Introduction

Equation: It means equality of two algebraic expressions involving one or more variables.

Linear equation: When an equation involves only one linear polynomial, it is linear equation.

There are some rules while solving linear equations:

(a) We can add same number on both sides of the equation.

(b) We can subtract the same number from both sides of the equation.

(c) We can multiply both sides of equation by the same nonzero number.

(d) We can divide both sides of the equation by the same nonzero number.

This is known as cross multiplication.

Examples:

Example 1 – Solve: $8x = 20 + 3x$

Solution: $8x = 20 + 3x$

$\Rightarrow 8x - 3x = 20$

$\Rightarrow x = \frac{20}{5} = 4$

Example 2 – Solve: $\frac{2}{3}x + 1 = \frac{7}{3}$

Solution: $\frac{2}{3}x + 1 = \frac{7}{3}$

$\Rightarrow \frac{2}{3}x = \frac{7}{3} - 1$

$\Rightarrow \frac{2}{3}x = \frac{7-3}{3} = \frac{4}{3}$

$\Rightarrow x = \frac{4}{3} \times \frac{3}{2}$

$\Rightarrow x = 2$

Example 3 - Solve: $\frac{1}{4}x + \frac{1}{6}x = x - 7$

Solution: $\frac{1}{4}x + \frac{1}{6}x = x - 7$

We multiply both sides of equation by LCM of (4, 6) which is 12

=> $3x + 2x = 12x - 84$

=> $12x - 5x = 84$

=>$x = \frac{84}{7} = 12$

Example 4 - Solve: $\frac{y+6}{4} + \frac{y-3}{5} = \frac{5y-4}{8}$

Solution: We multiply both sides of equation by LCM of (4, 5, 8) which is 40

=> $10(y + 6) + 8(y - 3) = 5(5y - 4)$

=> $10y + 60 + 8y - 24 = 25y - 20$

=> $10y + 8y - 25y = -20 - 60 + 24$

=> $-7y = -56$

=> $y = \frac{-56}{-7} = 8$

Example 5 – Solve: $\frac{3x+5}{2x+1} = \frac{1}{3}$

Solution: By cross multiplication, we get

=> $3(3x + 5) = 2x + 1$

=> $9x + 15 = 2x + 1$

=> $9x - 2x = 1 - 15$

=> $7x = -14$

=> $x = \frac{-14}{7} = -2$

Example 6- Solve: $\frac{6x+7}{3x+2} = \frac{4x+5}{2x+3}$

Solution: By cross multiplication, we get

=> $(6x + 7)(2x + 3) = (3x + 2)(4x + 5)$

=> $6x(2x + 3) + 7(2x + 3) = 3x(4x + 5) + 2(4x + 5)$

=> $12x^2 + 18x + 14x + 21 = 12x^2 + 15x + 8x + 10$

=> $12x^2 - 12x^2 + 18x + 14x - 15x - 8x = 10 - 21$

=> $9x = -11$

=> $x = \frac{-11}{9}$

Exercise 8A

Solve:

Question 1: $8x + 3 = 27 + 2x$

Solution: $8x - 2x = 27 - 3$

=> $6x = 24$

=> $x = \frac{24}{6}$

=> $x = 4$

Question 2: $5x + 7 = 2x - 8$

Solution: $5x - 2x = -8 - 7$

=> $3x = -15$

=> $x = \frac{-15}{3}$

=> $x = -5$

Question 3: $2z - 1 = 14 - z$

Solution: $2z + z = 14 + 1$

=> $3z = 15$

=> $z = \frac{15}{3}$

=> $z = 5$

Question 4: $9x + 5 = 4(x - 2) + 8$

Solution: $9x + 5 = 4x - 8 + 8$

=> $9x - 4x = -5$

=> $5x = -5$

=> $x = -1$

Question 5: $\frac{7y}{5} = y - 4$

Solution: By cross multiplication, we get

=> $7y = 5(y - 4)$

=> $7y = 5y - 20$

=> $7y - 5y = -20$

=> $2y = -20$

=> $y = -\frac{20}{2}$

=> $y = -10$

Question 6: $3x + \frac{2}{3} = 2x + 1$

Solution: Multiply both sides of equation by LCM of (1, 3) which is 3, we get

=> $3(3x) + 2 = 3(2x) + 3$

=> $9x + 2 = 6x + 3$

=> $9x - 6x = 3 - 2$

=> $3x = 1$

=> $x = \frac{1}{3}$

Question 7: $15(y - 4) - 2(y - 9) + 5(y + 6) = 0$

Solution: $15y - 60 - 2y + 18 + 5y + 30 = 0$

=> $15y - 2y + 5y = 60 - 18 - 30$

=> $18y = 12$

=> $y = \frac{12}{18}$

=> $y = \frac{2}{3}$

Question 8: $3(5x - 7) - 2(9x - 11) = 4(8x - 13) - 17$

Solution: $15x - 21 - 18x + 22 = 32x - 52 - 17$

=> $15x - 18x - 32x = -52 - 17 + 21 - 22$

=> $-35x = -70$

=> $x = \frac{-70}{-35}$

=> $x = 2$

Question 9: $\frac{x-5}{2} - \frac{x-3}{5} = \frac{1}{2}$

Solution: Multiply both sides of equation by LCM of (2, 5) which is 10

=> $5(x - 5) - 2(x - 3) = 5$

=> $5x - 25 - 2x + 6 = 5$

=> $5x - 2x = 5 + 25 - 6$

=> $3x = 24$

=> $x = \frac{24}{3}$

=> $x = 8$

Question 10: $\frac{3t-2}{4} - \frac{2t+3}{3} = \frac{2}{3} - t$

Solution: Multiply both sides of equation by LCM of (4, 3) which is 12

=> $3(3t - 2) - 4(2t + 3) = 4(2) - 12t$

=> $9t - 6 - 8t - 12 = 8 - 12t$

=> $9t - 8t + 12t = 8 + 6 + 12$

=> $13t = 26$

=> $t = \frac{26}{13}$

=> $t = 2$

Question 11: $\frac{2x+7}{5} - \frac{3x+11}{2} = \frac{2x+8}{3} - 5$

Solution: Multiply both sides of equation by LCM of (5, 2, 3) which is 30

=> $6(2x + 7) - 15(3x + 11) = 10(2x + 8) - 150$

=> $12x + 42 - 45x - 165 = 20x + 80 - 150$

=> $12x - 45x - 20x = 80 - 150 + 165 - 42$

=> $-53x = 53$

=> $x = -1$

Question 12: $\frac{5x-4}{6} = 4x + 1 - \frac{3x+10}{2}$

Solution: Multiply both sides of equation by LCM of (6, 2) which is 6

=> $5x - 4 = 6(4x) + 6 - 3(3x + 10)$

=> $5x - 4 = 24x + 6 - 9x - 30$

=> $5x - 24x + 9x = -30 + 6 + 4$

=> $-10x = -20$

=> $x = \frac{-20}{-10}$

=> $x = 2$

Question 13: $5x - \frac{1}{3}(x + 1) = 6\left(x + \frac{1}{30}\right)$

Solution: Multiply both sides of equation by LCM of (3, 30) which is 30

=> $30(5x) - 10(x + 1) = 180\left(x + \frac{1}{30}\right)$

=> $150x - 10x - 10 = 180x + 6$

=> $150x - 10x - 180x = 6 + 10$

=> $-40x = 16$

=> $x = \frac{16}{-40}$

=> $x = \frac{2}{-5}$

=> $x = -\frac{2}{5}$

Question 14: $\mathbf{4 - \frac{2(z-4)}{3} = \frac{1}{2}(2z + 5)}$

Solution: Multiply both sides of equation by LCM of (3, 2) which is 6

=> $6(4) - 4(z - 4) = 3(2z + 5)$

=> $24 - 4z + 16 = 6z + 15$

=> $-4z - 6z = 15 - 16 - 24$

=> $-10z = -25$

=> $z = \frac{-25}{-10}$

=> $z = \frac{5}{2}$

Question 15: $\mathbf{\frac{3(y-5)}{4} - 4y = 3 - \frac{(y-3)}{2}}$

Solution**:** Multiply both sides of equation by LCM of (4, 2) which is 4

=> $3(y - 5) - 4(4y) = 4(3) - 2(y - 3)$

=> $3y - 15 - 16y = 12 - 2y + 6$

=> $3y - 16y + 2y = 12 + 6 + 15$

=> $-11y = 33$

=> $y = \frac{33}{-11}$

=> $y = -3$

Question 16: $\frac{8x-3}{3x} = 2$

Solution: By cross multiplication, we get

=> $8x - 3 = 2(3x)$

=> $8x - 3 = 6x$

=> $8x - 6x = 3$

=> $2x = 3$

=> $x = \frac{3}{2}$

Question 17: $\frac{9x}{7-6x} = 15$

Solution: By cross multiplication, we get

=> $9x = 15(7 - 6x)$

=> $9x = 105 - 90x$

=> $9x + 90x = 105$

=> $99x = 105$

=> $x = \frac{105}{99}$

=> $x = \frac{35}{33}$

Question 18: $\frac{3x}{5x+2} = -4$

Solution: By cross multiplication, we get

=> $3x = -4(5x + 2)$

=> $3x = -20x - 8$

=> $3x + 20x = -8$

=> $23x = -8$

=> $x = -\frac{8}{23}$

Question 19: $\frac{6y-5}{2y} = \frac{7}{9}$

Solution: By cross multiplication, we get

=> $9(6y - 5) = 7(2y)$

=> $54y - 45 = 14y$

=> $54y - 14y = 45$

=> $40y = 45$

=> $y = \frac{45}{40}$

=> $y = \frac{9}{8}$

Question 20: $\frac{2-9z}{17-4z} = \frac{4}{5}$

Solution: By cross multiplication, we get

=> $5(2 - 9z) = 4(17 - 4z)$

=> $10 - 45z = 68 - 16z$

=> $-45z + 16z = 68 - 10$

=> $-29z = 58$

=> $z = \frac{58}{-29}$

=> $z = -2$

Question 21: $\frac{4x+7}{9-3x} = \frac{1}{4}$

Solution: By cross multiplication, we get

=> $4(4x + 7) = 9 - 3x$

=> $16x + 28 = 9 - 3x$

=> $16x + 3x = 9 - 28$

=> $19x = -19$

=> $x = -1$

Question 22: $\frac{7y+4}{y+2} = -\frac{4}{3}$

Solution: By cross multiplication, we get

=> $3(7y + 4) = -4(y + 2)$

=> $21y + 12 = -4y - 8$

=> $21y + 4y = -8 - 12$

=> $25y = -20$

=> $y = \frac{-20}{25}$

=> $y = -\frac{4}{5}$

Question 23: $\frac{15(2-y)-5(y+6)}{1-3y} = 10$

Solution: By cross multiplication, we get

=> $15(2 - y) - 5(y + 6) = 10(1 - 3y)$

=> $30 - 15y - 5y - 30 = 10 - 30y$

=> $-15y - 5y + 30y = 10$

=> $10y = 10$

=> $y = \frac{10}{10}$

=> $y = 1$

Question 24: $\frac{2x-(7-5x)}{9x-(3+4x)} = \frac{7}{6}$

Solution: By cross multiplication, we get

=> $6(2x - (7 - 5x)) = 7(9x - (3 + 4x)$

=> $6(2x - 7 + 5x) = 7(9x - 3 - 4x)$

=> $6(7x - 7) = 7(5x - 3)$

=> $42x - 42 = 35x - 21$

=> $42x - 35x = -21 + 42$

=> $7x = 21$

=> $x = \frac{21}{7}$

=> $x = 3$

Question 25: $\boldsymbol{m - \frac{(m-1)}{2} = 1 - \frac{(m-2)}{3}}$

Solution**:** Multiply both sides of equation by LCM of (2, 3) which is 6

=> $6m - 3(m - 1) = 6 - 2(m - 2)$

=> $6m - 3m + 3 = 6 - 2m + 4$

=> $6m - 3m + 2m = 6 + 4 - 3$

=> $5m = 7$

=> $m = \frac{7}{5}$

Question 26: $\boldsymbol{\frac{3x+5}{4x+2} = \frac{3x+4}{4x+7}}$

Solution: By cross multiplication, we get

=> $(3x + 5)(4x + 7) = (3x + 4)(4x + 2)$

=> $3x(4x + 7) + 5(4x + 7) = 3x(4x + 2) + 4(4x + 2)$

=> $12x^2 + 21x + 20x + 35 = 12x^2 + 6x + 16x + 8$

=> $12x^2 - 12x^2 + 21x + 20x - 6x - 16x = 8 - 35$

=> $19x = -27$

=> $x = -\frac{27}{19}$

Question 27: $\boldsymbol{\frac{9x-7}{3x+5} = \frac{3x-4}{x+6}}$

Solution: By cross multiplication, we get

=> $(9x-7)(x+6)=(3x-4)(3x+5)$

=> $9x(x+6)-7(x+6)=3x(3x+5)-4(3x+5)$

=> $9x^2+54x-7x-42=9x^2+15x-12x-20$

=> $9x^2-9x^2+54x-7x-15x+12x=-20+42$

=> $44x=22$

=> $x=\frac{22}{44}$

=> $x=\frac{1}{2}$

Question 28: $\frac{2-7x}{1-5x}=\frac{3+7x}{4+5x}$

Solution: By cross multiplication, we get

=> $(2-7x)(4+5x)=(3+7x)(1-5x)$

=> $2(4+5x)-7x(4+5x)=3(1-5x)+7x(1-5x)$

=> $8+10x-28x-35x^2=3-15x+7x-35x^2$

=> $-35x^2+35x^2-28x+10x+15x-7x=3-8$

=> $-10x=-5$

=> $x=\frac{-5}{-10}$

=> $x=\frac{1}{2}$

Applications of Linear Equations

Example 1: Two numbers are in the ratio 5:8. If the sum of the numbers is 182, find the numbers.

Solution: Let one number be 5x and other number be 8x

According to the given question,

$5x + 8x = 182$

=> $13x = 182$

=> $x = \frac{182}{13}$

=> $x = 14$

Therefore, numbers are 5 (14) = 80 and 8 (14) = 112

Example 2: The sum of the digits of a two digit number is 15. If the number formed by reversing the digits is less than the original number by 27, find the original number. Check your solution.

Solution: Let the unit digit be y and ten digit be x

Then, two digit number = 10x + y

It is given that, $x + y = 15$

=> $y = 15 - x$ $\longrightarrow$ 1

Number formed by reversing the digits = 10y + x

Now, according to given question,

$10x + y = 10y + x + 27$

=> $10x - x + y - 10y = 27$

=> $9x - 9y = 27$

=> $9(x - y) = 27$

=> $x - y = \frac{27}{9} = 3$

=> $x - y = 3 \longrightarrow 2$

Substituting 1 in 2, we get

$x - (15 - x) = 3$

=> $x - 15 + x = 3$

=> $2x = 3 + 15$

=> $x = \frac{18}{2}$

=> $x = 9$

Thus, y = $15 - 9 = 15 - 9 = 6$

Therefore, number = 10x + y = $10(9) + 6 = 90 + 6 = 96$

Check: We will check whether all the given conditions are satisfied or not

1) Sum of digits = x + y = 6 + 9 = 15 which is true

2) Number formed by reversing the digits = 69

Now, original number – new number = 96 – 69 = 27, which is also true.

Thus, original number = 96, is correct.

Example 3: The denominator of a rational number is greater than its numerator by 3. If 3 is subtracted from the numerator and 2 is added to its denominator, the new number becomes $\frac{1}{5}$. Find the original number. Check your solution.

Solution: Let the numerator of a rational number be 'x'.

Given that, denominator = $x + 3$

Then, ratio is $\frac{x}{x+3} \longrightarrow 1$

According to the given question, we have

$$\frac{x-3}{x+3+2} = \frac{1}{5}$$

=> $\frac{x-3}{x+5} = \frac{1}{5}$

=> $5(x-3) = x+5$

=> $5x - 15 = x + 5$

=> $5x - x = 5 + 15$

=> $4x = 20$

=> $x = \frac{20}{4}$

=> $x = 5$

Thus, numerator = 5

Denominator = $5 + 3 = 8$

Therefore, original fraction = $\frac{5}{8}$

Check: We will check whether all the given conditions are satisfied or not

1) Denominator – Numerator = 8-5 = 3, which is true

2)$\frac{x-3}{x+5} = \frac{5-3}{5+5} = \frac{2}{10} = \frac{1}{5}$, which is also true

Thus, $\frac{5}{8}$ satisfies all given conditions.

Example 4: The length of a rectangle exceeds its breadth by 9 cm. If the length and breadth are each increased by 3 cm, the area of the new rectangle will be 84 cm^2 more than that of the given rectangle. Find the length and breadth of the given rectangle. Check your solution.

Solution: Let the breadth of rectangle be 'x' cm

Then, Length of rectangle = (x + 9) cm

Area = Length × breadth

Area = $x(x+9)$

According to the given question,

New length = $x + 9 + 3 = (x + 12)$ cm

New breadth = $(x + 3)$ cm

New area = $(x + 12)(x + 3)$ $\longrightarrow$ 1

New area is given to be $(x(x + 9) + 84)cm^2$ $\longrightarrow$ 2

Equating 1 and 2, we get

$(x + 12)(x + 3) = (x(x + 9) + 84)$

=> $x(x + 3) + 12(x + 3) = x^2 + 9x + 84$

=> $x^2 + 3x + 12x + 36 = x^2 + 9x + 84$

=> $15x - 9x = 84 - 36$

=> $6x = 48$

=> $x = \frac{48}{6}$

=> $x = 8$ cm

Thus, breadth of rectangle = 8 cm

Length of rectangle = 8 + 9 = 17 cm

Check: We will check whether all the given conditions are satisfied or not

We have l =17 cm and b = 8 cm

Area of rectangle = $17 \times 8 = 136\ cm^2$

Area of new rectangle = $(8 + 12)(8 + 3) = 20 \times 11 = 220cm^2$

Difference =$220 - 136 = 84cm^2$, which is true.

Therefore, Length = 17 cm and breadth =8 cm are correct dimensions.

Example 5: A steamer goes downstream and covers the distance between two ports in 4 hours while it covers the same distance upstream in 5 hours. If the speed of the stream is 2 km/h, find the speed of the streamer in still water. Check your solution.

Solution: Let the speed of streamer in still water be x km/h.

Given the speed of stream = 2 km/h.

Then, upstream speed = $(x - 2)$ km/h

Downstream speed = $(x + 2)$ km/h

Distance covered while going downstream in 4 hours = $4(x + 2)$ km

Distance covered while going upstream in 5 hours = $5(x - 2)$

Now, it is given that distance remains the same in both the cases.

=> $4(x + 2) = 5(x - 2)$

=> $4x + 8 = 5x - 10$

=> $4x - 5x = -10 - 8$

=> $-x = -18$

=> $x = 18$ km/h

Therefore, speed of streamer in still water = 18 km/h

Check: We will check whether all the given conditions are satisfied or not

We have x = 18 km/h

Then, upstream speed = $18 - 2 = 16$ km/h

Downstream speed = $18 + 2 = 20$ km/h

Distance while going upstream = $5(18 - 2) = 5(16) = 80$ km

Distance while going downstream = $4(18 + 2) = 4(20) = 80$ km

Both distances are equal.

Hence, speed of streamer = 18 km/h is correct.

Example 6: The distance between two stations is 425 km. Two trains start simultaneously from these stations on parallel tracks to cross each other. The speed of one of them is greater than that of the other by 5 km/h. If the distance between the two trains after 3 hours of their start is 20 km. find the speed of each train. Check your solution.

Solution: Let the speed of one train be 'x' km/h

It is given that speed of one train is greater than other by 5 km/h

Thus, speed of second train = $(x + 5)$ km/h

Given that distance between two stations = 425 km

Distance covered by first train after 3 hours = $3x$ km

Distance covered by second train after 3 hours = $3(x + 5) = (3x + 15)$ km

Distance between them = 20 km

Distance covered by both trains = 425 – 20 = 405 km

According to given question,

$3x + 3x + 15 = 405$

=> $6x + 15 = 405$

=> $6x = 405 - 15$

=> $6x = 390$

=> $x = \frac{390}{6} = 65$

Thus, speed of first train = 65 km/h

Speed of second train = 65 + 5 =70 km/h

Check: We will check whether all the given conditions are satisfied or not

We have speed of first train = 65 km/h

Speed of second train = 70 km/h

Distance covered by first train = 3(65) = 195 km

Distance covered by second train = 3(70) = 210 km

Distance covered by both = 210+195 = 405 km, which is true

Hence, both speeds are correct.

Example 7: Two years ago, dilip was three times as old as his son and two years hence, twice his age will be equal to five times that of his son. Find their present ages. Check your solution.

Solution: Let the age of dilip's son two years ago be x years

Then, Age of dilip = 3x years

Thus, the present age of dilip's son = $x + 2$ years

The present age of dilip = $3x + 2$ years

Two years hence,

Age of dilip = $3x + 4$

Age of his son = $x + 4$

According to given question, we have

$2(3x + 4) = 5(x + 4)$

=> $6x + 8 = 5x + 20$

=> $6x - 5x = 20 - 8$

=> $x = 12$

Thus, present age of dilip = $3(12) + 2 = 36 + 2 = 38$ years

Present age of his son = $12 + 2 = 14$ years.

Check: We will check whether all the given conditions are satisfied or not

We have, present age of dilip $= 38$ years

Present age of his son $= 14$ years.

1) Son's age two years ago = $14 - 2 = 12$ yrs.

Dilip's age two years ago = $38 - 2 = 36$ yrs. = 3 times the son's age which is true

2) Son's age two years hence = $14 + 2 = 16$ yrs.

Dilip's age two years hence = $38 + 2 = 40$ yrs.

Now, twice the age of dilip => 2(40) = 80 years

Five times age of son = 5(16) = 80 years

Hence, all the conditions are satisfied with age of son = 14 years and age of dilip = 38 years.

Exercise 8B

Question 1: Two numbers are in the ratio 8: 3. If the sum of the numbers is 143, find the numbers.

Solution: Let one number be 8x and other number be 3x

According to the given question,

$8x + 3x = 143$

=> $11x = 143$

=> $x = \frac{143}{11}$

=> $x = 13$

Therefore, numbers are 8 (13) = 104 and 3 (13) = 39

Question 2: $\frac{2}{3}$ of a number is 20 less than the original number. Find the number.

Solution: Let the required number be 'x'

According to the given question, we have

$\frac{2}{3}x + 20 = x$

=> $20 = x - \frac{2x}{3}$

=> $20 = \frac{3x-2x}{3}$

=> $60 = x$

Therefore, required number = 60

Question 3: Four-fifths of a number is 10 more than two-thirds of the number. Find the number.

Solution: Let the required number be 'x'

According to the given question, we have

$\frac{4x}{5} - 10 = \frac{2x}{3}$

Multiply both sides of equation by LCM of (5, 3) which is 15

$3(4x) - 15(10) = 5(2x)$

=> $12x - 150 = 10x$

=> $12x - 10x = 150$

=> $2x = 150$

=> $x = \frac{150}{2}$

=> $x = 75$

Therefore, required number = 75

Question 4: Twenty-four is divided into two parts such that 7 times the first part added to 5 times the second part makes 146. Find each part.

Solution: Let one part be 'x'

Then, other part = $24 - x$

According to the given question, we have

$7x + 5(24 - x) = 146$

=> $7x + 120 - 5x = 146$

=> $2x = 146 - 120 = 26$

=> $x = \frac{26}{2} = 13$

Therefore, one part = 13

Other part = $24 - 13 = 11$

Question 5: Find the number whose fifth part increased by 5 is equal to its fourth part diminished by 5.

Solution: Let the required number be 'x'

Fifth part of number = $\frac{x}{5}$

Fourth part of number = $\frac{x}{4}$

According to given question, we have

$\frac{x}{5} + 5 = \frac{x}{4} - 5$

=> $5 + 5 = \frac{x}{4} - \frac{x}{5}$

=> $10 = \frac{5x-4x}{20}$

=> $10(20) = x$

=> $x = 200$

Therefore, required number = 200

Question 6: Three numbers are in the ratio of 4: 5: 6. If the sum of the largest and the smallest equals the sum of the third and 55, find the numbers.

Solution: Let the three numbers be '4x', '5x' and '6x'

According to given question, we have

$6x + 4x = 5x + 55$

=> $10x - 5x = 55$

=> $5x = 55$

=> $x = \frac{55}{5}$

=> $x = 11$

Therefore, numbers are 4(11) = 44, 5(11) = 55 and 6(11) = 66

Question 7: If 10 be added to four times a certain number, the result is 5 less than five times the number. Find the number.

Solution: Let the required number be 'x'

According to given question, we have

$10 + 4x = 5x - 5$

=> $10 + 5 = 5x - 4x$

=> $15 = x$

Therefore, required number = 15

Question 8: Two numbers are such that the ratio between them is 3:5. If each is increased by 10, the ratio between the new numbers so formed is 5:7. Find the original numbers.

Solution: Let the two numbers be '3x' and '5x'

Ratio = $\frac{3x}{5x}$

Now, according to given question, we have

$\frac{3x + 10}{5x + 10} = \frac{5}{7}$

=> $7(3x + 10) = 5(5x + 10)$

=> $21x + 70 = 25x + 50$

=> $70 - 50 = 25x - 21x$

=> $20 = 4x$

=> $x = \frac{20}{4}$

=> $x = 5$

Therefore, original numbers are 3(5) =15 and 5(5) = 25

Question 9: Find three consecutive odd numbers whose sum is 147

Solution: Let three consecutive odd numbers be (2x+1), (2x+3) and (2x+5) respectively

According to the given question,

$2x + 1 + 2x + 3 + 2x + 5 = 147$

=> $6x + 9 = 147$

=> $6x = 147 - 9$

=> $6x = 138$

=> $x = \frac{138}{6}$

=> $x = 23$

Therefore, numbers are: $2x + 1 = 2(23) + 1 = 46 + 1 = 47$

$2x + 3 = 2(23) + 3 = 46 + 3 = 49$

$2x + 5 = 2(23) + 5 = 46 + 5 = 51$

Question 10: Find three consecutive even numbers whose sum is 234.

Solution: Let three consecutive odd numbers be 2x, 2x+2 and 2x+4 respectively

According to the given question,

$2x + 2x + 2 + 2x + 4 = 234$

=> $6x + 6 = 234$

=> $6x = 234 - 6$

=> $6x = 228$

=> $x = \frac{228}{6}$

=> $x = 38$

Therefore, numbers are: $2x = 2(38) + 1 = 76$

$2x + 2 = 2(38) + 2 = 76 + 2 = 78$

$2x + 4 = 2(38) + 4 = 76 + 4 = 80$

Question 11: The sum of the digits of a two-digit number is 12. If the new number formed by reversing the digits is greater than the original number by 54, find the original number. Check your solution.

Solution: Let the unit digit be y and ten digit be x

Then, two digit number = 10x + y

It is given that, $x + y = 12$

=> $y = 12 - x$ $\longrightarrow$ 1

Number formed by reversing the digits = 10y + x

Now, according to given question,

$10y + x = 10x + y + 54$

=> $10y - y + x - 10x = 54$

=> $9y - 9x = 54$

=> $9(y - x) = 54$

=> $y - x = \frac{54}{9} = 6$

=> $y - x = 6$ $\longrightarrow$ 2

Substituting 1 in 2, we get

$12 - x - x = 6$

=> $12 - 2x = 6$

=> $12 - 6 = 2x$

=> $6 = 2x$

=> $x = 3$

Thus, y = $12 - 3 = 9$

Therefore, number = 10x + y = $10(3) + 9 = 30 + 9 = 39$

Check: We will check whether all the given conditions are satisfied or not

1) Sum of digits = x + y = 3 + 9 = 12 which is true

2) Number formed by reversing the digits = 93

Now, new number – original number = 93 – 36 = 54, which is also true.

Thus, original number = 39 is correct.

Question 12 – The digit in the tens place of a two-digit number is three times that in the units place. If the digits are reversed, the new number will be 36 less than the original number. Find the original number. Check your solution.

Solution: Let the unit digit be 'x'

Then, tens digit = 3x

Then, two digit number = 10(3x) + x

= 30x + x = 31 x

Number formed by reversing the digits = 10x + 3x = 13x

According to given question, we have

$13x + 36 = 31x$

=> $31x - 13x = 36$

=> $18x = 36$

=> $x = \frac{36}{18} = 2$

Thus, unit digit = 2

Tens digit = 3(2) = 6

Number = 31x = 31(2) = 62

Check: We will check whether all the given conditions are satisfied or not

1) Unit digit = 2, tens digit = 6 = 3 times the unit digit, which is true

2) Number formed by reversing the digits = 26

Now, original number – new number = 62 – 26 = 36, which is also true.

Thus, original number = 62 is correct.

Question 13: The denominator of a rational number is greater than its numerator by 7. If the numerator is increased by 17 and the denominator is decreased by 6, the new number becomes 2. Find the original number.

Solution: Let the numerator of a rational number be 'x'.

Given that, denominator = $x + 7$

Then, ratio is $\frac{x}{x+7}$ $\longrightarrow$ 1

According to the given question, we have

$$\frac{x+17}{x+7-6} = 2$$

=> $\frac{x+17}{x+1} = 2$

=> $x + 17 = 2(x + 1)$

=> $x + 17 = 2x + 2$

=> $17 - 2 = 2x - x$

=> $15 = x$

Thus, numerator = 15

Denominator = $15 + 7 = 22$

Therefore, original fraction = $\frac{15}{22}$

Check: We will check whether all the given conditions are satisfied or not

1) Denominator – Numerator = 22-15 = 7, which is true

2)$\frac{x+17}{x+1} = \frac{15+17}{15+1} = \frac{32}{16} = 2$, which is also true

Thus, $\frac{15}{22}$ satisfies all given conditions.

Question 14: In a fraction, twice the numerator is 2 more than the denominator. If 3 is added to the numerator and to the denominator, the new fraction is $\frac{2}{3}$. Find the original fraction.

Solution: Let the numerator and denominator in a fraction be x and y respectively

Then, according to the given question

$2x = y + 2$

$y = 2x - 2 \longrightarrow 1$

Also, $\frac{x+3}{y+3} = \frac{2}{3}$

=> $3(x + 3) = 2(y + 3)$

=> $3x + 9 = 2y + 6$

=> $3x - 2y = 6 - 9 = -3$

=> $3x - 2y = -3 \longrightarrow 2$

Substitute 1 in 2, we get

$3x - 2(2x - 2) = -3$

=> $3x - 4x + 4 = -3$

=> $-x + 4 + 3 = 0$

=> $-x + 7 = 0$

=> $x = 7$

Thus, $y = 2(7) - 2$

=> $y = 14 - 2$

=> $y = 12$

Therefore, original fraction = $\frac{7}{12}$

Question 15: The length of a rectangle exceeds its breadth by 7 cm. If the length is decreased by 4 cm and the breadth is increased by 3 cm, the area of the new rectangle is the

same as the area of the original rectangle. Find the length and the breadth of the original rectangle.

Solution: Let the breadth of rectangle be 'x' cm

Then, Length of rectangle = (x + 7) cm

Area = Length × breadth

Area = $x(x + 7)$

According to the given question,

New length = $x + 7 - 4 = (x + 3)$ cm

New breadth = $(x + 3)$ cm

New area = $(x + 3)(x + 3)$ $\longrightarrow$ 1

New area is given to be same as the original rectangle area which is $x(x + 7)$ $\longrightarrow$ 2

Equating 1 and 2, we get

$(x + 3)(x + 3) = x(x + 7)$

=> $x(x + 3) + 3(x + 3) = x^2 + 7x$

=> $x^2 + 3x + 3x + 9 = x^2 + 7x$

=> $7x - 6x = 9$

=> $x = 9$ cm

Thus, breadth of rectangle = 9 cm

Length of rectangle = 9 + 7 = 16 cm

Question 16: The width of a rectangle is two-thirds the length. If the perimeter is 180 meters, find the dimensions of the rectangle.

Solution: Let the length (l) of rectangle be 'x' m

Then, width (b) of rectangle = $\frac{2x}{3}$ m

Perimeter is given to be 180 m

Perimeter of rectangle = $2(l + b)$

=> $2(l + b) = 180$

=> $2\left(x + \frac{2x}{3}\right) = 180$

=> $x + \frac{2x}{3} = 90$

=> $\frac{3x+2x}{3} = 90$

=> $\frac{5x}{3} = 90$

=> $5x = 270$

=> $x = \frac{270}{5} = 54\ m$

Thus, length = 54 m

And width = $\frac{2}{3} \times 54 = 36\ m$

Question 17: An altitude of a triangle is five-thirds the length of its corresponding base. If the altitude be increased by 4 cm and the base decreased by 2 cm, the area of the triangle remains the same. Find the base and the altitude of the triangle.

Solution: Let the length of base of the triangle be 'x' cm

Then, altitude of triangle = $\frac{5x}{3}$cm

We know that, area of triangle = $\frac{1}{2} \times base \times altitude$

=> Area = $\frac{1}{2} \times x \times \frac{5x}{3}$ $\longrightarrow$ 1

According to the given question, we have

New altitude = $\frac{5x}{3} + 4$

New base = $x - 2$

Then, new area = $\frac{1}{2}(\frac{5x}{3} + 4)(x - 2)$ $\longrightarrow$ 2

Also, new area is given to be same as area of original triangle which is = $\frac{1}{2} \times x \times \frac{5x}{3}$ $\longrightarrow$ 3

Now, equating 2 and 3, we get

$\frac{1}{2}(\frac{5x}{3}+4)(x-2) = \frac{1}{2} \times x \times \frac{5x}{3}$

=> $\frac{5x}{3}(x-2)+4(x-2) = \frac{5x^2}{3}$

=> $\frac{5x^2}{3} - \frac{10x}{3} + 4x - 8 = \frac{5x^2}{3}$

=> $\frac{-10x+12x}{3} = 8$

=> $\frac{2x}{3} = 8$

=> $2x = 24$

=> $x = \frac{24}{2} = 12$

Thus, base = 12 cm

Altitude = $\frac{5(12)}{3} = 20\ cm$

Question 18: Two angles of a triangle are in the ratio 4:5. If the sum of these angles is equal to the third angle, find the angles of the triangle.

Solution: Let the two angles of a triangle be '4x' and '5x' respectively and the third angle be 'y'

According to the given question, we have

$4x + 5x = y$

=> $9x = y \longrightarrow 1$

We know that, the sum of all angles of a triangle is 180^o

=> $4x + 5x + y = 180^0$

=> $9x + y = 180^0$

=> $9x + 9x = 180^0$ (using 1)

=> $18x = 180^0$

=> $x = \frac{180}{18} = 10^0$

Thus, three angles of triangle are 4(10) = 40^o, 5(10) = 50^o and 9(10) = 90^o

Question 19: A streamer goes downstream from one port to another in 9 hours. It covers the same distance upstream in 10 hours. If the speed of the stream be 1 km/h, find the speed of the streamer in still water and the distance between the ports.

Solution: Let the speed of streamer in still water be x km/h.

Given the speed of stream = 1 km/h.

Then, upstream speed = $(x - 1)$ km/h

Downstream speed = $(x + 1)$ km/h

Distance covered while going downstream in 9 hours = $9(x + 1)$ km

Distance covered while going upstream in 10 hours = $10(x - 1)$

Now, it is given that distance remains the same in both the cases.

=> $9(x + 1) = 10(x - 1)$

=> $9x + 9 = 10x - 10$

=> $9x - 10x = -10 - 9$

=> $-x = -19$

=> $x = 19$ km/h

Therefore, speed of streamer in still water = 19 km/h

Distance between the ports = $9(x + 1)$

= $9(19 + 1) = 9(20)$

= 180 km

Question 20: The distance between two stations is 300 km. Two motorcyclists start simultaneously from these stations and move towards each other. The speed of one of them is 7 km/h more than that of the other. If the distance between them after 2 hours of their start is 34 km, find the speed of each motorcyclist. Check your solution.

Solution: Let the speed of one motorcyclist be 'x' km/h

It is given that speed of one motorcyclist is greater than other by 7 km/h

Thus, speed of second motorcyclist = $(x + 7)$ km/h

Given that distance between two stations = 300 km

Distance covered by first motorcyclist after 2 hours = $2x$ km

Distance covered by second motorcyclist after 2 hours = $2(x + 7) = (2x + 14)$ km

Distance between them = 34 km

Distance covered by both trains = 300 – 34 = 266 km

According to given question,

$2x + 2x + 14 = 266$

=> $4x + 14 = 266$

=> $4x = 266 - 14$

=> $4x = 252$

=> $x = \frac{252}{4} = 63$

Thus, speed of motorcyclist = 63 km/h

Speed of second motorcyclist = 63 + 7 =70 km/h

Check: We will check whether all the given conditions are satisfied or not

We have speed of first motorcyclist = 63 km/h

Speed of second motorcyclist = 70 km/h

Distance covered by first motorcyclist = 2(63) = 126 km

Distance covered by second motorcyclist = 2(70) = 140 km

Distance covered by both = 126+140 = 266 km, which is true

Hence, both speeds are correct.

Question 21: Divide 150 into three parts such that the second number is five-sixths the first and the third number is four-fifths the second.

Solution: Let the first number be 'x'

Then, second number = $\frac{5x}{6}$

Third number = $\frac{4}{5}\left(\frac{5x}{6}\right) = \frac{2x}{3}$

According to the given question, we have

$$x + \frac{5x}{6} + \frac{2x}{3} = 150$$

$$\Rightarrow \frac{6x+5x+4x}{6} = 150$$

$$\Rightarrow \frac{15x}{6} = 150$$

$$\Rightarrow x = 60$$

Thus, the numbers are x = 60

$$\frac{5x}{6} = \frac{5}{6}(60) = 50$$

$$\frac{2x}{3} = \frac{2}{3}(60) = 40$$

Question 22: Divide 4500 into two parts such that 5% of the first part is equal to 10% of the second part.

Solution: Let first part be 'x'

Then, second part = (4500 – x)

According to given question, we have

$$5\%\ of\ x = 10\%\ of\ (4500 - x)$$

$$\Rightarrow \frac{5x}{100} = \frac{10}{100}(4500 - x)$$

$$\Rightarrow 5x = 10(4500 - x)$$

$$\Rightarrow 5x = 45000 - 10x$$

=> $5x + 10x = 45000$

=> $15x = 45000$

=> $x = \frac{45000}{15}$

=> $x = 3000$

Thus, numbers are x = 3000

$(4500 - x) = (4500 - 3000) = 1500$

Question 23: Rakhi's mother is four times as old as rakhi. After 5 years, her mother will be three times as old as she will be then. Find their present ages.

Solution: Let the present age of rakhi be 'x' years

Then, age of her mother = '4x' years

After 5 years,

Age of rakhi = $(x + 5)$ years

Age of her mother = $(4x + 5)$ years

According to the given question, we have

$4x + 5 = 3(x + 5)$

=> $4x + 5 = 3x + 15$

=> $4x - 3x = 15 - 5$

=> $x = 10$

Thus, present age of rakhi = 10 years

Present age of her mother = 4(10) = 40 years

Question 24: Monu's Father is 26 years younger than Monu's grandfather and 29 years older than Monu. The sum of the ages of all the three is 135 years. What is the age of each one of them?

Solution: Let the present age of father of monu be 'x' years

Then, age of monu = $(x - 29)$ years

Age of grandfather of monu = $(x + 26)$ years

According to the given question, we have

$x + x + 26 + x - 29 = 135$

=> $3x - 3 = 135$

=> $3x = 135 + 3 = 138$

=> $x = \frac{138}{3} = 46$

Thus, age of father = 46 years

Age of monu = $x - 29 = 46 - 29 = 17$ years

Age of grandfather = $x + 26 = 46 + 26 = 72$ years

Question 25: A man is 10 times older than his grandson. He is also 54 years older than him. Find their present ages.

Solution: Let the present age of grandson be 'x' years and present age of man be 'y' years

According to given question, we have

$y = 10x$ and $y = x + 54$

Equating both equations, we get

$10x = x + 54$

=> $10x - x = 54$

=> $9x = 54$

=> $x = \frac{54}{9} = 6$

Thus, age of grandson = 6 years

Age of man = 10(6) = 60 years

Question 26: The difference between the ages of two cousins is 10 years.15 years ago, if the elder one was twice as old as the younger one, find their present ages.

Solution: Let the present age of younger cousin be 'x' years and the present age of elder cousin be 'y' years

According to given question, we have

$y - x = 10$

=> $y = 10 + x \longrightarrow 1$

15 years ago,

Age of younger cousin = $x - 15$

Age of elder cousin = $y - 15$

Given that, $y - 15 = 2(x - 15)$

=> $10 + x - 15 = 2x - 30$ (using 1)

=> $x - 5 = 2x - 30$

=> $-5 + 30 = 2x - x$

=> $25 = x$

Thus, present age of younger cousin = 25 years

Present age of elder cousin = $10 + 25 = 35$ years

Question 27: Half of a herd of deer are grazing in the field and three-fourths of the remaining is playing nearby. The rest 9 are drinking water from the pond. Find the number of deer in the herd

Solution: Let the total number of deer in the herd be 'x'

According to the given question, we have

Number of deer who are grazing = $\frac{x}{2}$

Remaining deer = $x - \frac{x}{2} = \frac{2x-x}{2} = \frac{x}{2}$

Number of deer playing nearby = $\frac{3}{4}\left(\frac{x}{2}\right) = \frac{3x}{8}$

Number of deer drinking water from pond = 9

Now, Total number of deer = $\frac{x}{2} + \frac{3x}{8} + 9$

=> $x = \frac{x}{2} + \frac{3x}{8} + 9$

=> $x - \frac{x}{2} - \frac{3x}{8} = 9$

=> $\frac{8x-4x-3x}{8} = 9$

=> $\frac{x}{8} = 9$

=> $x = 72$

Thus, total number of deer = 72

Exercise 8C

Question 1: If $2x - 3 = x + 2$, then x =?

Solution: $2x - 3 = x + 2$

=> $2x - x = 2 + 3$

=> $x = 5$

Question 2: If $5x + \frac{7}{2} = \frac{3x}{2} - 14$, then x =?

Solution: $5x + \frac{7}{2} = \frac{3x}{2} - 14$

=> $5x - \frac{3x}{2} = -14 - \frac{7}{2}$

=> $\frac{10x-3x}{2} = \frac{-28-7}{2}$

=> $\frac{7x}{2} = -\frac{35}{2}$

=> $7x = -35$

=> $x = \frac{-35}{7} = -5$

Question 3: If $z = \frac{4}{5}(z + 10)$, then z=?

Solution: $z = \frac{4}{5}(z + 10)$

=> $5z = 4(z + 10)$

=> $5z = 4z + 40$

=> $5z - 4z = 40$

=> $z = 40$

Question 4: If $3m = 5m - \frac{8}{5}$, then m =?

Solution: $3m = 5m - \frac{8}{5}$

=> $5m - 3m = \frac{8}{5}$

=> $2m = \frac{8}{5}$

=> $m = \frac{8}{10}$

=> $m = \frac{4}{5}$

Question 5: If $5t - 3 = 3t - 5$, then t =?

Solution: $5t - 3 = 3t - 5$

=> $5t - 3t = -5 + 3$

=> $2t = -2$

=> $t = -1$

Question 6: If $2y + \frac{5}{3} = \frac{26}{3} - y$, then y =?

Solution: $2y + y = \frac{26}{3} - \frac{5}{3}$

=> $3y = \frac{26-5}{3} = \frac{21}{3} = 7$

=> $y = \frac{7}{3}$

Question 7: If $\frac{6x+1}{3} + 1 = \frac{x-3}{6}$, then x =?

Solution: $\frac{6x+1}{3} + 1 = \frac{x-3}{6}$

Multiply both sides of equation by LCM of (3,6) which is 6

$2(6x + 1) + 6 = (x - 3)$

=> $12x + 2 + 6 = x - 3$

=> $12x - x = -3 - 2 - 6$

=> $11x = -11$

=> $x = -1$

Question 8: If $\frac{n}{2} - \frac{3n}{4} + \frac{5n}{6} = 21$, then n =?

Solution: $\frac{n}{2} - \frac{3n}{4} + \frac{5n}{6} = 21$

Multiply both sides of equation by LCM of (2, 4, 6) which is 12

$6n - 3(3n) + 2(5n) = 252$

=> $6n - 9n + 10n = 252$

=> $7n = 252$

=> $n = \frac{252}{7} = 36$

Question 9: If $\frac{x+1}{2x+3} = \frac{3}{8}$, then x =?

Solution: By cross multiplication, we get

$8(x + 1) = 3(2x + 3)$

=> $8x + 8 = 6x + 9$

=> $8x - 6x = 9 - 8$

=> $2x = 1$

=> $x = \frac{1}{2}$

Question 10: If $\frac{4x+8}{5x+8} = \frac{5}{6}$, then x =?

Solution: By cross multiplication, we get

$6(4x + 8) = 5(5x + 8)$

=> $24x + 48 = 25x + 40$

=> $24x - 25x = 40 - 48$

=> $-x = -8$

=> $x = 8$

Question 11: If $\frac{n}{n+15} = \frac{4}{9}$, then n =?

Solution: By cross multiplication, we have

$9n = 4(n + 15)$

=> $9n = 4n + 60$

=> $9n - 4n = 60$

=> $5n = 60$

=> $n = \frac{60}{5} = 12$

Question 12: If $3(t - 3) = 5(2t + 1)$, then t =?

Solution: $3t - 9 = 10t + 5$

=> $3t - 10t = 5 + 9$

=> $-7t = 14$

=> $t = \frac{14}{-7}$

=> $t = -2$

Question 13: Four-fifths of a number is greater than three-fourths of the number by 4. The number is?

Solution: Let the required number be 'x'

According to the given question, we have

$\frac{4x}{5} = \frac{3x}{4} + 4$

=> $\frac{4x}{5} - \frac{3x}{4} = 4$

=> $\frac{16x-15x}{20} = 4$

=> $\frac{x}{20} = 4$

=> $x = 80$

Question 14: The ages of A and B are in the ration 5:7. Four years from now the ratio of their ages will be 3:4. The present age of B is?

Solution: Let the ages of A and B be '5x' and '7x' respectively

After 4 years,

Age of A = 5x + 4

Age of B = 7x + 4

Ratio = $\frac{5x+4}{7x+4} = \frac{3}{4}$

By cross multiplication, we get

$4(5x + 4) = 3(7x + 4)$

=> $20x + 16 = 21x + 12$

=> $20x - 21x = 12 - 16$

=> $-x = -4$

=> $x = 4$

Thus, age of A = 5 (4) =20 years

Age of B = 7 (4) = 28 years

Question 15: The base of an isosceles triangle is 6 cm and its perimeter is 16 cm. Length of each of the equal sides is?

Solution: Let the length of equal sides of isosceles triangle be 'x' cm

Given that, Base of triangle = 6 cm

Perimeter of triangle = 16 cm

=> $x + x + 6 = 16$

=> $2x + 6 = 16$

=> $2x = 16 - 6 = 10$

=> $x = \frac{10}{2} = 5$

Thus, length of each equal side is 5 cm

Question 16: Sum of three consecutive integers is 51. The middle one is?

Solution: Let the three consecutive integers be x, x+1 and x+2 respectively

According to given question, we have

$x + x + 1 + x + 2 = 51$

=> $3x + 3 = 51$

=> $3x = 51 - 3 = 48$

=> $x = \frac{48}{3} = 16$

Thus middle one = $x + 1 = 16 + 1 = 17$

Question 17: The sum of two numbers is 95. If one exceeds the other by 15, then the smaller of the two is?

Solution: Let the first number be 'x'

Then, second number = 95 – x

According to the given question, we have

$x = 95 - x + 15$

=> $x + x = 95 + 15$

=> $2x = 110$

=> $x = \frac{110}{2} = 55$

Thus, smaller number = $95 - 55 = 40$

Question 18: Number of boys and girls in a class are in the ratio 7:5. The number of boys is 8 more than the number of girls. The total class strength is?

Solution: Let the number of boys and girls be '7x' and '5x' respectively

According to the given question, we have

$7x = 5x + 8$

=> $7x - 5x = 8$

=> $2x = 8$

=> $x = \frac{8}{2} = 4$

Thus, number of boys = 7(4) = 28

Number of girls = 5(4) = 20

Total strength of class = 28 + 20 = 48 students