r + 8 \\ + \quad - \quad - \quad - \\ \hline 7p - 3q - 8r - 16 \\ \hline \end{array}$$

Question 15: $x^3 + 3x^2 - 5x + 4$ from $3x^3 - x^2 + 2x - 4$

Solution -

$$\begin{array}{r} 3x^3 - x^2 + 2x - 4 \\ x^3 + 3x^2 - 5x + 4 \\ - \quad - \quad + \quad - \\ \hline 2x^3 - 4x^2 + 7x - 8 \\ \hline \end{array}$$

Question 16: $5y^4 - 3y^3 + 2y^2 + y - 1$ from $4y^4 - 2y^3 - 6y^2 - y + 5$

Solution -

$$\begin{array}{r} 4y^4 - 2y^3 - 6y^2 - y + 5 \\ 5y^4 - 3y^3 + 2y^2 + y - 1 \\ - \quad + \quad - \quad - \quad + \\ \hline -y^4 + y^3 - 8y^2 - 2y + 6 \\ \hline \end{array}$$

Question 17: $4p^2 + 5q^2 - 6r^2 + 7$ from $3p^2 - 4q^2 - 5r^2 - 6$

Solution -

$$3p^2 - 4q^2 - 5r^2 - 6$$

$$4p^2 + 5q^2 - 6r^2 + 7$$

$$- \quad - \quad + \quad -$$

$$-p^2 - 9q^2 + r^2 - 13$$

Question 18: What must be subtracted from $3a^2 - 6ab - 3b^2 - 1$ to get $4a^2 - 7ab - 4b^2 + 1$?

Solution - Let x be subtracted from $3a^2 - 6ab - 3b^2 - 1$ to get $4a^2 - 7ab - 4b^2 + 1$

Then, $(3a^2 - 6ab - 3b^2 - 1) - (x) = (4a^2 - 7ab - 4b^2 + 1)$

=> x = $(3a^2 - 6ab - 3b^2 - 1) - (4a^2 - 7ab - 4b^2 + 1)$

$$3a^2 - 6ab - 3b^2 - 1$$

$$4a^2 - 7ab - 4b^2 + 1$$

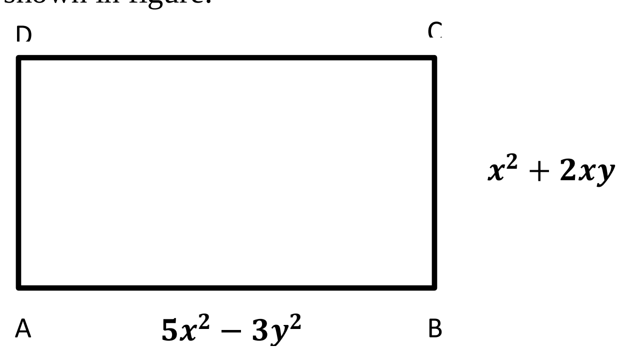
$$- \quad + \quad + \quad -$$

$$-a^2 + ab + b^2 - 2$$

Thus, $x = -a^2 + ab + b^2 - 2$

Question 19 - The two adjacent sides of a rectangle are $5x^2 - 3y^2$ and $x^2 + 2xy$. Find the perimeter.

Solution - We are given the adjacent sides of rectangle as shown in figure:



We know that perimeter = sum of all the sides

Since AB = CD = $5x^2 - 3y^2$ (Opposite sides of rectangle are equal)

Also, AD = BC = $x^2 + 2xy$

Thus, perimeter of rectangle = AB+BC+CD+AD

$$5x^2 - 3y^2 + 0xy$$

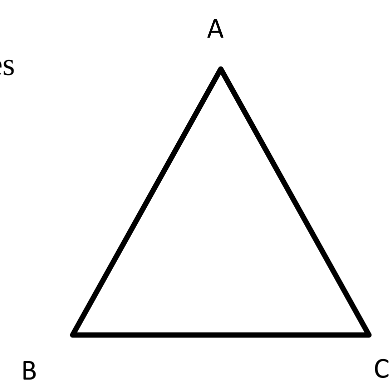
$$5x^2 - 3y^2 + 0xy$$

$$x^2 + 0y^2 + 2xy$$

$$x^2 + 0y^2 + 2xy$$

$$12x^2 - 6y^2 + 4xy$$

Question 20 - The perimeter of triangle is $6p^2 - 4p + 9$ and two of its sides are $p^2 - 2p + 1$ and$3p^2 - 5p + 3$. Find the third side of the triangle.



Solution -We know that perimeter of triangle = sum of all three sides

Perimeter = AB+BC+AC

Here, perimeter = $6p^2 - 4p + 9$

AB = $p^2 - 2p + 1$

BC = $3p^2 - 5p + 3$

AC =?

Since, perimeter = AB+BC+AC

=> $6p^2 - 4p + 9$ = $(p^2 - 2p + 1)$ + $(3p^2 - 5p + 3)$ + AC

=> $6p^2 - 4p + 9$ = ($(p^2 + 3p^2) + (-2p - 5p) + (1 + 3)$) + AC

=> $6p^2 - 4p + 9$ = ($4p^2 - 7p + 4$)+AC

=> AC =$(6p^2 - 4p + 9)$ - $(4p^2 - 7p + 4)$

$$6p^2 - 4p + 9$$

$$4p^2 - 7p + 4$$

$$- \quad + \quad -$$

$$2p^2 + 3p + 5$$

Therefore, third side AC = $2p^2 + 3p + 5$

Multiplication of algebraic expressions

Two simple rules:

(a) The product of two factors with like signs is positive, and the product of two factors with unlike signs is negative

(b) If x is a variable and m, n are positive integers, then $(x^m \times x^n) = x^{(m+n)}$

Let us understand by examples:

Example 1 - Find the product of:

(a) $6xy$ and $-3x^2y^3$

Solution - Product of two monomials = (product of their numerical coefficients) ×(product of their variable parts)

$(6xy) \times (-3x^2y^3)$ = $(6 \times (-3)) \times (xy \times x^2y^3)$ = $-18x^{(1+2)}y^{(1+3)}$ = $-18x^3y^4$

(b) $7ab^2, -4a^2b$ and $-5abc$

Solution -$(7ab^2) \times (-4a^2b) \times (-5abc)$

= $(7 \times (-4) \times (-5)) \times (a^{(1+2+1)}b^{(2+1+1)}c)$

= $140a^4b^4c$

Example 2 - Find each of the following products:

(a) $5a^2b^2 \times (3a^2 - 4ab + 6b^2)$

Solution - Since $a \times (b + c) = a \times b + a \times c$

Thus, $5a^2b^2 \times (3a^2 - 4ab + 6b^2)$

= $(5a^2b^2) \times (3a^2) + (5a^2b^2) \times (-4ab) + (5a^2b^2) \times (6b^2)$

= $15a^4b^2 - 20a^3b^3 + 30a^2b^4$

(b) $(\mathbf{-3x^2y}) \times (\mathbf{4x^2y - 3xy^2 + 4x - 5y})$

Solution - Since $a \times (b + c) = a \times b + a \times c$

Thus, $(-3x^2y) \times (4x^2y - 3xy^2 + 4x - 5y)$

= $(-3x^2y) \times (4x^2y) + (-3x^2y) \times (-3xy^2) + (-3x^2y) \times (4x) + (-3x^2y) \times (-5y)$

= $-12x^4y^2 + 9x^3y^3 - 12x^3y + 15x^2y^2$

Example 3 - Multiply $(3x + 5y)$ and $(5x - 7y)$

Solution -Since by distributive law, we have $(a + b) \times (c + d) = a \times (c + d) + b \times (c + d)$

Thus, $(3x + 5y) \times (5x - 7y)$

= $3x(5x - 7y) + 5y(5x - 7y)$

= $(3x \times 5x - 3x \times 7y) + (5y \times 5x - 5y \times 7y)$

= $(15x^2 - 21xy) + (25xy - 35y^2)$

= $15x^2 - 21xy + 25xy - 35y^2$

= $15x^2 + 4xy - 35y^2$

Example 4 - Multiply $(3x^2 + y^2)$ by $(2x^2 + 3y^2)$

Solution - $(3x^2 + y^2) \times (2x^2 + 3y^2)$

= $3x^2(2x^2 + 3y^2) + y^2(2x^2 + 3y^2)$

= $6x^4 + 9x^2y^2 + 2x^2y^2 + 3y^4$

= $6x^4 + 11x^2y^2 + 3y^4$

Example 5 - Multiply $(5x^2 - 6x + 9)$ by $(2x - 3)$

Solution -$(2x - 3) \times (5x^2 - 6x + 9)$

= $2x(5x^2 - 6x + 9) - 3(5x^2 - 6x + 9)$

$= 10x^3 - 12x^2 + 18x - 15x^2 + 18x - 27$

$= 10x^3 - 12x^2 - 15x^2 + 18x + 18x - 27$

$= 10x^3 - 27x^2 + 36x - 27$

Example 6 - Multiply $(2x^2 - 5x + 4)$ by $(x^2 + 7x - 8)$

Solution -$(2x^2 - 5x + 4) \times (x^2 + 7x - 8)$

$= 2x^2(x^2 + 7x - 8) - 5x(x^2 + 7x - 8) + 4(x^2 + 7x - 8)$

$= 2x^4 + 14x^3 - 16x^2 - 5x^3 - 35x^2 + 40x + 4x^2 + 28x - 32$

$= 2x^4 + 14x^3 - 5x^3 - 16x^2 - 35x^2 + 4x^2 + 40x + 28x - 32$

$= 2x^4 + 9x^3 - 47x^2 + 68x - 32$

Example 7 - Multiply $(2x^3 - 5x^2 - x + 7)$ by $(3 - 2x + 4x^2)$

Solution - $(3 - 2x + 4x^2) \times (2x^3 - 5x^2 - x + 7)$

$= 3(2x^3 - 5x^2 - x + 7) - 2x(2x^3 - 5x^2 - x + 7) + 4x^2(2x^3 - 5x^2 - x + 7)$

$= 6x^3 - 15x^2 - 3x + 21 - 4x^4 + 10x^3 + 2x^2 - 14x + 8x^5 - 20x^4 - 4x^3 + 28x^2$

$= 8x^5 - 4x^4 - 20x^4 + 6x^3 + 10x^3 - 4x^3 - 15x^2 + 2x^2 + 28x^2 - 3x - 14x + 21$

$= 8x^5 - 24x^4 + 12x^3 + 15x^2 - 17x + 21$

Exercise 6B

Find each of the following products:

Question 1: $(5x + 7) \times (3x + 4)$

Solution - $5x(3x + 4) + 7(3x + 4)$

$= 15x^2 + 20x + 21x + 28$

$= 15x^2 + 41x + 28$

Question 2: $(4x + 9) \times (x - 6)$

Solution - $4x(x - 6) + 9(x - 6)$

$= 4x^2 - 24x + 9x - 54$

= $4x^2 - 15\text{x} - 54$

Question 3: $(\mathbf{2x + 5}) \times (\mathbf{4x - 3})$

Solution -$2x(4x - 3) + 5(4x - 3)$

= $8x^2 - 6x + 20x - 15$

= $8x^2 + 14x - 15$

Question 4: $(\mathbf{3y - 8}) \times (\mathbf{5y - 1})$

Solution - $3y(5y - 1) - 8(5y - 1)$

= $15y^2 - 3y - 40y + 1$

= $15y^2 - 43y + 1$

Question 5: $(\mathbf{7x + 2y}) \times (\mathbf{x + 4y})$

Solution - $7x(x + 4y) + 2y(x + 4y)$

= $7x^2 + 28xy + 2xy + 8y^2$

= $7x^2 + 30xy + 8y^2$

Question 6: $(\mathbf{9x + 5y}) \times (\mathbf{4x + 3y})$

Solution - $9x(4x + 3y) + 5y(4x + 3y)$

= $36x^2 + 27xy + 20xy + 15y^2$

= $36x^2 + 47xy + 15y^2$

Question 7: $(\mathbf{3m - 4n}) \times (\mathbf{2m - 3n})$

Solution - $3m(2m - 3n) - 4n(2m - 3n)$

= $6m^2 - 9mn - 8mn + 12n^2$

= $6m^2 - 17mn + 12n^2$

Question 8: $(\mathbf{x^2 - a^2}) \times (\mathbf{x - a})$

Solution - $x(x^2 - a^2) - a(x^2 - a^2)$

= $x^3 - xa^2 - ax^2 + a^3$

Question 9: $(\mathbf{x^2 - y^2}) \times (\mathbf{x + 2y})$

Solution - $x(x^2 - y^2) + 2y(x^2 - y^2)$

= $x^3 - xy^2 + 2x^2y - 2y^3$

Question 10: $(\mathbf{3p^2 + q^2}) \times (\mathbf{2p^2 - 3q^2})$

Solution - $3p^2(2p^2 - 3q^2) + q^2(2p^2 - 3q^2)$

= $6p^4 - 9p^2q^2 + 2p^2q^2 - 3q^4$

= $6p^4 - 7p^2q^2 - 3q^4$

Question 11: $(\mathbf{2x^2 - 5y^2}) \times (\mathbf{x^2 + 3y^2})$

Solution - $2x^2(x^2 + 3y^2) - 5y^2(x^2 + 3y^2)$

= $2x^4 + 6x^2y^2 - 5x^2y^2 - 15y^4$

= $2x^4 + x^2y^2 - 15y^4$

Question 12: $(\mathbf{x^3 - y^3}) \times (\mathbf{x^2 + y^2})$

Solution - $x^3(x^2 + y^2) - y^3(x^2 + y^2)$

= $x^5 + x^3y^2 - x^2y^3 - y^5$

Question13: $(\mathbf{x^4 + y^4}) \times (\mathbf{x^2 - y^2})$

Solution - $x^4(x^2 - y^2) + y^4(x^2 - y^2)$

= $x^6 - x^4y^2 + x^2y^4 - y^6$

Question 14: $(\mathbf{x^4 + \frac{1}{x^4}}) \times (\mathbf{x + \frac{1}{x}})$

Solution - $x^4\left(x + \frac{1}{x}\right) + \frac{1}{x^4}(x + \frac{1}{x})$

= $x^5 + \frac{x^4}{x} + \frac{x}{x^4} + \frac{1}{x^5}$

= $x^5 + x^3 + \frac{1}{x^3} + \frac{1}{x^5}$

Find each of the following products:

Question 15: $(\mathbf{x^2 - 3x + 7}) \times (\mathbf{2x + 3})$

Solution - $2x(x^2 - 3x + 7) + 3(x^2 - 3x + 7)$

= $2x^3 - 6x^2 + 14x + 3x^2 - 9x + 21$

= $2x^3 - 6x^2 + 3x^2 + 14x - 9x + 21$

= $2x^3 - 3x^2 + 5x + 21$

Question 16: $(\mathbf{3x^2 + 5x - 9}) \times (\mathbf{3x - 5})$

Solution - $3x(3x^2 + 5x - 9) - 5(3x^2 + 5x - 9)$

= $9x^3 + 15x^2 - 27x - 15x^2 - 25x + 45$

= $9x^3 + 15x^2 - 15x^2 - 27x - 25x + 45$

= $9x^3 - 52x + 45$

Question 17: $(\boldsymbol{x^2 - xy + y^2}) \times (\boldsymbol{x + y})$

Solution: $x(x^2 - xy + y^2) + y(x^2 - xy + y^2)$

= $x^3 - x^2y + xy^2 + x^2y - xy^2 + y^3$

= $x^3 - x^2y + x^2y + xy^2 - xy^2 + y^3$

= $x^3 + y^3$

Question 18: $(\boldsymbol{x^2 + xy + y^2}) \times (\boldsymbol{x - y})$

Solution: $x(x^2 + xy + y^2) - y(x^2 + xy + y^2)$

= $x^3 + x^2y + xy^2 - x^2y - xy^2 - y^3$

= $x^3 + x^2y - x^2y + xy^2 - xy^2 - y^3$

= $x^3 - y^3$

Question 19: $(\boldsymbol{x^3 - 2x^2 + 5}) \times (\boldsymbol{4x - 1})$

Solution: $4x(x^3 - 2x^2 + 5) - 1(x^3 - 2x^2 + 5)$

= $4x^4 - 8x^3 + 20x - x^3 + \ 2x^2 - 5$

= $4x^4 - 8x^3 - x^3 + \ 2x^2 + 20x - 5$

= $4x^4 - 9x^3 + \ 2x^2 + 20x - 5$

Question 20: $(\boldsymbol{9x^2 - x + 15}) \times (\boldsymbol{x^2 - 3})$

Solution: $x^2(9x^2 - x + 15) - 3(9x^2 - x + 15)$

= $9x^4 - x^3 + 15x^2 - 27x^2 + 3x - 45$

= $9x^4 - x^3 - 12x^2 + 3x - 45$

Question 21: $(\boldsymbol{x^2 - 5x + 8}) \times (\boldsymbol{x^2 + 2})$

Solution: $x^2(x^2 - 5x + 8) + 2(x^2 - 5x + 8)$

= $x^4 - 5x^3 + 8x^2 + 2x^2 - 10x + 16$

= $x^4 - 5x^3 + 10x^2 - 10x + 16$

Question 22: $(\boldsymbol{x^3 - 5x^2 + 3x + 1}) \times (\boldsymbol{x^2 - 3})$

Solution: $x^2(x^3 - 5x^2 + 3x + 1) - 3(x^3 - 5x^2 + 3x + 1)$

= $x^5 - 5x^4 + 3x^3 + x^2 - 3x^3 + 15x^2 - 9x - 3$

= $x^5 - 5x^4 + 3x^3 - 3x^3 + x^2 + 15x^2 - 9x - 3$

= $x^5 - 5x^4 + 16x^2 - 9x - 3$

Question 23: $(\boldsymbol{3x + 2y - 4}) \times (\boldsymbol{x - y + 2})$

Solution: $x(3x + 2y - 4) - y(3x + 2y - 4) + 2(3x + 2y - 4)$

= $3x^2 + 2xy - 4x - 3xy - 2y^2 + 4y + 6x + 4y - 8$

= $3x^2 + 2xy - 3xy - 4x + 6x - 2y^2 + 4y + 4y - 8$

= $3x^2 - xy + 2x - 2y^2 + 8y - 8$

Question 24: $(\boldsymbol{x^2 - 5x + 8}) \times (\boldsymbol{x^2 + 2x - 3})$

Solution: $x^2(x^2 + 2x - 3) - 5x(x^2 + 2x - 3) + 8(x^2 + 2x - 3)$

= $x^4 + 2x^3 - 3x^2 - 5x^3 - 10x^2 + 15x + 8x^2 + 16x - 24$

= $x^4 + 2x^3 - 5x^3 - 3x^2 - 10x^2 + 8x^2 + 15x + 16x - 24$

= $x^4 - 3x^3 - 5x^2 + 31x - 24$

Question 25: $(\boldsymbol{2x^2 + 3x - 7}) \times (\boldsymbol{3x^2 - 5x + 4})$

Solution: $2x^2(3x^2 - 5x + 4) + 3x(3x^2 - 5x + 4) - 7(3x^2 - 5x + 4)$

= $6x^4 - 10x^3 + 8x^2 + 9x^3 - 15x^2 + 12x - 21x^2 + 35x - 21$

= $6x^4 - 10x^3 + 9x^3 + 8x^2 - 15x^2 - 21x^2 + 12x + 35x - 21$

$= 6x^4 - x^3 - 28x^2 + 47x - 21$

Question 26: $(\mathbf{9x^2 - x + 15}) \times (\mathbf{x^2 - x - 1})$

Solution**:** $9x^2(x^2 - x - 1) - x(x^2 - x - 1) + 15(x^2 - x - 1)$

$= 9x^4 - 9x^3 - 9x^2 - x^3 + x^2 + x + 15x^2 - 15x - 15$

$= 9x^4 - 9x^3 - x^3 - 9x^2 + x^2 + 15x^2 + x - 15x - 15$

$=9x^4 - 10x^3 + 7x^2 - 14x - 15$

Division of algebraic expressions:

If x is a variable and m, n are positive integers such that m > n then

$(x^m \div x^n) = x^{m-n}$

Example 1 - Divide

(a) $\mathbf{8x^2y^3}$ **By** $\mathbf{-2xy}$

Solution - We have: $\frac{\mathbf{8x^2y^3}}{\mathbf{-2xy}} = (\frac{8}{-2})x^{2-1}y^{3-1}$

$= -4xy^2$

(b) $\mathbf{-15x^3yz^3}$ **By** $\mathbf{-5xyz^2}$

Solution - We have: $\frac{\mathbf{-15x^3yz^3}}{\mathbf{-5xyz^2}} = (\frac{-15}{-5})x^{3-1}z^{3-2}$

$= 3x^2z$

Example 2 - Divide:

(a) $\mathbf{6x^5 + 18x^4 - 3x^2}$ **By** $\mathbf{3x^2}$

Solution - We have: $\frac{\mathbf{6x^5+18x^4-3x^2}}{\mathbf{3x^2}}$

Note: when we divide a polynomial by a monomial, divide each term of the polynomial by the monomial.

=> $\frac{\mathbf{6x^5}}{\mathbf{3x^2}} + \frac{\mathbf{18x^4}}{\mathbf{3x^2}} - \frac{\mathbf{3x^2}}{\mathbf{3x^2}}$

$= 2x^{5-2} + 6x^{4-2} - 1$

$= 2x^3 + 6x^2 - 1$

(b) $20x^3y + 12x^2y^2 - 10xy$ By $2xy$

Solution - We have: $\frac{20x^3y+12x^2y^2-10xy}{2xy}$

$= \frac{20x^3y}{2xy} + \frac{12x^2y^2}{2xy} - \frac{10xy}{2xy}$

$= 10x^{3-1} + 6x^{2-1}y^{2-1} - 5$

$= 10x^2 + 6xy - 5$

Division of a polynomial by a polynomial

There is a procedure to solve this as follows:

Step1: Arrange the terms of the dividend and divisor in descending order of their degrees.

Step2: Divide the first term of the dividend by the first term of the divisor to obtain the first term of the quotient.

Step3: Multiply all the terms of the divisor by the first term of the quotient and subtract the result from the dividend.

Step4: Consider the remainder (if any) as a new dividend and proceed as before.

Step5: Repeat this process till we obtain a remainder which is either 0 or a polynomial of degree less than that of the divisor.

Example 3 - Divide $2x^2 + 3x + 1$ by $x + 1$

Solution -

$x + 1$) $\mathbf{2x^2 + 3x + 1}$ ($2x + 1$

$2x^2 + 2x$

$-\quad -$

$x + 1$

$x + 1$

$-\quad -$

0

Thus, we get quotient = $2x + 1$

Example 4 - Divide $9x - 6x^2 + x^3 - 2$ by $x - 2$

Solution - First we will arrange dividend in the descending order,

Dividend = $9x - 6x^2 + x^3 - 2$ = $x^3 - 6x^2 + 9x - 2$

$$x - 2 \;\overline{)\; x^3 - 6x^2 + 9x - 2 \;(}\; x^2 - 4x+1$$

$$x^3 - 2x^2$$

$$- \quad +$$

$$-4x^2 + 9x$$

$$-4x^2 + 8x$$

$$+ \quad -$$

$$x - 2$$

$$x - 2$$

$$- \quad +$$

$$0$$

Thus, we get quotient = $x^2 - 4x + 1$

Example 5 - Divide $29x - 6x^2 - 28$ by $3x - 4$

Solution - First we will arrange dividend in the descending order,

Dividend =$29x - 6x^2 - 28$ =$-6x^2 + 29x - 28$

$$3x - 4 \;\overline{)\; -6x^2 + 29x - 28 \;(}\; -2x + 7$$

$$-6x^2 + 8x$$

$$+ \quad -$$

$$21x - 28$$

$$21x - 28$$

$$- \quad +$$

$$0$$

Thus, we get quotient = $-2x + 7$

Example 6 - Divide $5x^3 - 4x^2 + 3x + 18$ by $3 - 2x + x^2$

Solution - First we will arrange divisor in the descending order,

Divisor $= 3 - 2x + x^2 = x^2 - 2x + 3$

$x^2 - 2x + 3$) $5x^3 - 4x^2 + 3x + 18$ ($5x + 6$

$5x^3 - 10x^2 + 15x$

$- \quad + \quad -$

$6x^2 - 12x + 18$

$6x^2 - 12x + 18$

$- \quad + \quad -$

0

Thus, we get quotient = $5x + 6$

Example 7 - Using division, show that $(x - 1)$ is a factor of $(x^3 - 1)$

Solution -

$x - 1$) $x^3 + 0x^2 + 0x - 1$ ($x^2 + x + 1$

$x^3 - x^2$

$- \quad +$

$x^2 + 0x - 1$

$x^2 - x$

$+$

$x - 1$

$x - 1$

$- \quad +$

0

Since $x^3 - 1$ is completely divisible by $x - 1$

Therefore, $(x - 1)$ is a factor of $(x^3 - 1)$

Example 8 - Find the quotient and remainder when $(7 + 15x - 13x^2 + 5x^3)$ is divided by $4 - 3x + x^2$

Solution - First we will arrange the terms of dividend and divisor in the descending order

Dividend = $5x^3 - 13x^2 + 15x + 7$

Divisor = $x^2 - 3x + 4$

$$
\begin{array}{r|l|l}
x^2 - 3x + 4 & 5x^3 - 13x^2 + 15x + 7 & 5x + 2 \\
& 5x^3 - 15x^2 + 20x & \\
& - \quad + \quad - & \\
\hline
& 2x^2 - 5x + 7 & \\
& 2x^2 - 6x + 8 & \\
& - \quad + \quad - & \\
\hline
& x - 1 &
\end{array}
$$

We get quotient = $5x + 2$

Remainder = $x - 1$

Example 9 - Divide $10x^4 + 17x^3 - 62x^2 + 30x - 3$ by $2x^2 + 7x - 1$

Solution -

$2x^2 + 7x - 1$) $10x^4 + 17x^3 - 62x^2 + 30x - 3$ ($5x^2 - 9x + 3$

$10x^4 + 35x^3 - 5x^2$

$- \quad - \quad +$

$-18x^3 - 57x^2 + 30x$

$-18x^3 - 63x^2 + 9x$

$+ \quad + \quad -$

$6x^2 + 21x - 3$

$6x^2 + 21x - 3$

$- \quad - \quad +$

0

Exercise 6C

Question 1 - Divide

(a) $24x^2y^3$ By $3xy$

Solution - We have: $\frac{24x^2y^3}{3xy} = (\frac{24}{3})x^{2-1}y^{3-1}$

$= 8xy^2$

(b) $36xyz^2$ By $-9xz$

Solution - We have: $\frac{36xyz^2}{-9xz} = (\frac{36}{-9})yz^{2-1}$

$= -4yz$

(c) $-72x^2y^2z$ By $-12xyz$

Solution - We have: $\frac{-72x^2y^2z}{-12xyz} = (\frac{-72}{-12})x^{2-1}y^{2-1}$

$= 6xy$

(d) $-56mnp^2$ By $7mnp$

Solution - We have: $\frac{-56mnp^2}{7mnp} = (\frac{-56}{7})p^{2-1}$

$= -8p$

Question 2 - Divide:

(a) $5m^3 - 30m^2 + 45m$ By 5m

Solution - We have: $\frac{5m^3-30m^2+45m}{5m}$

$= \frac{5m^3}{5m} - \frac{30m^2}{5m} + \frac{45m}{5m}$

$= m^{3-1} - 6m^{2-1} + 9$

$= m^2 - 6m + 9$

(b) $8x^2y^2 - 6xy^2 + 10x^2y^3$ By $2xy$

Solution - We have: $\frac{8x^2y^2-6xy^2+10x^2y^3}{2xy}$

$= \frac{8x^2y^2}{2xy} - \frac{6xy^2}{2xy} + \frac{10x^2y^3}{2xy}$

$= 4x^{2-1}y^{2-1} - 3y^{2-1} + 5x^{2-1}y^{3-1}$

$= 4xy - 3y + 5xy^2$

(c) $9x^2y - 6xy + 12xy^2$ By – $3xy$

Solution - We have: $\frac{9x^2y-6xy+12xy^2}{-3xy}$

$= \frac{9x^2y}{-3xy} - \frac{6xy}{-3xy} + \frac{12xy^2}{-3xy}$

$= -3x^{2-1} + 2 - 4y^{2-1}$

$= -3x + 2 - 4y$

(d) $12x^4 + 8x^3 - 6x^2$ By $-2x^2$

Solution - We have: $\frac{\mathbf{12x^4+8x^3-6x^2}}{-2x^2}$

= $\frac{12x^4}{-2x^2} + \frac{8x^3}{-2x^2} - \frac{6x^2}{-2x^2}$

= $-6x^{4-2} - 4x + 3$

= $-6x^2 - 4x + 3$

Write the quotient and remainder when we divide:

Question 3 - $(\boldsymbol{x^2 - 4x + 4})$ by $(\boldsymbol{x - 2})$

Solution -

$x - 2$) $x^2 - 4x + 4$ ($x - 2$

$x^2 - 2x$

$-\quad +$

$-2x + 4$

$-2x + 4$

$+\quad -$

0

Thus, we get quotient = $x - 2$ and remainder = 0

Question 4: $(\boldsymbol{x^2 - 4})$ by $(\boldsymbol{x + 2})$

Solution $x + 2$) $x^2 + 0x - 4$ ($x - 2$

$x^2 + 2x$

$-\quad -$

$-2x - 4$

$-2x - 4$

$+\quad +$

0

Thus, we get quotient = $x - 2$

Remainder = 0

Question 5: $(x^2 + 12x + 35)$ **by** $(x + 7)$

Solution

$$
\begin{array}{r|l|l}
x+7 & x^2+12x+35 & x+5 \\
 & x^2+7x & \\
 & - \quad - & \\
\hline
 & 5x+35 & \\
 & 5x+35 & \\
 & - \quad - & \\
\hline
 & 0 &
\end{array}
$$

Thus, we get quotient = $x + 5$ and remainder = 0

Question 6: $15x^2 + x - 6$ **by** $3x + 2$

Solution

$$
\begin{array}{r|l|l}
3x+2 & 15x^2+x-6 & 5x-3 \\
 & 15x^2+10x & \\
 & - \quad - & \\
\hline
 & -9x-6 & \\
 & -9x-6 & \\
 & + \quad + & \\
\hline
 & 0 &
\end{array}
$$

Thus, we get quotient = $5x - 3$ and remainder = 0

Question 7: $14x^2 - 53x + 45$ **by** $7x - 9$

Solution

$$
\begin{array}{r|l|l}
7x-9 & 14x^2-53x+45 & 2x-5 \\
 & 14x^2-18x & \\
 & - \quad + & \\
\hline
 & -35x+45 & \\
 & -35x+45 &
\end{array}
$$

$+ \quad -$

Thus, we get quotient = $2x - 5$ and remainder = 0

Question 8: $6x^2 - 31x + 47$ by $2x - 5$

Solution

$$2x - 5 \,)\, 6x^2 - 31x + 47 \,(\, 3x - 8$$

$$6x^2 - 15x$$

$$- \quad +$$

$$-16x + 47$$

$$-16x + 40$$

$$+ \quad -$$

$$7$$

Thus, we get quotient = $3x - 8$

Remainder = 7

Question 9: $2x^3 + x^2 - 5x - 2$ by $2x + 3$

Solution

$$2x + 3 \,)\, 2x^3 + x^2 - 5x - 2 \,(\, x^2 - x - 1$$

$$2x^3 + 3x^2$$

$$- \quad -$$

$$-2x^2 - 5x - 2$$

$$-2x^2 - 3x$$

$$+ \quad +$$

$$-2x - 2$$

$$-2x - 3$$

$$+ \quad +$$

$$1$$

Thus, we get quotient = $x^2 - x - 1$

Remainder = 1

Question 10: $x^3 + 1$ by $x + 1$

Solution

$x + 1$) $x^3 + 0x^2 + 0x + 1$ ($x^2 - x + 1$

$x^3 + x^2$

$- \quad -$

$-x^2 + 0x + 1$

$-x^2 - x$

$+ \quad +$

$x + 1$

$x + 1$

$+ \quad +$

0

Thus, we get quotient = $x^2 - x + 1$ and remainder = 0

Question 11: $x^4 - 2x^3 + 2x^2 + x + 4$ by $x^2 + x + 1$

Solution -

$x^2 + x + 1$) $x^4 - 2x^3 + 2x^2 + x + 4$ ($x^2 - 3x + 4$

$x^4 + x^3 + x^2$

$- \quad - \quad -$

$-3x^3 + x^2 + x + 4$

$-3x^3 - 3x^2 - 3x$

$+ \quad + \quad +$

$4x^2 + 4x + 4$

$4x^2 + 4x + 4$

$- \quad - \quad -$

0

Thus, we get quotient = $x^2 - 3x + 4$ and remainder = 0

Question 12: $x^3 - 6x^2 + 11x - 6$ by $x^2 - 5x + 6$

Solution -

$$
\begin{array}{r|l|l}
x^2 - 5x + 6 & x^3 - 6x^2 + 11x - 6 & x - 1 \\
 & x^3 - 5x^2 + 6x & \\
 & - \quad + \quad - & \\
\hline
 & -x^2 + 5x - 6 & \\
 & -x^2 + 5x - 6 & \\
 & + \quad - \quad + & \\
\hline
 & 0 &
\end{array}
$$

Thus, we get quotient = $x - 1$ and remainder = 0

Question 13: $5x^3 - 12x^2 + 12x + 13$ by $x^2 - 3x + 4$

Solution -

$$
\begin{array}{r|l|l}
x^2 - 3x + 4 & 5x^3 - 12x^2 + 12x + 13 & 5x + 3 \\
 & 5x^3 - 15x^2 + 20x & \\
 & - \quad + \quad - & \\
\hline
 & 3x^2 - 8x + 13 & \\
 & 3x^2 - 9x + 12 & \\
 & - \quad + \quad - & \\
\hline
 & x + 1 &
\end{array}
$$

We get quotient = $5x + 3$

Remainder = $x + 1$

Question 14: $2x^3 - 5x^2 + 8x - 5$ by $2x^2 - 3x + 5$

Solution -

$2x^2 - 3x + 5$) $2x^3 - 5x^2 + 8x - 5$ ($x - 1$

$2x^3 - 3x^2 + 5x$

$- \quad + \quad -$

$-2x^2 + 3x - 5$

$-2x^2 + 3x - 5$

$+ \quad - \quad +$

0

We get quotient = $x - 1$ and remainder = 0

Question 15: $8x^4 + 10x^3 - 5x^2 - 4x + 1$ by $2x^2 + x - 1$

Solution -

$2x^2 + x - 1$) $8x^4 + 10x^3 - 5x^2 - 4x + 1$ ($4x^2 + 3x - 2$

$8x^4 + 4x^3 - 4x^2$

$- \quad - \quad +$

$6x^3 - x^2 - 4x + 1$

$6x^3 + 3x^2 - 3x$

$- \quad - \quad +$

$-4x^2 - x + 1$

$-4x^2 - 2x + 2$

$+ \quad + \quad -$

$x - 1$

Thus, we get quotient = $4x^2 + 3x - 2$ and remainder = $x - 1$

Some special products

Identity1: $(a+b)^2 = a^2 + 2ab + b^2$

Identity2: $(a-b)^2 = a^2 - 2ab + b^2$

Identity3: $(a-b)(a+b) = a^2 - b^2$

Example 1 - Find each of the following products:

(a) $(\mathbf{3x+2y})(\mathbf{3x+2y})$

Solution - We can write $(3x+2y)(3x+2y) = (3x+2y)^2$

Since $(a+b)^2 = a^2 + 2ab + b^2$

=> $(3x+2y)^2 = (3x)^2 + 2(3x)(2y) + (2y)^2$

=> $9x^2 + 12xy + 4y^2$

(b) $(\mathbf{4x^2+5})(\mathbf{4x^2+5})$

Solution - We can write $(4x^2+5)(4x^2+5) = (4x^2+5)^2$

Since $(a+b)^2 = a^2 + 2ab + b^2$

=> $(4x^2+5)^2 = (4x^2)^2 + 2(4x^2)(5) + (5)^2$

=> $16x^4 + 40x^2 + 25$

Example 2 - Expand

(a) $(\mathbf{2x+5y})^{\mathbf{2}}$

Solution - Since $(a+b)^2 = a^2 + 2ab + b^2$

=> $(2x+5y)^2 = (2x)^2 + 2(2x)(5y) + (5y)^2$

=> $4x^2 + 20xy + 25y^2$

(b) $(\frac{\mathbf{2}}{\mathbf{3}}\boldsymbol{a} + \frac{\mathbf{3}}{\mathbf{4}}\boldsymbol{b})^{\mathbf{2}}$

Solution - Since $(a+b)^2 = a^2 + 2ab + b^2$

=> $(\frac{2}{3}a + \frac{3}{4}b)^2 = (\frac{2}{3}a)^2 + 2\left(\frac{2}{3}a\right)\left(\frac{3}{4}b\right) + (\frac{3}{4}b)^2$

$= \frac{4}{9}a^2 + ab + \frac{9}{16}b^2$

Example 3 - Find each of the following products:

(a) $(\mathbf{4x - 7y})(\mathbf{4x - 7y})$

Solution - We can write $(4x - 7y)(4x - 7y) = (4x - 7y)^2$

Since $(a - b)^2 = a^2 - 2ab + b^2$

=> $(4x - 7y)^2 = (4x)^2 - 2(4x)(7y) + (7y)^2$

$= 16x^2 - 56xy + 49y^2$

(b) $(\mathbf{3x^2 - 4y^2})(\mathbf{3x^2 - 4y^2})$

Solution - We can write $(3x^2 - 4y^2)(3x^2 - 4y^2) = (3x^2 - 4y^2)^2$

Since $(a - b)^2 = a^2 - 2ab + b^2$

=> $(3x^2 - 4y^2)^2 = (3x^2)^2 - 2(3x^2)(4y^2) + (4y^2)^2$

$= 9x^4 - 24x^2y^2 + 16y^4$

Example 4 - Expand:

(a) $(\mathbf{3x - 2y})^{\mathbf{2}}$

Solution - Since $(a - b)^2 = a^2 - 2ab + b^2$

=>$(3x - 2y)^2 = (3x)^2 - 2(3x)(2y) + (2y)^2$

$= 9x^2 - 12xy + 4y^2$

(b) $(\mathbf{\frac{3}{4}p - \frac{5}{6}q})^{\mathbf{2}}$

Solution - Since $(a - b)^2 = a^2 - 2ab + b^2$

=>$(\frac{3}{4}p - \frac{5}{6}q)^2 = (\frac{3}{4}p)^2 - 2\left(\frac{3}{4}p\right)\left(\frac{5}{6}q\right) + (\frac{5}{6}q)^2$

$= \frac{9}{16}p^2 - \frac{5}{4}pq + \frac{25}{36}q^2$

Example 5 - Find each of the following product:

(a) $(\mathbf{4x + 5y})(\mathbf{4x - 5y})$

Solution - Since $(a-b)(a+b)=a^2-b^2$

=> $(4x+5y)(4x-5y)=(4x)^2-(5y)^2$

= $16x^2-25y^2$

(b) $(3x^2+2y^2)(3x^2-2y^2)$

Solution - Since $(a-b)(a+b)=a^2-b^2$

=> $(3x^2+2y^2)(3x^2-2y^2)=(3x^2)^2-(2y^2)^2$

= $9x^4-4y^4$

Example 6 - Evaluate the following, using identities:

(a) $(105)^2$

Solution - Since $(a+b)^2=a^2+2ab+b^2$

Thus, $(105)^2=(100+5)^2=(100)^2+2(100)(5)+5^2$

= $10000+1000+25=11025$

(b) $(47)^2$

Solution - Since $(a-b)^2=a^2-2ab+b^2$

Thus, $(47)^2=(50-3)^2=(50)^2-2(50)(3)+3^2$

= $2500-300+9=2209$

(c) 8.3×7.7

Solution - 8.3 × 7.7 = $(8+0.3)(8-0.3)$

Since $(a-b)(a+b)=a^2-b^2$

=> $(8+0.3)(8-0.3)=(8)^2-(0.3)^2$

= $64-0.09=63.91$

Example 7 - Find the value of the expression$25x^2+9y^2+30xy$, when $x=8$ and $y=10$

Solution - Using $(a+b)^2=a^2+2ab+b^2$

$25x^2+9y^2+30xy=(5x)^2+(3y)^2+2(5x)(3y)$

=> $(5x+3y)^2$

Now putting x = 8 and y = 10, we get

$(5x + 3y)^2 = (5(8) + 3(10))^2$

= $(40 + 30)^2 = (70)^2 = 4900$

Example 8 - Find the value of the expression$(81x^2 + 16y^2 - 72xy)$, when $x = \frac{2}{3}$ and $y = \frac{3}{4}$

Solution - Using $(a - b)^2 = a^2 - 2ab + b^2$

$81x^2 + 16y^2 - 72xy = (9x)^2 + (4y)^2 - 2(9x)(4y)$

= $(9x - 4y)^2$

Now putting x = $\frac{2}{3}$ and y =$\frac{3}{4}$, we get

$(9x - 4y)^2 = (9\left(\frac{2}{3}\right) - 4\left(\frac{3}{4}\right))^2$

= $(6 - 3)^2 = 3^2 = 9$

Example 9 - If$x + \frac{1}{x} = 5$, find the values of:

- **(a)** $x^2 + \frac{1}{x^2}$
- **(b)** $x^4 + \frac{1}{x^4}$

Solution - Given that**:** $x + \frac{1}{x} = 5$

Squaring both sides of the equation, we get

$\left(x + \frac{1}{x}\right)^2 = 25$

=> $x^2 + \frac{1}{x^2} + 2(x)\left(\frac{1}{x}\right) = 25$

=> $x^2 + \frac{1}{x^2} + 2 = 25$

=> $x^2 + \frac{1}{x^2} = 25 - 2$

=> $x^2 + \frac{1}{x^2} = 23$

Now again squaring both sides of equation,

$(x^2 + \frac{1}{x^2})^2 = (23)^2$

=> $(x^2)^2 + \frac{1}{(x^2)^2} + 2(x^2)\left(\frac{1}{x^2}\right) = 529$

=> $x^4 + \frac{1}{x^4} + 2 = 529$

=>$x^4 + \frac{1}{x^4} = 529 - 2$

=>$x^4 + \frac{1}{x^4} = 527$

Exercise 6D

Question 1 - Find each of the following products:

(a) $(\boldsymbol{x} + \mathbf{6})(\boldsymbol{x} + \mathbf{6})$

Solution - We can write $(x + 6)(x + 6) = (x + 6)^2$

Since $(a + b)^2 = a^2 + 2ab + b^2$

=> $(x + 6)^2 = (x)^2 + 2(x)(6) + (6)^2$

=> $x^2 + 12x + 36$

(b) $(\mathbf{4}\boldsymbol{x} + \mathbf{5}\boldsymbol{y})(\mathbf{4}\boldsymbol{x} + \mathbf{5}\boldsymbol{y})$

Solution - We can write $(4x + 5y)(4x + 5y) = (4x + 5y)^2$

Since $(a + b)^2 = a^2 + 2ab + b^2$

=> $(4x + 5y)^2 = (4x)^2 + 2(4x)(5y) + (5y)^2$

=> $16x^2 + 40xy + 25y^2$

(c) $(\mathbf{7}\boldsymbol{a} + \mathbf{9}\boldsymbol{b})(\mathbf{7}\boldsymbol{a} + \mathbf{9}\boldsymbol{b})$

Solution - We can write $(7a + 9b)(7a + 9b) = (7a + 9b)^2$

Since $(a + b)^2 = a^2 + 2ab + b^2$

=> $(7a + 9b)^2 = (7a)^2 + 2(7a)(9b) + (9b)^2$

=> $49a^2 + 126ab + 81b^2$

(d) $\left(\frac{2}{3}x + \frac{4}{5}y\right)\left(\frac{2}{3}x + \frac{4}{5}y\right)$

Solution - We can write $\left(\frac{2}{3}x + \frac{4}{5}y\right)\left(\frac{2}{3}x + \frac{4}{5}y\right) = (\frac{2}{3}x + \frac{4}{5}y)^2$

Since $(a + b)^2 = a^2 + 2ab + b^2$

=> $(\frac{2}{3}x + \frac{4}{5}y)^2 = \left(\frac{2}{3}x\right)^2 + 2\left(\frac{2}{3}x\right)\left(\frac{4}{5}y\right) + \left(\frac{4}{5}y\right)^2$

=> $\frac{4}{9}x^2 + \frac{16}{15}xy + \frac{16}{25}y^2$

(e) $(x^2 + 7)(x^2 + 7)$

Solution - We can write $(x^2 + 7)(x^2 + 7) = (x^2 + 7)^2$

Since $(a + b)^2 = a^2 + 2ab + b^2$

=> $(x^2 + 7)^2 = (x^2)^2 + 2(x^2)(7) + (7)^2$

=> $x^4 + 14x^2 + 49$

(f) $(\frac{5}{6}a^2 + 2)(\frac{5}{6}a^2 + 2)$

Solution - We can write $\left(\frac{5}{6}a^2 + 2\right)\left(\frac{5}{6}a^2 + 2\right) = (\frac{5}{6}a^2 + 2)^2$

Since $(a + b)^2 = a^2 + 2ab + b^2$

=> $\left(\frac{5}{6}a^2 + 2\right)^2 = \left(\frac{5}{6}a^2\right)^2 + 2\left(\frac{5}{6}a^2\right)(2) + (2)^2$

=> $\frac{25}{36}a^4 + \frac{10}{3}a^2 + 4$

Question 2 - Find each of the following products:

(a) $(x - 4)(x - 4)$

Solution - We can write $(x - 4)(x - 4) = (x - 4)^2$

Since $(a - b)^2 = a^2 - 2ab + b^2$

=> $(x - 4)^2 = (x)^2 - 2(x)(4) + (4)^2$

= $x^2 - 8x + 16$

(b) $(\boldsymbol{2x-3y})(\boldsymbol{2x-3y})$

Solution - We can write $(2x-3y)(2x-3y)=(2x-3y)^2$

Since $(a-b)^2=a^2-2ab+b^2$

=> $(2x-3y)^2=(2x)^2-2(2x)(3y)+(3y)^2$

= $4x^2-12xy+9y^2$

(c) $(\boldsymbol{\frac{3}{4}x-\frac{5}{6}y})(\boldsymbol{\frac{3}{4}x-\frac{5}{6}y})$

Solution - We can write $\left(\frac{3}{4}x-\frac{5}{6}y\right)\left(\frac{3}{4}x-\frac{5}{6}y\right)=(\frac{3}{4}x-\frac{5}{6}y)^2$

Since $(a-b)^2=a^2-2ab+b^2$

=> $(\frac{3}{4}x-\frac{5}{6}y)^2=\left(\frac{3}{4}x\right)^2-2\left(\frac{3}{4}x\right)\left(\frac{5}{6}y\right)+\left(\frac{5}{6}y\right)^2$

=> $\frac{9}{16}x^2-\frac{5}{4}xy+\frac{25}{36}y^2$

(d) $(\boldsymbol{x-\frac{3}{x}})\ (\boldsymbol{x-\frac{3}{x}})$

Solution - We can write $\left(x-\frac{3}{x}\right)\left(x-\frac{3}{x}\right)=(x-\frac{3}{x})^2$

Since $(a-b)^2=a^2-2ab+b^2$

=> $(x-\frac{3}{x})^2=(x)^2-2(x)\left(\frac{3}{x}\right)+\left(\frac{3}{x}\right)^2$

=> $x^2-6+\frac{9}{x^2}$

(e) $(\boldsymbol{\frac{1}{3}x^2-9})(\boldsymbol{\frac{1}{3}x^2-9})$

Solution - We can write $\left(\frac{1}{3}x^2-9\right)\left(\frac{1}{3}x^2-9\right)=(\frac{1}{3}x^2-9)^2$

Since $(a-b)^2=a^2-2ab+b^2$

=> $(\frac{1}{3}x^2-9)^2=\left(\frac{1}{3}x^2\right)^2-2\left(\frac{1}{3}x^2\right)(9)+(9)^2$

=> $\frac{1}{9}x^4-6x^2+81$

(f) $(\frac{1}{2}y^2 - \frac{1}{3}y)(\frac{1}{2}y^2 - \frac{1}{3}y)$

Solution - We can write $(\frac{1}{2}y^2 - \frac{1}{3}y)(\frac{1}{2}y^2 - \frac{1}{3}y) = (\frac{1}{2}y^2 - \frac{1}{3}y)^2$

Since $(a-b)^2 = a^2 - 2ab + b^2$

=> $(\frac{1}{2}y^2 - \frac{1}{3}y)^2 = \left(\frac{1}{2}y^2\right)^2 - 2\left(\frac{1}{2}y^2\right)\left(\frac{1}{3}y\right) + \left(\frac{1}{3}y\right)^2$

=> $\frac{1}{4}y^4 - \frac{1}{3}y^3 + \frac{1}{9}y^2$

Question 3 - Expand:

(a) $(8a + 3b)^2$

Solution - Since $(a+b)^2 = a^2 + 2ab + b^2$

=> $(8a+3b)^2 = (8a)^2 + 2(8a)(3b) + (3b)^2$

=> $64a^2 + 48ab + 9b^2$

(b) $(7x + 2y)^2$

Solution - Since $(a+b)^2 = a^2 + 2ab + b^2$

=> $(7x+2y)^2 = (7x)^2 + 2(7x)(2y) + (2y)^2$

=> $49x^2 + 28xy + 4y^2$

(c) $(5x + 11)^2$

Solution - Since $(a+b)^2 = a^2 + 2ab + b^2$

=> $(5x+11)^2 = (5x)^2 + 2(5x)(11) + (11)^2$

=> $25x^2 + 110x + 121$

(d) $(\frac{a}{2} + \frac{2}{a})^2$

Solution - Since $(a+b)^2 = a^2 + 2ab + b^2$

=> $(\frac{a}{2} + \frac{2}{a})^2 = (\frac{a}{2})^2 + 2\left(\frac{a}{2}\right)\left(\frac{2}{a}\right) + (\frac{2}{a})^2$

= $\frac{a^2}{4} + 2 + \frac{4}{a^2}$

(e) $(\frac{3}{4}x + \frac{2}{9}y)^2$

Solution - Since $(a+b)^2 = a^2 + 2ab + b^2$

=> $(\frac{3}{4}x + \frac{2}{9}y)^2 = (\frac{3}{4}x)^2 + 2\left(\frac{3}{4}x\right)\left(\frac{2}{9}y\right) + (\frac{2}{9}y)^2$

= $\frac{9}{16}x^2 + \frac{xy}{3} + \frac{4}{81}y^2$

(f) $(9x - 10)^2$

Solution - Since $(a-b)^2 = a^2 - 2ab + b^2$

=>$(9x-10)^2 = (9x)^2 - 2(9x)(10) + (10)^2$

= $81x^2 - 180x + 100$

(g) $(x^2y - yz^2)^2$

Solution - Since $(a-b)^2 = a^2 - 2ab + b^2$

=>$(x^2y - yz^2)^2 = (x^2y)^2 - 2(x^2y)(yz^2) + (yz^2)^2$

= $x^4y^2 - 2x^2y^2z^2 + z^4y^2$

(h) $(\frac{x}{y} - \frac{y}{x})^2$

Solution - Since $(a-b)^2 = a^2 - 2ab + b^2$

=> $(\frac{x}{y} - \frac{y}{x})^2 = (\frac{x}{y})^2 - 2\left(\frac{x}{y}\right)\left(\frac{y}{x}\right) + (\frac{y}{x})^2$

= $\frac{x^2}{y^2} - 2 + \frac{y^2}{x^2}$

(g) $\left(3m - \frac{4}{5}n\right)^2$

Solution - Since $(a-b)^2 = a^2 - 2ab + b^2$

=> $(3m - \frac{4}{5}n)^2 = (3m)^2 - 2(3m)\left(\frac{4}{5}n\right) + (\frac{4}{5}n)^2$

= $9m^2 - \frac{24}{5}mn + \frac{16}{25}n^2$

Question 4 - Find each of the following products:

(a) $(\boldsymbol{x}+\mathbf{3})(\boldsymbol{x}-\mathbf{3})$

Solution - Since $(a-b)(a+b)=a^2-b^2$

=> $(x+3)(x-3)=(x)^2-(3)^2$

= x^2-9

(b) $(\mathbf{2}\boldsymbol{x}+\mathbf{5})(\mathbf{2}\boldsymbol{x}-\mathbf{5})$

Solution - Since $(a-b)(a+b)=a^2-b^2$

=> $(2x+5)(2x-5)=(2x)^2-(5)^2$

= $4x^2-25$

(c) $(\mathbf{8}+\boldsymbol{x})(\mathbf{8}-\boldsymbol{x})$

Solution - Since $(a-b)(a+b)=a^2-b^2$

=> $(8-x)(8+x)=(8)^2-(x)^2$

= $64-x^2$

(d) $(\mathbf{7}\boldsymbol{x}+\mathbf{11}\boldsymbol{y})(\mathbf{7}\boldsymbol{x}-\mathbf{11}\boldsymbol{y})$

Solution - Since $(a-b)(a+b)=a^2-b^2$

=> $(7x+11y)(7x-11y)=(7x)^2-(11y)^2$

= $49x^2-121y^2$

(e) $(\mathbf{5}\boldsymbol{x}^{\mathbf{2}}+\frac{\mathbf{3}}{\mathbf{4}}\boldsymbol{y}^{\mathbf{2}})(\mathbf{5}\boldsymbol{x}^{\mathbf{2}}-\frac{\mathbf{3}}{\mathbf{4}}\boldsymbol{y}^{\mathbf{2}})$

Solution - Since $(a-b)(a+b)=a^2-b^2$

=> $\left(5x^2+\frac{3}{4}y^2\right)(5x^2-\frac{3}{4}y^2)=(5x^2)^2-\left(\frac{3}{4}y^2\right)^2$

= $25x^4-\frac{9}{16}y^4$

(f) $(\frac{\mathbf{4}\boldsymbol{x}}{\mathbf{5}}-\frac{\mathbf{5}\boldsymbol{y}}{\mathbf{3}})(\frac{\mathbf{4}\boldsymbol{x}}{\mathbf{5}}+\frac{\mathbf{5}\boldsymbol{y}}{\mathbf{3}})$

Solution - Since $(a-b)(a+b)=a^2-b^2$

=> $(\frac{4x}{5} - \frac{5y}{3})(\frac{4x}{5} + \frac{5y}{3}) = \left(\frac{4x}{5}\right)^2 - \left(\frac{5y}{3}\right)^2$

= $\frac{16}{25}x^2 - \frac{25}{9}y^2$

(g) $(\boldsymbol{x} + \frac{\mathbf{1}}{\boldsymbol{x}})(\boldsymbol{x} - \frac{\mathbf{1}}{\boldsymbol{x}})$

Solution - Since $(a - b)(a + b) = a^2 - b^2$

=>$(x + \frac{1}{x})(x - \frac{1}{x}) = (x)^2 - (\frac{1}{x})^2$

= $x^2 - \frac{1}{x^2}$

(h) $(\frac{\mathbf{1}}{\boldsymbol{x}} + \frac{\mathbf{1}}{\boldsymbol{y}})(\frac{\mathbf{1}}{\boldsymbol{x}} - \frac{\mathbf{1}}{\boldsymbol{y}})$

Solution - Since $(a - b)(a + b) = a^2 - b^2$

=>$(\frac{1}{x} + \frac{1}{y})(\frac{1}{x} - \frac{1}{y}) = \left(\frac{1}{x}\right)^2 - (\frac{1}{y})^2$

= $\frac{1}{x^2} - \frac{1}{y^2}$

(i) $(\mathbf{2}\boldsymbol{a} + \frac{\mathbf{3}}{\boldsymbol{b}})(\mathbf{2}\boldsymbol{a} - \frac{\mathbf{3}}{\boldsymbol{b}})$

Solution - Since $(a - b)(a + b) = a^2 - b^2$

=>$(2a + \frac{3}{b})(2a - \frac{3}{b}) = (2a)^2 - (\frac{3}{b})^2$

= $4a^2 - \frac{9}{b^2}$

Question 5 - Using the formula for squaring a binomial, evaluate the following:

(a) $(\mathbf{54})^2$

Solution - Since $(a + b)^2 = a^2 + 2ab + b^2$

Thus, $(54)^2 = (50 + 4)^2 = (50)^2 + 2(50)(4) + 4^2$

= $2500 + 400 + 16 = 2916$

(b) $(\mathbf{82})^2$

Solution **-l** Since $(a + b)^2 = a^2 + 2ab + b^2$

Thus, $(82)^2 = (80 + 2)^2 = (80)^2 + 2(80)(2) + 2^2$

$= 6400 + 320 + 4 = 6724$

(c) $(\mathbf{103})^{\mathbf{2}}$

Solution - Since $(a+b)^2 = a^2 + 2ab + b^2$

Thus, $(103)^2 = (100+3)^2 = (100)^2 + 2(100)(3) + 3^2$

$= 10000 + 600 + 9 = 10609$

(d) $(\mathbf{704})^{\mathbf{2}}$

Solution - Since $(a+b)^2 = a^2 + 2ab + b^2$

Thus, $(704)^2 = (700+4)^2 = (700)^2 + 2(700)(4) + 4^2$

$= 490000 + 5600 + 16 = 495616$

Question 6 - Using the formula for squaring a binomial, evaluate the following:

(a) $(\mathbf{69})^{\mathbf{2}}$

Solution - Since $(a-b)^2 = a^2 - 2ab + b^2$

Thus, $(69)^2 = (70-1)^2 = (70)^2 - 2(70)(1) + 1^2$

$= 4900 - 140 + 1 = 4761$

(b) $(\mathbf{78})^{\mathbf{2}}$

Solution - Since $(a-b)^2 = a^2 - 2ab + b^2$

Thus, $(78)^2 = (80-2)^2 = (80)^2 - 2(80)(2) + 2^2$

$= 6400 - 320 + 4 = 6084$

(c) $(\mathbf{197})^{\mathbf{2}}$

Solution - Since $(a-b)^2 = a^2 - 2ab + b^2$

Thus, $(197)^2 = (200-3)^2 = (200)^2 - 2(200)(3) + 3^2$

$= 40000 - 1200 + 9 = 38809$

(d) $(\mathbf{999})^{\mathbf{2}}$

Solution - Since $(a-b)^2 = a^2 - 2ab + b^2$

Thus, $(999)^2 = (1000-1)^2 = (1000)^2 - 2(1000)(1) + 1^2$

$= 1000000 - 2000 + 1 = 998001$

Question 7 - Find the value of:

(a) $(\mathbf{82})^{\mathbf{2}} - (\mathbf{18})^{\mathbf{2}}$

Solution - Using identity $(a - b)(a + b) = a^2 - b^2$

=> $(82)^2 - (18)^2 = (82 + 18)(82 - 18) = 100 \times 64 = 6400$

(b) $(\mathbf{128})^{\mathbf{2}} - (\mathbf{72})^{\mathbf{2}}$

Solution - Using identity $(a - b)(a + b) = a^2 - b^2$

=> $(128)^2 - (72)^2 = (128 + 72)(128 - 72) = 200 \times 56 = 11200$

(c) $\mathbf{197} \times \mathbf{203}$

Solution - $197 \times 203 = (200 - 3)(200 + 3)$

Since $(a - b)(a + b) = a^2 - b^2$

=> $(200 - 3)(200 + 3) = (200)^2 - (3)^2$

$= 40000 - 9 = 39991$

(d) $\frac{\mathbf{198} \times \mathbf{198} - \mathbf{102} \times \mathbf{102}}{\mathbf{96}}$

Solution - We can write it as follows:

$$\frac{(198)^2 - (102)^2}{96}$$

Now using $(a - b)(a + b) = a^2 - b^2$

=> $\frac{(198)^2 - (102)^2}{96} = \frac{(198+102)(198-102)}{96}$

$= \frac{300 \times 96}{96} = 300$

(e) $(\mathbf{14.7} \times \mathbf{15.3})$

Solution**:** $14.7 \times 15.3 = (15 - 0.3)(15 + 0.3)$

Since $(a - b)(a + b) = a^2 - b^2$

=> $(15 - 0.3)(15 + 0.3) = (15)^2 - (0.3)^2$

$= 225 - 0.09 = 224.91$

(f) $(8.63)^2 - (1.37)^2$

Solution - Since $(a-b)(a+b) = a^2 - b^2$

=> $(8.63)^2 - (1.37)^2 = (8.63 + 1.37)(8.63 - 1.37)$

$= 10 \times 7.26 = 72.6$

Question 8 - Find the value of the expression$(9x^2 + 24x + 16)$, when $x = 12$

Solution - Using $(a+b)^2 = a^2 + 2ab + b^2$

$9x^2 + 24x + 16 = (3x)^2 + (4)^2 + 2(3x)(4)$

=> $(3x+4)^2$

Now putting x = 12, we get

$(3x+4)^2 = (3(12)+4)^2$

$= (36+4)^2 = (40)^2 = 1600$

Question 9 - Find the value of the expression$(64x^2 + 81y^2 + 144xy)$, when $x = 11$ and $y = \frac{4}{3}$

Solution - Using $(a+b)^2 = a^2 + 2ab + b^2$

$64x^2 + 81y^2 + 144xy = (8x)^2 + (9y)^2 + 2(8x)(9y)$

=> $(8x+9y)^2$

Now putting x = 11 and$y = \frac{4}{3}$, we get

$(8x+9y)^2 = (8(11) + 9\left(\frac{4}{3}\right))^2$

$= (88+12)^2 = (100)^2 = 10000$

Question 10 - Find the value of the expression$36x^2 + 25y^2 - 60xy)$, when $x = \frac{2}{3}$ and$y = \frac{1}{5}$.

Solution - Using $(a-b)^2 = a^2 - 2ab + b^2$

$36x^2 + 25y^2 - 60xy = (6x)^2 + (5y)^2 - 2(6x)(5y)$

=> $(6x-5y)^2$

Now putting x =$\frac{2}{3}$ and$y = \frac{1}{5}$, we get

$(6x - 5y)^2 = (6\left(\frac{2}{3}\right) - 5\left(\frac{1}{5}\right))^2$

= $(4 - 1)^2 = (3)^2 = 9$

Question 11 - If$x + \frac{1}{x} = 4$, find the values of:

- **(a)** $x^2 + \frac{1}{x^2}$
- **(b)** $x^4 + \frac{1}{x^4}$

Solution -Given that**:** $x + \frac{1}{x} = 4$

Squaring both sides of the equation, we get

$\left(x + \frac{1}{x}\right)^2 = 16$

=> $x^2 + \frac{1}{x^2} + 2(x)\left(\frac{1}{x}\right) = 16$

=> $x^2 + \frac{1}{x^2} + 2 = 16$

=> $x^2 + \frac{1}{x^2} = 16 - 2$

=> $x^2 + \frac{1}{x^2} = 14$

Now again squaring both sides of equation,

$(x^2 + \frac{1}{x^2})^2 = (14)^2$

=> $(x^2)^2 + \frac{1}{(x^2)^2} + 2(x^2)\left(\frac{1}{x^2}\right) = 196$

=> $x^4 + \frac{1}{x^4} + 2 = 196$

=>$x^4 + \frac{1}{x^4} = 196 - 2$

=>$x^4 + \frac{1}{x^4} = 194$

Question 12 - If $x - \frac{1}{x} = 5$, find the values of:

(a) $x^2 + \frac{1}{x^2}$
(b) $x^4 + \frac{1}{x^4}$

Solution - Given that: $x - \frac{1}{x} = 5$

Squaring both sides of the equation, we get

$\left(x - \frac{1}{x}\right)^2 = 25$

=> $x^2 + \frac{1}{x^2} - 2(x)\left(\frac{1}{x}\right) = 25$

=> $x^2 + \frac{1}{x^2} - 2 = 25$

=> $x^2 + \frac{1}{x^2} = 25 + 2$

=> $x^2 + \frac{1}{x^2} = 27$

Now again squaring both sides of equation,

$(x^2 + \frac{1}{x^2})^2 = (27)^2$

=> $(x^2)^2 + \frac{1}{(x^2)^2} + 2(x^2)\left(\frac{1}{x^2}\right) = 729$

=> $x^4 + \frac{1}{x^4} + 2 = 729$

=>$x^4 + \frac{1}{x^4} = 729 - 2$

=>$x^4 + \frac{1}{x^4} = 727$

Question 13 - Find the continued product:

(a) $(x + 1)(x - 1)(x^2 + 1)$

Solution - Using identity: $(a - b)(a + b) = a^2 - b^2$

=> $(x + 1)(x - 1)(x^2 + 1) = (x^2 - 1)(x^2 + 1)$

= $(x^2)^2 - 1 = x^4 - 1$

(b) $(\boldsymbol{x}-\mathbf{3})(\boldsymbol{x}+\mathbf{3})(\boldsymbol{x}^{\mathbf{2}}+\mathbf{9})$

Solution - Using identity: $(a-b)(a+b)=a^2-b^2$

=> $(x-3)(x+3)(x^2+9)=(x^2-3^2)(x^2+9)$

=$(x^2-9)(x^2+9)=(x^2)^2-9^2=x^4-81$

(c) $(\mathbf{3}\boldsymbol{x}-\mathbf{2}\boldsymbol{y})(\mathbf{3}\boldsymbol{x}+\mathbf{2}\boldsymbol{y})(\mathbf{9}\boldsymbol{x}^{\mathbf{2}}+\mathbf{4}\boldsymbol{y}^{\mathbf{2}})$

Solution - Using identity: $(a-b)(a+b)=a^2-b^2$

=> $(3x-2y)(3x+2y)(9x^2+4y^2)=((3x)^2-(2y)^2)(9x^2+4y^2)$

=$(9x^2-4y^2)(9x^2+4y^2)=(9x^2)^2-(4y^2)^2=81x^4-16y^4$

(d) $(\mathbf{2}\boldsymbol{p}+\mathbf{3})(\mathbf{2}\boldsymbol{p}-\mathbf{3})(\mathbf{4}\boldsymbol{p}^{\mathbf{2}}+\mathbf{9})$

Solution - Using identity: $(a-b)(a+b)=a^2-b^2$

=> $(2p+3)(2p-3)(4p^2+9)=((2p)^2-3^2)(4p^2+9)$

= $(4p^2-9)(4p^2+9)=(4p^2)^2-9^2=16p^4-81$

Question 14 - If $\boldsymbol{x}+\boldsymbol{y}=\mathbf{12}$ and$\boldsymbol{xy}=\mathbf{14}$, find the value of $(\boldsymbol{x}^{\mathbf{2}}+\boldsymbol{y}^{\mathbf{2}})$

Solution - Since $(x+y)^2=x^2+2xy+y^2$

Now putting the given values, we get

=> $(12)^2=x^2+y^2+2(14)$

=> $144=x^2+y^2+28$

=> $x^2+y^2=144-28=116$

Question 15 - If $\boldsymbol{x}-\boldsymbol{y}=\mathbf{7}$ and$\boldsymbol{xy}=\mathbf{9}$, find the value of $(\boldsymbol{x}^{\mathbf{2}}+\boldsymbol{y}^{\mathbf{2}})$

Solution - Since $(x-y)^2=x^2-2xy+y^2$

Now putting the given values, we get

=> $(7)^2=x^2+y^2-2(9)$

=> $49=x^2+y^2-18$

=> $x^2+y^2=49+18=67$

Exercise 6E

Question1- The sum of
$(\boldsymbol{6a+4b-c+3}), (\boldsymbol{2b-3c+4}), (\boldsymbol{11b-7a+2c-1}), \boldsymbol{and}\ (\boldsymbol{2c-5a-6})$ **is:**

Solution -

$$6a + 4b - c + 3$$

$$0a + 2b - 3c + 4$$

$$-7a + 11b + 2c - 1$$

$$-5a + 0b + 2c - 6$$

$$-6a + 17b$$

Question 2: $(\boldsymbol{3q + 7p^2 - 2r^3 + 4}) - (\boldsymbol{4p^2 - 2q + 7r^3 - 3})$ =?

Solution -

$$7p^2 + 3q - 2r^3 + 4$$

$$4p^2 - 2q + 7r^3 - 3$$

$$- \quad + \quad - \quad +$$

$$3p^2 + 5q - 9r^3 + 7$$

Question 3: $(\boldsymbol{x + 5})(\boldsymbol{x - 3})$ =?

Solution - $(x + 5)(x - 3) = x(x - 3) + 5(x - 3)$

= $x^2 - 3x + 5x - 15 = x^2 + 2x - 15$

Question 4: $(\boldsymbol{2x + 3})(\boldsymbol{3x - 1})$ =?

Solution - $(2x + 3)(3x - 1) = 2x(3x - 1) + 3(3x - 1)$

=6 $x^2 - 2x + 9x - 3 = 6x^2 + 7x - 3$

Question 5: $(\boldsymbol{x + 4})(\boldsymbol{x + 4})$ =?

Solution - Using identity: $(x + y)^2 = x^2 + 2xy + y^2$

$(x + 4)(x + 4) = (x + 4)^2 = x^2 + 2(x)(4) + 4^2$

= $x^2 + 8x + 16$

Question 6: $(\boldsymbol{x}-\mathbf{6})(\boldsymbol{x}-\mathbf{6})$ =?

Solution - Using identity: $(x-y)^2 = x^2 - 2xy + y^2$

$(x-6)(x-6) = (x-6)^2 = x^2 - 2(x)(6) + 6^2$

= $x^2 - 12x + 36$

Question 7: $(\mathbf{2}\boldsymbol{x}+\mathbf{5})(\mathbf{2}\boldsymbol{x}-\mathbf{5})$ =?

Solution -Using identity: $(x-y)(x+y) = x^2 - y^2$

$(2x+5)(2x-5) = (2x)^2 - 5^2 = 4x^2 - 25$

Question 8: $\mathbf{8}\boldsymbol{a}^{\mathbf{2}}\boldsymbol{b}^{\mathbf{3}} \div (-\mathbf{2}\boldsymbol{ab})$ =?

Solution - $\frac{8a^2b^3}{-2ab} = \left(\frac{8}{-2}\right)a^{2-1}b^{3-1} = -4ab^2$

Question 9: $(\mathbf{2}\boldsymbol{x}^{\mathbf{2}}+\mathbf{3}\boldsymbol{x}+\mathbf{1}) \div (\boldsymbol{x}+\mathbf{1})$ =?

Solution - $x+1$) $2x^2 + 3x + 1$ (

$2x^2 + 2x$

$-\quad -$

$x+1$

$x+1$

$-\quad -$

0

Thus, we get quotient = $2x + 1$

Question 10: $(\boldsymbol{x^2-4x+4}) \div (\boldsymbol{x-2})$ =?

Solution - $x-2$) x^2-4x+4 ($x-2$

x^2-2x

$-\quad +$

$-2x+4$

$-2x+4$

$+\quad -$

0

Thus, we get quotient = $x-2$

Question 11: $(\boldsymbol{a+1})(\boldsymbol{a-1})(\boldsymbol{a^2+1})$ =?

Solution - Using identity: $(a-b)(a+b)=a^2-b^2$

=> $(a+1)(a-1)(a^2+1)=(a^2-1)(a^2+1)$

= $(a^2)^2-1=a^4-1$

Question 12: $(\frac{\boldsymbol{1}}{\boldsymbol{x}}+\frac{\boldsymbol{1}}{\boldsymbol{y}})(\frac{\boldsymbol{1}}{\boldsymbol{x}}-\frac{\boldsymbol{1}}{\boldsymbol{y}})$

Solution - Using identity: $(a-b)(a+b)=a^2-b^2$

=> $\left(\frac{1}{x}+\frac{1}{y}\right)\left(\frac{1}{x}-\frac{1}{y}\right)=(\frac{1}{x})^2-\left(\frac{1}{y}\right)^2=\frac{1}{x^2}-\frac{1}{y^2}$

Question 13: If $\boldsymbol{x+\frac{1}{x}=5}$**, then** $\boldsymbol{x^2+\frac{1}{x^2}}$ =?

Solution - Given that**:** $x+\frac{1}{x}=5$

Squaring both sides of the equation, we get

$$\left(x+\frac{1}{x}\right)^2=25$$

=> $x^2+\frac{1}{x^2}+2(x)\left(\frac{1}{x}\right)=25$

=> $x^2+\frac{1}{x^2}+2=25$

=> $x^2 + \frac{1}{x^2} = 25 - 2$

=> $x^2 + \frac{1}{x^2} = 23$

Question 14: If $x - \frac{1}{x} = 6$, then $x^2 + \frac{1}{x^2}$ =?

Solution -Given that**:** $x - \frac{1}{x} = 6$

Squaring both sides of the equation, we get

$\left(x - \frac{1}{x}\right)^2 = 6^2$

=> $x^2 + \frac{1}{x^2} - 2(x)\left(\frac{1}{x}\right) = 36$

=> $x^2 + \frac{1}{x^2} - 2 = 36$

=> $x^2 + \frac{1}{x^2} = 36 + 2$

=> $x^2 + \frac{1}{x^2} = 38$

Question 15: $(82)^2 - (18)^2$ =?

Solution - Using identity $(a - b)(a + b) = a^2 - b^2$

=> $(82)^2 - (18)^2 = (82 + 18)(82 - 18) = 100 \times 64 = 6400$

Question 16: (197×203) =?

Solution - $197 \times 203 = (200 - 3)(200 + 3)$

Since $(a - b)(a + b) = a^2 - b^2$

=> $(200 - 3)(200 + 3) = (200)^2 - (3)^2$

= $40000 - 9 = 39991$

Question 17: If $(a + b) = 12$ and $ab = 14$, then $a^2 + b^2$ =?

Solution - Since $(a + b)^2 = a^2 + 2ab + b^2$

Now putting the given values, we get

=> $(12)^2 = a^2 + b^2 + 2(14)$

=> $144 = a^2 + b^2 + 28$

=> $a^2 + b^2 = 144 - 28 = 116$

Question 18 - If $a - b = 7$ and $ab = 9$, find the value of $(a^2 + b^2)$

Solution - Since $(a - b)^2 = a^2 - 2ab + b^2$

Now putting the given values, we get

=> $(7)^2 = a^2 + b^2 - 2(9)$

=> $49 = a^2 + b^2 - 18$

=> $a^2 + b^2 = 49 + 18 = 67$

Question 19 - If $x = 10$, then the value of $(4x^2 + 20x + 25)$ =?

Solution - Since $(4x^2 + 20x + 25) = ((2x)^2 + 2(2x)(5) + 5^2)$

Now using identity: $(a + b)^2 = a^2 + 2ab + b^2$

$(4x^2 + 20x + 25) = (2x + 5)^2$

Now put $x = 10$, we get

= $(2(10) + 5)^2 = (20 + 5)^2 = (25)^2 = 625$